Kinematic Analysis of Lower Mobility Cooperative Arms by Screw Theory

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Abstract: This paper studies the kinematic modeling and analysis of a system with two cooperative manipulators, working together on a common task. The task is defined as the transportation of an object in space. The cooperative system is the dual armed humanoid Nao robot, where the serial architecture of each arm has five degrees of freedom. The mobility of the closed loop system is analyzed and the nature of the possible motion explored. The serial singular configurations of the system are considered. Furthermore the parallel singularities of the closed loop system associated with each actuation scheme are analyzed.

1 INTRODUCTION

The capability of dual independent arms when processing parts reduces the need for custom fixtures and permits the use of a simpler end effector. The system can then execute sophisticated tasks that may be difficult for a single arm system. For example, rather than using a large serial robot a cooperative system distributes a heavy load among several smaller robots. Similarly if the object is of an unwieldy, non-rigid or awkward composition, the single arm robot may struggle to manipulate it.

By using a cooperative system of two or more manipulators, both the location and the internal forces of the object can be controlled. The two principal approaches that avail of force sensors on the robot are: hybrid position/force control (Uchiyama and Dauchez, 1988) and impedance control (Sadati and Ghaffarkhah, 2008), (Caccavale et al., 2008).

Another approach is to formulate kinematic relations that create a task space describing the multi-arm system while grasping an object. The main methods are known as *Symmetric formulation* (Uchiyama and Dauchez, 1988) and *Task orientated formulation* (Chiacchio et al., 1996; Caccavale et al., 2000). Both create a cooperative task space of velocity variables, describing the object motion in space and the relative motion between the end effectors (i.e forces applied on object). On the other hand, the system can also be viewed as a redundantly actuated parallel manipulator. In this case kinematic constraint equations are derived that establish a relationship between the chosen independent and dependent joint variables (Yeo et al., 1999; Liu et al., 1999; Cheng et al., 2003; Özkan and Özgören, 2001). The dependent joint variables adopt values that ensure loop closure at each instant.

Most of the preceding work has been carried out with dual arm systems, where both arms are either of 3-DOF or 6-DOF spatial composition. Thus away from singularities, the grasped object has a mobility of 3 or 6 respectively. On the other hand the study of lower mobility cooperative manipulators has been limited. In (Yeo et al., 1999) the cooperation between a 5-DOF and 4-DOF robot is used in conjunction with a passive joint in order to execute a 4-DOF position/force task. In (Zielinski and Szynkiewicz, 1996) admissible path planning for two 5-DOF robots is explored. In (Bicchi et al., 1995) a generalized method based on the Jacobian matrix of each arm, and their constraint relations with the object, is formulated. Analysis of these matrices permits the calculation of the mobility and possible first order differential motions of general multiple limb robots.

Lower mobility systems suffer from three types of singularities, limb (serial) singularities, actuation and constraint (parallel) singularities (Amine et al., 2011). In (Liu et al., 1999) the presence of parallel singularities of a cooperative system with passive joints is explored. The issue of a valid selection of actuators is addressed in (Özkan and Özgören, 2001). In both cases an analysis of the Jacobian matrix is carried out. Conversely screw theory can be used to locate and un-

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derstand parallel singularities in closed chain mechanisms (Zlatanov et al., 2002).

In this paper the cooperative system, defined by the two arms of Aldebaran NAO T14 humanoid robot and a grasped object, is examined. The system has been modeled as a closed chain mechanism (Sections 2 and 3). The originality of this paper lies in the use of screw theory techniques to analyze the system's mobility, singularities and motion type. The benefit of this approach is that special configurations such as the loss of stiffness, loss of DOF etc., can be determined without the complex derivation of the Jacobian matrices (or their inverses) (Sections 4 and 5).

