Modified Evolutionary Strategies Algorithm in Linear Dynamic System Identification

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Abstract: The approach to dynamic systems modelling in the form of the linear differential equation that uses only the system output and the control sample is presented. To develop a linear dynamic model as an ordinary differential equation we need to know the structure of differential equation and its order, so then it would be possible to identify parameters. It is common that measurements of the system output are distorted with a noise. In case of the non-uniform sample we would need a special output function approximation approach so the unit step function can be estimated. The dynamic system identification with an ordinary linear differential equation allows solving different control tasks, determining the system state with another control function.

1 INTRODUCTION

The solution for the given problem can be obtained with neural networks, fuzzy logic systems or other methods with universal structure. However, following models would not fit if we need an analytical form of model. There is also a possibility, in general, to build the solution using exponential, trigonometric and other functions that describe the ordinary differential equation (ODE) solution, but the control function could be given in non-analytical form. The static model that was build as an approximation with these functions is not as useful and flexible as the dynamic model. Moreover, the task would be reduced to the enumerative technique for different combination of functions, since we do not know the order of equation and multiplicity of characteristic equation roots. In article (Janiczek and Janiczek, 2010) we can see an identification method in terms of fractional derivatives and the frequency domain. The information about the plant is taken from the given frequency domain and not from the output observations. Having the model in fractional derivatives requires special control and regulation methods. We can also use stochastic difference equations as in (Zoteev, 2008), and build a model using the output observations, observations of reaction on step excitation. This approach is partially parameterized: the order and the functional relation between the system state and previous states are commonly unknown. In article (Parmar et al., 2007) the dynamic system approximation with the second order linear differential equations is examined. The coefficients are determined with the genetic algorithm. In this paper, there is the description of the structure and parameters identification task solution, reduction the identification task to the real value optimization with the modified evolutionary strategies method. The goal of approach presented in this study is finding the order of the differential equation and its parameters using only the distorted output data and the optimization technique.

2 STRUCTURE AND PARAMETERS ESTIMATION PROBLEMS

Let us have the sample $\{y_i, u_i, t_i\}$, $i = \overline{1,s}$, where $y_i \in R$ is the dynamic system output measurements at a time point t_i, $u_i = u(t_i)$ is a control action. It is also known, that the system is linear and dynamic one, so it can be described with ODE:

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$$a_k \cdot x^{(k)} + a_{k-1} \cdot x^{(k-1)} + \ldots + a_0 \cdot x = b \cdot u(t), \quad x(0) = x_0. \tag{1}$$

Here x_0 is supposed to be known. In case of the transition observing, we can put forward a hypothesis about initial conditions: the system output is known at the initial time point and the derivative values can be set to zero, if the nature of the problem is such or can be so approximated. Using the sample data, we need to identify parameters and the system order m, which is assumed to be limited, so $m \le M, M \in N$. M is a parameter that is set by the user. It is also assumed, that there is an additive noise $\xi : E(\xi) = 0, D(\xi) < \infty$, which results on output measurements:

$$y_i = x(t_i) + \xi_i. \tag{2}$$

Without loss of generality, one may assume that the system is described with following equation:

$$x^{(k)} + \frac{a_{k-1}}{a_k} \cdot x^{(k-1)} + \ldots + \frac{a_0}{a_k} \cdot x = \frac{b}{a_k} \cdot u(t) \tag{3}$$

or

$$x^{(k)} + \tilde{a}_k \cdot x^{(k-1)} + \ldots + \tilde{a}_1 \cdot x = \tilde{b} \cdot u(t). \tag{4}$$

Then we can seek the solution of the identification task as a linear differential equation with the order $m \le M, M \in N$:

$$\hat{x}^{(m)} + \hat{a}_m \cdot \hat{x}^{(m-1)} + \ldots + \hat{a}_1 \cdot \hat{x} = \hat{a}_0 \cdot u(t), \quad \hat{x}(0) = x_0. \tag{5}$$

