

A COMPRESSED DATA MODEL FOR A BITMAPPED XML STRUCTURE

Mohammed Al-Badawi

Department of Computer Science, Sultan Qaboos University, Muscat, Oman

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Abstract: Developments on XML processing usually produce tools to formulate both the XML data storage and the associated query processor. PACD is one of such developments that stores the XML structure into a set of $n \times n$ bitmap matrices each of which encodes a specific XML structure related to an XPath axis. The amount of space and the complexity of storing uncompressed version of these matrices is large for huge XML databases; and such requirements may go beyond the HW/SW capabilities; this justify the need for the data compression model discussed in this paper.

1 INTRODUCTION

PACD, as a new bitmapped XML processing technique, was initially introduced in (Al-Badawi et al, 2009). The technique encodes the entire XML structure into ten bitmap matrices each of which corresponds to a certain XPath (Berglund et al., 2010) axis. This paper discusses the specifications of the data compression model (DCM) used by PACD to reduce the amount of the storage space required for storing the XML structure. The DCM illustrated in Fig. 1, performs three compression processes to the XML structure (labelled by I.2, I.3 and I.4 in Fig. 1) in order to reduce its complexity in terms of storage requirements, representations layout and data manipulations. The first process reduces the number of the matrices encoded from ten to five matrices by using a simple matrix transformation algebra which facilitates *2-in-1* representations for some related matrices. The second compression process provides a method by which two or more matrices can be combined into a single matrix called a 'master' matrix. The process may generate multiple master matrices which should preserve specific characteristics of their composing matrices. The third process is the sparse-matrix compression process which employs one or more sparse-matrix compression techniques to act over the master matrices in order to compact their physical representations. The sparse-matrix compression should allow the compressed data to be managed

efficiently during the course of the query execution and data update transactions.

The rest of this paper is organized as follows. Section 2 briefly restates the specifications of PACD and its data representation in Sections 2.1 and 2.2 respectively. The need for the proposed DCM is justified in Section 2.3 while Section 3 discusses the specifications of the PACD's DCM including the three compression processes that are performed over the encoded XML structure. The paper is concluded in Section 4.

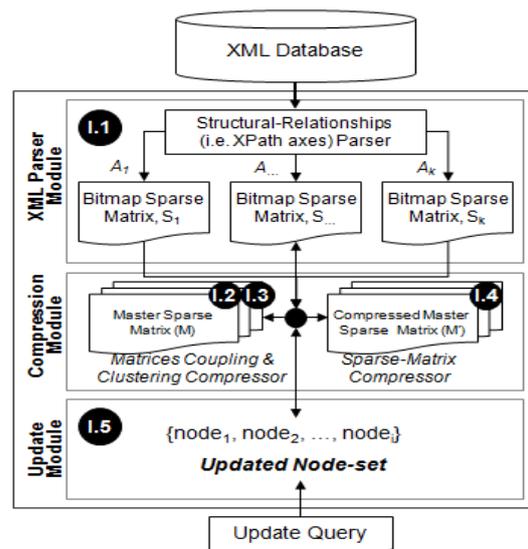


Figure 1: PACD's Index Builder (IB).

2 BACKGROUND

2.1 PACD Technique

PACD, introduced in (Al-Badawi et al, 2009), is a bitmap XML processing technique consists of two subsystems: the Index Builder (IB) and the Query Processor (QP). The IB shreds XML data (including the textual contents and the XML structure) into the underlying data storage while the QP translates and executes XML queries (expressed in XQuery syntax) over the shredded XML data. Of the former, the component performs three main operations including the conversion of XML structural relationships into a set of binary relations (bitmap format), the compression of bitmapped XML structure and the XML updates handling (see Fig 1). During the first operation (process I.1 in Fig. 1), the XML structural relationships (derived by the XPath's thirteen axes and their extension; the Next and Previous axes(Al-Badawi et al, 2009) are organized into a set of $n \times n$ matrices each is representing a specific XML structure corresponding to an XPath axis. The entries of the generated matrices are binary where '1' is indicating the existence of the structural relationship (e.g. child, parent, ...etc) while '0' is indicating the absence of such relationship between any node pairs (Wang et al., 2006; He et al., 2005). In the XML databases context, these matrices are sparse (Sun et al., 2008; George et al., 1993) and their dimensions are huge can go beyond any hardware and software limits. This justifies the need of the proposed Data Compression Model (DCM).