2 SYSTEM DESCRIPTION

The system is described by the Modified Denavit-Hartenberg (MDH) notation (Khalil and Kleinfinger, 1986),in Table 1. The right arm consists of joints 1-5 and the left arm consists of joints 6-10. The transformation matrix ${}^{a(j)}\mathbf{T}_j$, from frame $a(j)$, the antecedent of j, to frame j is the 4×4 matrix given by:

$$ {}^{a(j)}\mathbf{T}_j = rot_z(\gamma_j)\cdot trans_z(b_j)\cdot rot_x(\alpha_j)\cdot trans_x(d_j)\cdot rot_{zj}(\theta_j)\cdot trans_z(r_j) \quad (1) $$

where $rot_i(\theta)$ indicates a rotation of θ radians about the ith axis and $trans_i(l)$ a translation of l meters along the ith axis. It should be noted that $\gamma_j = b_j = 0$ when $\mathbf{x}_{(a(j))}$ is perpendicular to $\mathbf{z}_j$. Once the object is grasped a closed loop is formed. As shown in Fig. 1, link 5 of the closed chain is composed of link 5, link 10 of the open chain and the object. Frame 10 is fixed on link 5. We introduce frame 11, which is equivalent to frame 10, but its antecedent is frame 5. The parameters of frame 11 are defined once the robot has grasped the object. The system has, in this case, only nine bodies. Hence joint 10 is denoted as the cut joint.

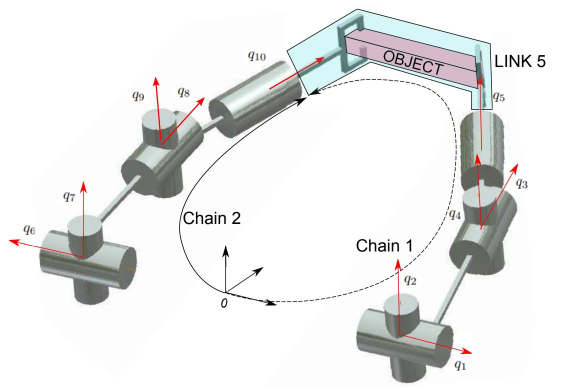

Figure 1: Closed Loop Formulation.

Table 1: MDH Parameters of the closed loop chain.

j	$a(j)$	γ	b	d	α	θ	r
1	0	0	b_1	0	$-\frac{\pi}{2}$	θ_1	$-r_1$
2	1	0	0	0	$\frac{\pi}{2}$	θ_2	0
3	2	0	0	$-d_3$	$\frac{\pi}{2}$	θ_3	r_3
4	3	0	0	0	$\frac{-\pi}{2}$	θ_4	0
5	4	0	0	0	$\frac{\pi}{2}$	θ_5	r_5
6	0	0	b_1	0	$\frac{-\pi}{2}$	θ_6	r_1
7	6	0	0	0	$\frac{\pi}{2}$	θ_7	0
8	7	0	0	d_3	$\frac{\pi}{2}$	θ_8	r_3
9	8	0	0	0	$\frac{-\pi}{2}$	θ_9	0
10	9	0	0	0	$\frac{\pi}{2}$	θ_{10}	r_5
11	5	γ_{11}	b_{11}	d_{11}	α_{11}	θ_{11}	r_{11}

3 KINEMATIC CONSTRAINTS

The location and velocity of the cut joint frame must be equivalent when calculated via either chain. This ensures a constant object grasp throughout the trajectory. In the closed loop formulation, joints are designated either as actuated or passive. $\mathbf{q}_a$ contains the joint variables that are actuated, $\mathbf{q}_p$ contains the passive joint variables and $\mathbf{q}_c$ contains the joints that are cut. The passive and cut joint variables can be obtained in terms of the active joint variables using the following geometric constraint equations:

$$ {}^0\mathbf{T}_1\,{}^1\mathbf{T}_2\,{}^2\mathbf{T}_3\,{}^3\mathbf{T}_4\,{}^4\mathbf{T}_5\,{}^5\mathbf{T}_{11} = {}^0\mathbf{T}_6\,{}^6\mathbf{T}_7\,{}^7\mathbf{T}_8\,{}^8\mathbf{T}_9\,{}^9\mathbf{T}_{10} \quad (2) $$

Similarly, the kinematic constraints are given by:

$$ \begin{bmatrix} \omega_{11} \\ \mathbf{v}_{11} \end{bmatrix} = \begin{bmatrix} \omega_{10} \\ \mathbf{v}_{10} \end{bmatrix} \quad (3) $$

By substituting in the joint velocity vectors:

$$ {}^0\mathbf{J}_{11}\,\dot{\mathbf{q}}_r = {}^0\mathbf{J}_{10}\,\dot{\mathbf{q}}_l \quad (4) $$

where $\dot{\mathbf{q}}_r$ and $\dot{\mathbf{q}}_l$ contain the joint velocities of the right arm and the left arm, respectively. ${}^0\mathbf{v}_j$ is the linear velocity and ${}^0\omega_j$ the angular velocity of frame j in frame 0, ${}^0\mathbf{J}_j$ is the 6×5 kinematic Jacobian matrix of frame j w.r.t. frame 0. By rearranging the rows and columns of (4), a relationship is obtained between the passive joint velocities and the actuated joint velocities

$$ \begin{bmatrix} \mathbf{G}_a & \mathbf{G}_p & \mathbf{0} \\ \mathbf{G}_{ac} & \mathbf{G}_{pc} & \mathbf{G}_c \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_a \\ \dot{\mathbf{q}}_p \\ \dot{\mathbf{q}}_c \end{bmatrix} = \mathbf{0} \quad (5) $$

From (5), we obtain:

$$ \dot{\mathbf{q}}_p = -\mathbf{G}_p^{-1}\,\mathbf{G}_a\,\dot{\mathbf{q}}_a \quad (6) $$

$$ \dot{\mathbf{q}}_c = -\mathbf{G}_c^{-1}\left(\mathbf{G}_{ac} - \mathbf{G}_{pc}\,\mathbf{G}_p^{-1}\,\mathbf{G}_a\right)\dot{\mathbf{q}}_a \quad (7) $$

The mobility from section 4 is found to be equal to 4, hence the dimension of $\mathbf{G}_a$ is 5×4 , $\mathbf{G}_p$ is 5×5 ,$\mathbf{G}_c$ is a scalar (4, 5, 1 being the number of active, passive and cut joints respectively). $\mathbf{G}_p$ degenerates at configurations corresponding to parallel singularities as seen in section 5.2.

4 MOBILITY ANALYSIS

The degree of freedom (DOF) of the system, is equal to the number of independent coordinates required to define the pose of the end-effector. The DOF can be obtained by several methods for example Chebychev-Grübler-Kutzbach, or Gogu's Method (Gogu, 2007). In order to elucidate the motion type, *screw theory* is used. In summary, each serial arm has 5-DOF however once the object is firmly grasped by each arm, a closed chain is formed and the object DOF becomes four.

4.1 Screw Theory

4.1.1 Mobility Analysis based on Screw Theory

Screw theory can be used to analyze the instantaneous motions of complex mechanisms (Hunt, 1978; Ball, 1900). A screw of pitch λ is defined as:

$$\$_\lambda = \begin{bmatrix} \mathbf{s} \\ \mathbf{s} \times \mathbf{r} + \lambda \mathbf{s} \end{bmatrix} \tag{8}$$

$\mathbf{s}$ is the unit vector of the rotational axis of the screw. $\mathbf{r}$ is the vector from any point on the axis to the origin. A zero-pitch screw and an infinite-pitch screw are given respectively by:

$$\$_0 = \begin{bmatrix} \mathbf{s} \\ \mathbf{s} \times \mathbf{r} \end{bmatrix} \quad , \quad \$_\infty = \begin{bmatrix} \mathbf{0}_{3\times1} \\ \mathbf{s} \end{bmatrix} \tag{9}$$

For every screw system, consisting of n linearly independent screws, there exists a reciprocal screw system of dimension $6-n$. Two screws $\$_1$ and $\$_2$ are reciprocal if their instantaneous power is zero, namely,

$$\left(\begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_3 \\ \mathbf{I}_3 & \mathbf{0}_{3\times3} \end{bmatrix} \$_1 \right)^T \$_2 = 0 \tag{10}$$

The following reciprocity conditions are defined from (10):

1. $\$_0$ is reciprocal to $\$_\infty$ if their axes $\perp$;
2. $\$_\infty$ is always reciprocal to another $\$_\infty$;
3. Two $\$_0$ are reciprocal if their axes are coplanar;