2.2 Encoding XML Structure in PACD

As mentioned above, PACD explicitly encodes the whole XML structure represented by XPath axes into a set of bitmap matrices. Two main advantages of such encoding are: a) to increase the QP coverage and b) to reduce the overhead workload caused by deriving some XML structures from others (Pettovello and Fotouhi, 2006).

XPath/XQuery (Berglund et al., 2010; Boag et al., 2010) specification describes 13 axes for any context node in the XML tree. These are the self, child, parent, descendant, self-or-descendant, ancestor, self-or-ancestor, preceding, following, preceding-siblings, following-siblings, attributes and namespace axes. Additionally, PACD introduced the 'next' and 'previous' axes to substitute the following-sibling and preceding-sibling in preserving the document order (Al-Badawi et al,

2009). The process of mapping XML structure into bitmap matrices explicitly excludes the 'namespace' axis due its popularity in XML database transactions, combines the 'self-or-descendant' with the 'descendant (and 'self-or-ancestor' with the 'ancestor) into a single matrix respectively, and encodes attributes as ordinary child-entries in all matrices. Furthermore, PACD deals with the 'self' axis at the XML processing level (i.e. querying and updating) instead of encoding this structure explicitly in a separate matrix

As for illustration, Fig. 2 depicts a sample XML database while Fig. 3 and Fig. 4 respectively show the childOf and descendantOf matrices for the XML tree as an example of the XML/matrix transformation.

```
<dblp>
  <book year="2001">
    <title>Data on the Web</title>
  </book>
  <book key="110">
    <author>
      <first>John</first>
      <last>Smith</last>
    </author>
  </book>
  <thesis key="500">
    <title>A Bit...</title>
  </thesis>
</dblp>
```

Figure 2: An XML database example.

2.3 The Need for Data Compression

It is clear from the above discussion that the size of the PACD's data storage for large XML databases will be huge in terms of matrix dimensions and physical storage space. This could result in several performance issues such as HW/SW failure and processing deficiencies. One way to overcome such limitation is to have the PACD's data storage reduced by the all means while keeping its XML structure coverage still comprehensive.

A such proposal can also benefit the following aspects:

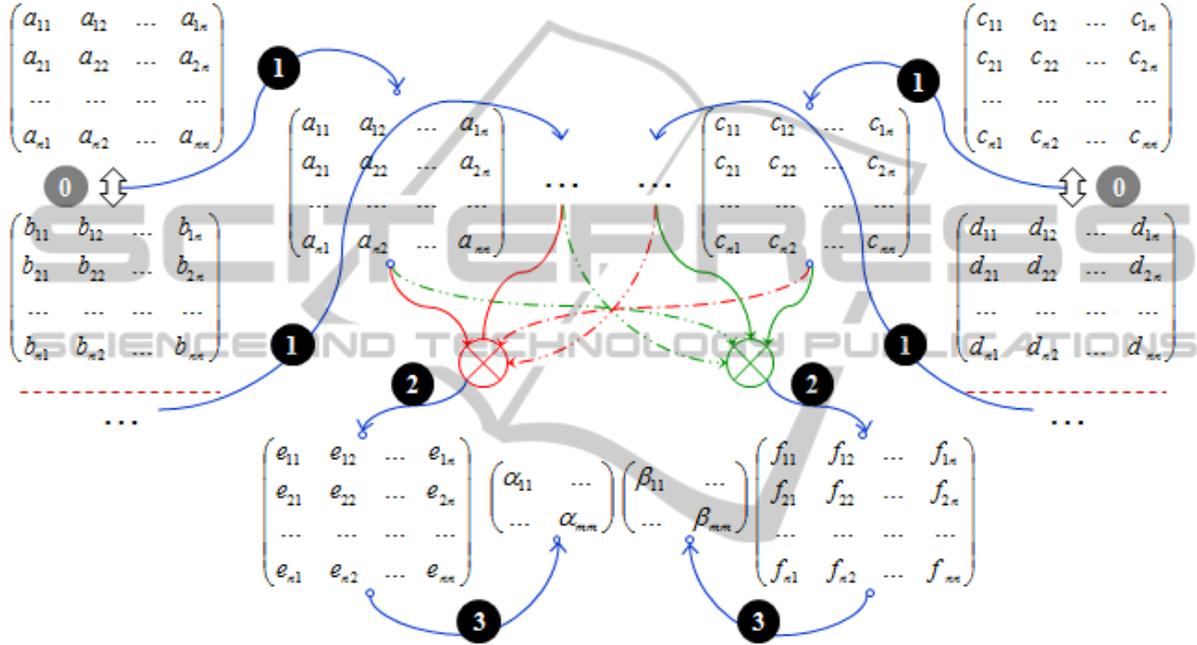
➤ **Memory Based Management:** is an attractive approach since it eliminated overhead caused by perform I/O operations. To achieve this, XML literature has produced different data compression techniques including data guide summaries (Goldman and Widom, 1997; Haw and Lee, 2009), adaptive indexes (Chung et al., 2002), and selective indexes (Milo and Suciu, 1999; Hammerschmidt, 2005). The compression architecture of these techniques takes place at the XML encoding level