A zero-pitch twist ε_0 corresponds to a pure rotation about its axis. An ∞-pitch twist ε_∞ corresponds to a pure translation along its axis. A zero-pitch wrench ζ_0 corresponds to a pure force along its axis. An ∞-pitch wrench ζ_∞ corresponds to a pure moment about its axis. The twist system $\mathtt{T}^i$ and the wrench system $\mathtt{W}^i$ of a serial kinematic chain i composed of f joints are given by:

$$\mathtt{T}^i = \bigoplus_{j=1}^{f} \mathtt{T}_j \quad , \quad \mathtt{W}^i = \bigcap_{j=1}^{f} \mathtt{W}_j \tag{11}$$

The twist system $\mathtt{T}$ and the wrench system $\mathtt{W}$ of a parallel kinematic chain composed of m serial chains are given by:

$$\mathtt{T} = \bigcap_{i=1}^{m} \mathtt{T}^i \quad , \quad \mathtt{W} = \bigoplus_{i=1}^{m} \mathtt{W}^i \tag{12}$$

$\mathbf{A} = \bigoplus_{j=1}^{f} \mathbf{B}_j$ means $\mathbf{A}$ is spanned by vectors $\mathbf{B}_{1\cdots f}$ while $\mathbf{A} = \bigcap_{j=1}^{f} \mathbf{B}_j$ means $\mathbf{A}$ is intersection of vectors of $\mathbf{B}_{1\cdots f}$

Table 1 shows that NAO T14 is composed of five revolute joints. From (11), the twist systems $\mathtt{T}^r$ and $\mathtt{T}^l$ of its right and left arms are defined as[1]:

$$\mathtt{T}^r = span(\varepsilon_{01}, \varepsilon_{02}, \varepsilon_{03}, \varepsilon_{04}, \varepsilon_{05}) \tag{13}$$

$$\mathtt{T}^l = span(\varepsilon_{06}, \varepsilon_{07}, \varepsilon_{08}, \varepsilon_{09}, \varepsilon_{010}) \tag{14}$$

The first two joints of each arm intersect and constitute a universal (U)-joint. The last three joints also intersect and are equivalent to a spherical (S)-joint. Since the twist system of each arm is a 5-system, its reciprocal wrench system is a 1-system. By reciprocity condition 3, it can be shown that each arm applies one pure force (a zero-pitch wrench $\zeta_{0i}, i = 1,2$) to the object along an axis intersecting the U- and S-joint axes. Thus, the wrench systems of the two arms are defined as:

$$\mathtt{W}_r = \zeta_{01} \quad , \quad \mathtt{W}_l = \zeta_{02} \tag{15}$$

From (12), the constraint wrench system applied to the object is a 2-system given by:

$$\mathtt{W}^c = \mathtt{W}_r \oplus \mathtt{W}_l = span(\zeta_{01}, \zeta_{02}) \tag{16}$$

The object twist system is reciprocal to $\mathtt{W}^c$. Thus, it is a 4-system and the object has four DOF.

4.1.2 Motion Analysis based on Screw Theory

When the constraint forces are parallel, i.e., $\zeta_{01} \parallel \zeta_{02}$, reciprocity condition 1 states that there are two independent ∞-pitch twists, $\varepsilon_{\infty1}$ and $\varepsilon_{\infty2}$, reciprocal (normal) to ζ_{01} and ζ_{02}. Reciprocity condition 3 states

[1] r and l stand for right and left.

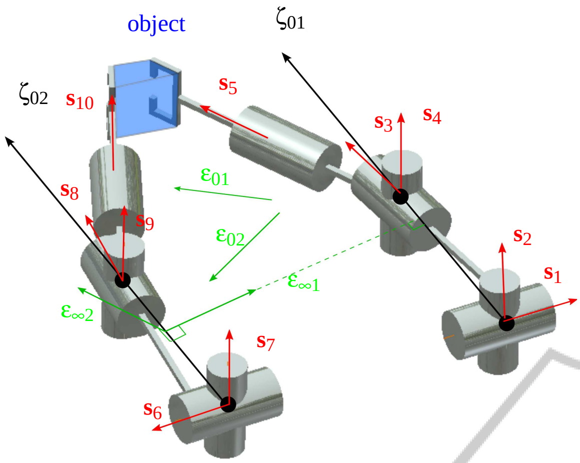

Figure 2: Reciprocal twists to parallel constraint forces (2T2R motion mode).

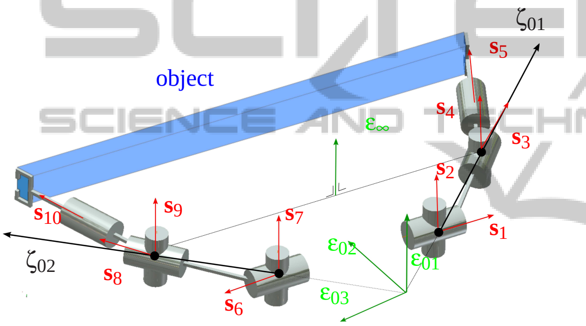

Figure 3: Reciprocal twists to intersecting constraint forces (1T3R motion mode).

that there are also two independent zero-pitch twists, ε_{01} and ε_{02}, reciprocal (coplanar) to ζ_{01} and ζ_{02} as shown in Fig. 2. Therefore, the corresponding motion mode is 2T2R[2].

When $\zeta_{01} \nparallel \zeta_{02}$, reciprocity condition 1 states there is one ∞-pitch twist $\varepsilon_{\infty 1}$ reciprocal (normal) to both ζ_{01} and ζ_{02} and condition 3 states there are three independent zero-pitch twists, ε_{01}, ε_{02} and ε_{03}, reciprocal (coplanar) to both ζ_{01} and ζ_{02} as shown in Fig. 3. Consequently, the corresponding motion mode is 1T3R.

5 SINGULARITY ANALYSIS

This section deals with the singularity analysis of the NAO 14 when it firmly grasps an object. Limb singularities can be characterized by a loss of DOF of the limb, while a gain of DOF or a lack of stiffness is known as a parallel singular configuration (Amine et al., 2011; Amine et al., 2012).

[2]T and R stand for Translation and Rotation, respectively.

5.1 Limb Singularities

A limb singularity is similar to the singularity of a serial manipulator. It occurs for the dual-arm system when the limb kinematic screw system (twist system) degenerates. Consequently, the grasped object loses one or more DOF in such a configuration. From (9) and (13) the kinematic Jacobian matrix of the right arm can be written as:

$$\mathbf{J}_r = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3 & \mathbf{s}_4 & \mathbf{s}_5 \\ \mathbf{s}_1 \times \mathbf{r}_1 & \mathbf{s}_2 \times \mathbf{r}_2 & \mathbf{s}_3 \times \mathbf{r}_3 & \mathbf{s}_4 \times \mathbf{r}_4 & \mathbf{s}_5 \times \mathbf{r}_5 \end{bmatrix} \tag{17}$$

Let the wrist center be the origin of the frame where vectors $\mathbf{r}_i$, $i = 1, \ldots, 5$ are expressed. Equation (17) becomes

$$\mathbf{J}_r = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3 & \mathbf{s}_4 & \mathbf{s}_5 \\ \mathbf{s}_1 \times \mathbf{r}_1 & \mathbf{s}_2 \times \mathbf{r}_2 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \end{bmatrix} \tag{18}$$

The right arm reaches a limb singularity when $\mathbf{J}_r$ is rank deficient. The relations leading to rank deficiency can be examined. Since $\mathbf{s}_1 \perp \mathbf{s}_2$, $\mathbf{s}_3 \perp \mathbf{s}_4$ and $\mathbf{s}_4 \perp \mathbf{s}_5$ these degeneracies can be eliminated. Furthermore, the architecture of the arm means the S-joint cannot lie on $\mathbf{s}_2$, leaving two singular configurations. Firstly when $\mathbf{s}_3 \parallel \mathbf{s}_5$ obtained at $q_4 = 0 \pm \pi$, secondly when $\mathbf{s}_1 \times \mathbf{r}_1 = \mathbf{0}_3$ obtained at $q_2 = \text{atan}\left(\frac{-r_3}{d_3}\right)$.