$$\begin{pmatrix} & \&0 & \&1 & \&2 & \&3 & \&4 & \&5 & \dots & \&11 \\ \&0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \&1 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \&2 & 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ \&3 & 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ \&4 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \&5 & 0 & 0 & 0 & 0 & 1 & 0 & \dots & 0 \\ \dots & \dots \\ \&11 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

Figure 3: The childOf Matrix.

$$\begin{pmatrix} & \&0 & \&1 & \&2 & \&3 & \&4 & \&5 & \dots & \&11 \\ \&0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \&1 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \&2 & 1 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ \&3 & 1 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ \&4 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \&5 & 1 & 0 & 0 & 0 & 1 & 0 & \dots & 0 \\ \dots & \dots \\ \&11 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

Figure 4: The descendantOf (or descOf) Matrix.



- Level 0: Original Matrix-set (No Compression)
- Level 1: Matrix Transformation (two matrices are represented by one)
- Level 2: Matrix Clustering (at this level a matrix can be clustered in different ways; this explains the dotted arrows)
- Level 3: Sparse-Matrix Compression (different sparse-matrix compression can be applied to individual matrix)

Figure 5: The framework of the PACD's data compression model for the XML structure.

rather than the data representation level which in turn results in some other performance issues such as summary expressiveness. PACD redeems such shortcoming by encoding the full XML structure and incorporates well established compression techniques to reduce the size of encoded information.

➤ **Database Sizing:** In this context, database sizing describes the process of controlling the size of the database in order to utilize the computer system's resources during the database management (McCord, 1981; O'Neil and O'Neil, 2001). Loading 'x' storage units (e.g. byte) is often faster than loading 'x+y' storage units (N.B.: other factors, such as the desirable data location, are also involved but the same logic applies). Therefore, there is no doubt

that the compressed data will outperform the uncompressed data provided that the same database's functionalities are obtainable from both versions, and the data compression complexity is kept to the minimum.

➤ **Nature of Data Presentation:** Finally, the amount and type of information stored by PACD necessitates the use of data compression techniques. PACD aims to store the XML structures using a bitmap based representation which is encoded into sparse-matrices. The system creates ten sparse matrices (*see next section*) for this purpose, each of which requires 'n×n' storage units. The total figure for storing these matrices, as well as XML's textual contents, becomes huge for large XML databases and could be beyond the capabilities of the

underlying hardware and software specifications; or, it could result in other performance issues.

3 A DATA COMPRESSION MODEL

To achieve a better data storage performance PACD maintains separate models for compressing the XML textual contents and the XML structure respectively. This paper only describes the mechanism used to compress the XML structure.

3.1 Model Overview

The PACD's matrix based storage (or XML structure storage) can be compressed using three different types of matrix operations. These are matrix transformation operations to derive some matrices from other matrices; matrix clustering operations to combine a set of matrices into one matrix; and sparse-matrix compression techniques to further reduce the storage space requirement of the final matrix set. The layout of the overall XML structure compression is illustrated in Fig. 5 while each of these levels is briefly described in the subsequent sections.

3.2 A Matrix Algebra

To formulate data compression model transactions, each matrix will be represented as a *set* and the group of *sets* are then manipulated (e.g. transformed) using *set*-theory notations. A set, corresponding to a specific PACD's matrix, contains the positive entries of that matrix encoded as node pairs. For example, the entries of the *childOf* set of the above XML tree are $\{(&1,&0), (&4,&0), (&9,&0), (&2,&1), \dots, (&11,&9)\}$.

On the other hand, XML structures reflected by the thirteen XPath's axes and their extensions the 'next' and 'previous' axes, are encoded into ten sparse matrices as proposed earlier. According to the XPath's specification (Berglund et al., 2010), these axes are found in *invertible* and/or *inclusive* pairs and hence their sparse-matrix and set representations. Based on this logic, the first compression level of the bitmapped XML structure uses the *invertible* characteristic of the XML structure in order to reduce the number of matrices explicitly encoded in the underlying storage while the second level of the compression uses the *inclusive* characteristics to combine two or more

matrices into a single matrix. The following two sections formulate the first two compressions while the third compression process is described then after.

3.3 Invertible-matrix Transformation

Invertible XPath's axes are those axes which their XML structural relationships are the *inverse* of each other. There are five axis-pairs of this type; the *child*, *descendent*, *following*, *following-sibling* and *next* axes are the inverse of the *parent*, *ancestor*, *preceding*, *preceding-sibling* and *previous* axes respectively. The notion of compression in this level is to encode (consider) only one axis of each axis-pair in the underlying data storage while the image-axis can be calculated using simple geometric transformations on the set (or matrix) representation of the corresponding XML structure. Thus, the number of matrices (sets) that need to be encoded in the underlying data storage is reduced from ten to five only which in turn halves the amount of storage space required for the entire XML structure.

As for illustration, the *childOf* and *parentOf* matrices can be algebraically calculated from each other using the formula $\text{childOf}[i,j] = \text{parentOf}[j,i]$. In terms of geometric transformation, the *parentOf* matrix can be obtained by rotating the *childOf* matrix by 90° clockwise and then reflecting the matrix on the y-axis. In practice this is simply done by inverting the first matrix indices to obtain the entries of the second matrix. The five possible geometric transformation of XPath's axes are formulated in Fig. 6.

3.4 Matrix Clustering

Invertible-matrix transformation were only able to reduce the number of encoded matrices (XML structure) from ten to five matrices. Therefore, the amount of storage space required to store the five matrices remains big for large XML databases. This necessitates advances in storage space reduction which could be achieved by further reduction in the number of matrices and/or using sophisticated compression algorithms to store the matrices inside the computer's storage media.

In this context, the matrix clustering process maps the corresponding entries of two or more matrices to a single entry producing a matrix which represents the all combined matrices. To do so, two conditions must apply. Firstly, all combined matrices must have the same *degree* of rows and columns. Secondly, entries of all combined matrices must have a finite set of *values*; in other words,

- a. $\overset{parentOf}{\Phi} : childOf \xrightarrow{90^\circ} M \xrightarrow{Y|Y'} parentOf$
- b. $\overset{anceOf}{\Phi} : descOf \xrightarrow{90^\circ} M \xrightarrow{Y|Y'} anceOf$
- c. $\overset{precOf}{\Phi} : follOf \xrightarrow{90^\circ} M \xrightarrow{Y|Y'} precOf$
- d. $\overset{precSibOf}{\Phi} : follSibOf \xrightarrow{90^\circ} M \xrightarrow{Y|Y'} precSibOf$
- e. $\overset{prevOf}{\Phi} : nextOf \xrightarrow{90^\circ} M \xrightarrow{Y|Y'} prevOf$

Figure 6: Invertible Transformations of the XML Structure.