5.2 Parallel Singularities

The loss of stiffness due to a parallel singularity can be characterized by a degeneracy of screw system representing the wrenches. Examples are given in Table 2 (Hunt, 1978).

Table 2: Linear Dependence of Screws.

No.	ζ_λ	Condition
≥ 2	ζ_0	collinear axes
≥ 2	ζ_∞	parallel axes
> 3	ζ_0	intersect the same point
≥ 6	ζ_0	intersect the same line

5.2.1 Constraint Singularities

A constraint singularity occurs when the constraint wrench system (16) degenerates, i.e., when ζ_{01} and ζ_{02} are linearly dependent (condition 1 in Table 2). The closed loop system reaches such a configuration when the two S-joint centers lie on $\mathbf{s}_1$ and $\mathbf{s}_6$, from the geometric model:

$$q_2 = \text{atan}\left(\frac{-r_3}{d_3}\right) \quad \text{and} \quad q_7 = \text{atan}\left(\frac{r_3}{d_3}\right) \tag{19}$$

From section 5.1 it is noted that when the closed loop system reaches a constraint singularity, both arms reach a limb singularity. Four wrenches forming a 3-system as described in Fig. 4 are applied on the object: the constraint wrenches ζ_{01} and ζ_{02} and the wrenches due to the serial singularity of each arm ζ_{0s1} and ζ_{0s2}. As a consequence, the object has 3-DOF in this configuration.

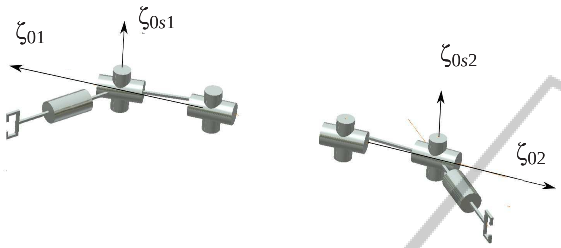

Figure 4: Constraint singularity of the dual arm system.

5.2.2 Actuation Singularities

In this section a selection criterion is given for suitable actuated joints. Once an actuator is locked, it imposes a wrench on the object. The actuator wrench for joint i is denoted as $\zeta^a_{\lambda j}$. This wrench is reciprocal to all the twists of the arm except the actuator twist itself and furthermore it should not lie in $\mathtt{W}^c$. The actuation wrench system $\mathtt{W}_a$ applied on the object is the span of actuation wrenches from both arms, namely,

$$\mathtt{W}^a = span\,(\mathtt{W}^a_r, \mathtt{W}^a_l) \quad (20)$$

The constraint wrenches and actuation wrenches of both arms should form a 6-system. This system is denoted as the global wrench system and is defined as:

$$\mathtt{W} = \mathtt{W}^c \oplus \mathtt{W}^a \quad (21)$$

An actuation singularity occurs when (21) degenerates while (16) does not. Since it is possible to actuate any four of the ten joints there are $\frac{10!}{4!(10-4)!}$ $= 210$ possible actuation schemes. By excluding non-symmetrical actuation schemes, 110 actuation schemes remain. It should be noted that since all the joints in the arm can be actuated, a redundant actuation scheme can decrease the likelihood of actuation singularities.

Example 1: $\mathbf{q}_a = [q_1\ \ q_2\ \ q_6\ \ q_7]$. Since it is generally preferable to actuate joints close to the base, the case when the base U-joints are actuated is examined. The global wrench system is derived:

- A pure force $\zeta^a_{01} \parallel \mathbf{s}_2$ and intersecting $\mathbf{s}_3, \mathbf{s}_4, \mathbf{s}_5$ is coplanar to all twists except that generated by q_1
- A pure force $\zeta^a_{02} \parallel \mathbf{s}_1$ and intersecting $\mathbf{s}_3, \mathbf{s}_4, \mathbf{s}_5$ is coplanar to all twists except that generated by q_2
- A pure force $\zeta^a_{06} \parallel \mathbf{s}_6$ and intersecting $\mathbf{s}_8, \mathbf{s}_9, \mathbf{s}_{10}$ is coplanar to all twists except that generated by q_6
- A pure force $\zeta^a_{07}, \parallel \mathbf{s}_7$ and intersecting $\mathbf{s}_8, \mathbf{s}_9, \mathbf{s}_{10}$ is coplanar to all twists except that generated by q_7