- f. $childOf \subseteq descOf \Rightarrow \overset{childOf}{S^\circ} \leq \overset{descOf}{S^\circ}$
- g. $\& \quad parentOf \subseteq anceOf \Rightarrow \overset{parentOf}{S^\circ} \leq \overset{anceOf}{S^\circ}$
- h. $\& \quad precSibOf \subseteq precOf \Rightarrow \overset{precSibOf}{S^\circ} \leq \overset{precOf}{S^\circ}$
- i. $\& \quad follSibOf \subseteq follOf \Rightarrow \overset{follSibOf}{S^\circ} \leq \overset{follOf}{S^\circ}$
- j. $\& \quad prevOf \subseteq precSibOf \Rightarrow \overset{prevOf}{S^\circ} \leq \overset{precSibOf}{S^\circ}$
- k. $\& \quad nextOf \subseteq follSibOf \Rightarrow \overset{nextOf}{S^\circ} \leq \overset{follSibOf}{S^\circ}$
- l. $descOf \cap childOf = childOf$
- m. $\Rightarrow \overset{descOf \oplus childOf}{S^\circ} = \overset{descOf}{S^\circ}$
- n. $childOf \cap nextOf = \phi$
- o. $\Rightarrow \overset{childOf \oplus nextOf}{S^\circ} = \overset{childOf}{S^\circ} + \overset{nextOf}{S^\circ} - 1$
- p. $descOf \cap nextOf = \phi$
- q. $\Rightarrow \overset{descOf \oplus nextOf}{S^\circ} = \overset{descOf}{S^\circ} + \overset{nextOf}{S^\circ} - 1$

$A S^\circ$ is the sparsity-degree of the matrix A; calculated by dividing the number of non-zero elements by n^2

\oplus is the addition operation (clustering) of two matrices

Figure 7: More XML structure semantic based on Matrix/Set representation.

entries must have limited domain of values and the domains must be identical; e.g. $\{0, 1\}$ or $\{a, b, c\}$ for all matrices. The value of the $[i, j]^{\text{th}}$ entries from all matrices are combined somehow to produce the $[i, j]^{\text{th}}$ entry in the resulting matrix. For example, if we have two matrices A and B such that $A[i, j] \in \{0, 1\}$ and $B[i, j] \in \{0, 1\}$, then we can use '0', 'a', 'b' and 'c' to respectively map '00', '01', '10' and '11'. The resulting matrix, say C, will then have entries all belong to $\{0, a, b, c\}$ (i.e. $\forall c_{ij} \in C; C[i, j] \in \{0, a, b, c\}$). We call output matrix a *Master* matrix and its *degree* (i.e. dimension) must be the same as the composing matrices.

In the PACD's data representation, the above two conditions are imbedded in the PACD's storage specifications (see Section 2.2). So, all matrices

generated are of power ' $n \times n$ ', and the entries of these matrices have either '0' or '1' values.

Complexity-wise, the cost of the clustering process itself is determined by the size of the values' set and the number of clustered matrices. In practice, the complexity is found low because the process may cluster a maximum of five matrices with only two possible values '0' or '1'. However, the matrix clustering process should also consider the trade-off between the storage space reduction and the performance issues that incurred by the querying and updating processes. Due the space limitation, the discussion of this topic is omitted from this paper.

In general, the clustering process is guided by the *set*-based XML semantic listed in Fig. 7. Fig. 8 provides an example of clustering the child and descendent matrices using a simple-clustering table.

	Case 1	Case 2	Case 3 [*]	Case 4
childOf (A)	0	0	1	1
descOf (B)	0	1	0	1
A&B	'00'	'01'	'10'	'11'
f(A & B)	0	1	2	3

* Impossible case (see Fig. 6)

	&0	&1	&2	&3	&4	&5	...	&11
&0	0	0	0	0	0	0	...	0
&1	3	0	0	0	0	0	...	0
&2	1	3	0	0	0	0	...	0
&3	1	3	0	0	0	0	...	0
&4	3	0	0	0	0	0	...	0
&5	1	0	0	0	3	0	...	0
...
&11	1	0	0	0	0	0	...	0

$$\overset{childOf}{S^\circ} = 0.924, \quad \overset{descOf}{S^\circ} = 0.854, \quad \overset{childOf \oplus descOf}{S^\circ} = 0.854$$

 Figure 8: A clustering example ($childOf \oplus descOf$).