The global wrench system as illustrated in Fig. 5, is given by $\mathtt{W} = span(\zeta_{01}\ \zeta_{02}\ \zeta^a_{01}\ \zeta^a_{02}\ \zeta^a_{06}\ \zeta^a_{07})$. A line between point A and point B (the S-Joints centers of either arm) can be drawn which is intersected by all constraint forces regardless of the configuration of the robot. Thus this actuation scheme is not admissible, for any configuration the global wrench system degenerates, i.e. rank$(\mathtt{W}) = 5$, rank$(\mathtt{W}^c) = 2$, rank$(\mathtt{W}^a) =$ 4, resulting in an uncontrollable closed chain mechanism.

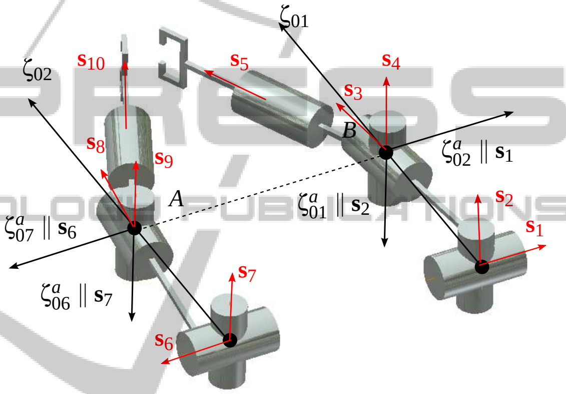

Figure 5: Non-admissible actuation scheme.

Example 2: $\mathbf{q}_a = [q_1\ \ q_2\ \ q_3\ \ q_7]$. To find the actuation wrench applied by q_3, the planes Π_{12}, Π_{45} spanned by ($\mathbf{s}_1$, $\mathbf{s}_2$) and ($\mathbf{s}_4$, $\mathbf{s}_5$) respectively, are examined:

1. if $\Pi_{12} \nparallel \Pi_{45}$, there is a pure force ζ^a_{03} acting along the line formed by the two planes
2. if $\Pi_{12} \parallel \Pi_{45}$, there is a pure moment $\zeta^a_{\infty 3}$ acting around the line normal to both two planes

The global wrench system for case 1 is shown in Fig.6, $\mathtt{W} = span(\zeta_{01}\ \zeta_{02}\ \zeta^a_{01}\ \zeta^a_{02}\ \zeta^a_{03}\ \zeta^a_{07})$. An actuation singularity occurs when the line ζ^a_{03} contains points A or B. Other singularities can be obtained by observing when the wrench system formed by two or more wrenches degenerates (Table 2), for instance when ζ^a_{02} and ζ^a_{07} become collinear.

6 CONCLUSIONS

This work has presented a study of the NAO robot's two arms engaged in a cooperative task. The DOF of an object, simultaneously grasped by both arms, was explored. The serial singularities are straightforward,

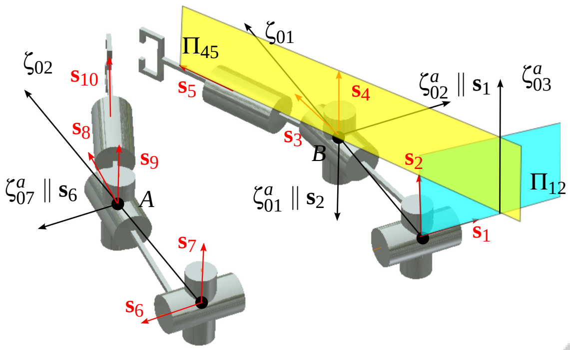

Figure 6: Admissible actuation scheme.

more interesting is the presence of constraint singularities. The issue of actuation singularities, present due to dependent joint variables, was investigated. By considering the wrenches exerted by the actuators, both an admissible and inadmissible actuation scheme were shown.

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