3.5 Sparse-matrix Compression

The maximum compression degree of the first two levels can reduce the storage requirement of storing k number of matrices (corresponding to a single XML tree of size 'n' nodes) up to n^2 storage units. This remains an issue for large XML databases and multi-document XML databases, where the value of 'n' is high with respect to the available system's resources. For example, a tree of 10^6 nodes requires 10^{12} bytes ($\cong 1000$ GB) when each matrix's entry is stored using 1-byte storage. Thus, this justify the need for an additional compression level which acts on the resulting matrices considering their unique characteristics; e.g. the sparsity-degree and the zero's distribution.

"A matrix is sparse if many of its coefficients are zero ... (and) ... Generally, we say that a matrix is

sparse if there is an advantage in exploiting its zeros” (Duff et al., 1986). In (George et al., 1993), a sparse matrix is also defined as the matrix which is populated mainly with zeros while some references are more specific, limiting the definition to those matrices with certain amount of 0's; e.g. 50% of the entries are 0 as in (Mackay and Neal, 1995).

According to the above, our matrices for representing the XML structures are considered to be sparse. This is mainly reflected by the analysis given in (Al-Badawi, 2010) which shows that the number of zeros in the childOf and nextOf matrices reaches n^2-n , and the numbers of zeros in the descOf matrix may exceed $n^2-h \times n$ where 'n' is the number of nodes and 'h' is the number of levels in the underlying XML tree. When n goes high, the number of 0 entries easily exceeds 90% of the total entries

From a technical point of view, storing matrices of this size in the computer system is a trade off between the high storage size and storage performance (Tarjan and Yao, 1979). For example, to store a matrix when $n=10^6$ into two-dimensional array of type character (one byte stores one character), we need 10^{12} bytes of memory which may defeat the HW/SW capabilities. One way to address this issue is using sparse matrix compression (SMC) techniques to compact the matrix's storage.

The architecture of any SMC technique depends on the computation to be performed, the pattern of the non-zero entries, and even the architecture of the computer system itself (Duff et al., 1986; Willcock and Lumsdaine, 2006). Among these three factors, we are only concerned with the computation constraints in this stage of our research; having that achieving optimum storage with good performance is the main goal of the compression process. The investigation of other issues is a subject for further research.

To align the choice of the used SMC with the cost reduction of the XML querying and updating operations, PACD categorizes the existing sparse-matrix techniques into two groups; the first includes the techniques which do not necessitate any decompression/recompression process during the XML querying and updating operations so the overhead complexity incurred by these processes will be avoided during the XML querying and updating. The second category contains those techniques which defeat the XML operations by the cost of decompressing/recompressing processes done to the underlying storage. Detailed discussion of this aspect plus the empirical proof lays down outside the scope of this paper due the space

limitation.

4 CONCLUSIONS

To conclude, this paper described the PACD's DCM which uses three data compression processes to compact the XML structures. As introduced in (Al-Badawi et al., 2009), the XML structures are theoretically encoded into ten $n \times n$ matrices each of which represents a structural relationship which corresponds to an XPath's axis or an extension. Each structural relationship is encoded into a set of node pairs where such relationship applies between them. So, each matrix represents the corresponding structural relationship between all nodes in the XML tree.

PACD's matrices are found in invertible pairs and inclusive pairs, and are sparse. The first compression phase uses the first characteristic; that is each invertible pair is represented by only one matrix. This process can reduce the number of matrices from ten to five matrices. The next compression phase uses the inclusiveness characteristic; that is two or more matrices are clustered into a single matrix such that the full architecture of all composing matrices is preserved in the clustered matrix. The last compression phase is based on using one or more sparse-matrix compression techniques to compact the layout of the resulting matrix from the first two compressions.

The strength and efficiency of the PACD's overall storage is determined by the specification of the clustering and SMC methods used. A complete discussion about this topic including the experimental proof is the subject for further publications.

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