Here the vector of equation parameters $\hat{a} = (0, \ldots, 0, \hat{a}_m, \ldots, \hat{a}_1, \hat{a}_0)^T \in R^n$, $n = M + 1$, has to deliver the extremum for the functional

$$I(a) = \sum_{i=1}^{N} \left| y_i - \hat{x}(t_i) \right|_{\hat{a}=a} \to \min_{a \in R^n}. \tag{6}$$

In general case, the solution $\hat{x}(t)$ is computed with a numerical integration method as the control function may have not analytical but algorithmic form. For the correct numerical scheme realization, let us have a coefficient restriction for the equation (3), $a_k > 0.05$. Otherwise, this parameter is going to be equal to zero, so $a_k = 0, m = m - 1$. This condition prevents extra computational efforts of the numerical evaluation scheme.

3 MODIFIED HYBRID EVOLUTIONARY STRATEGIES ALGORITHM FOR ORDINARY DIFFERENTIAL EQUATION IDENTIFICATION

The reason why the modification of an evolutionary strategies algorithm was used is that the identification problem leads to solve multimodal optimization problem. The specific representation of the equation structure results in searching not only parameters but also the structure at the same time that makes the criteria (6) complex. As a method for finding the solution for ODE identification, the hybrid modified evolutionary strategies method was developed. Original evolutionary strategies approach can be found in (Schwefel, 1995). Let every individual be represented with the tuple

$$H_i = \left\langle op^i, sp^i, fitness(op^i) \right\rangle, i = \overline{1, N_I},$$

where

$op^i_j \in R, j = \overline{1,k}$ is the set of objective parameters described the differential equation;

$sp^i_j \in R^+, j = \overline{1,k}$ is the set of method strategic parameters;

N_I is the population size;

$fitness(x) : R^k \to (0, 1], fitness(x) = \frac{1}{1 + I(x)}$ is the fitness function. The bigger fitness function value is, i.e., the fewer criterion (6) is, the more chances would have the individual to survive.

Proportional, rank and tournament selection operators were chosen as selection operator types. The algorithm produces one offspring from two parents. The population has the same size for all generations. Actually, these kinds of selection were borrowed from the conventional genetic algorithm. Let (op^1, sp^1) be the chromosome of the first parent that takes part in recombination and (op^2, sp^2) be the chromosome of the second parent. We consider different recombination types for the objective parameters (for strategic parameters it would be the same):

- intermediate crossover (here and further $i = \overline{1,n}$):

$$op_i^{offspring} = \frac{op_i^1 + op_i^2}{2};$$

- weighed intermediate crossover:

$$op_i^{offspring} = \frac{fitness(op^1)\cdot op_i^1 + fitness(op^2)\cdot op_i^2}{fitness(op^1) + fitness(op^2)};$$

- discrete crossover:

$$op_i^{offspring} = (1-z)\cdot op_i^1 + z\cdot op_i^2;$$

- randomly weighted crossover. Let $Rv = U(0,1)$ be the uniformly distributed random value:

$$op_i^{offspring} = op_i^1 \cdot Rv + op_i^2 \cdot (1-Rv).$$

The mutation of every offspring's gene is executed with the chosen probability p_m. If we have the random value $z = \{0,1\}$, $P(z=1) = p_m$, which is generated for every objective gene and its strategic parameter then

$$op_i^{offspring} = op_i^{offspring} + z\cdot N(0, sp_i^{offspring});$$

$$sp_i^{offspring} = \left| sp_i^{offspring} + z\cdot N(0,1) \right|,$$

where $N(m,\sigma^2)$ is normally distributed random value with the mean m and the variance σ^2.

We suggest a new operation that could increase the efficiency of the given algorithm. For every individual, the real value is rounded to integer. That provides searching for solutions with near the same structure. This modification is made to decrease the destructive effect of the mutation on the forming the structure.

Also for N_1 randomly chosen individuals and for N_2 randomly chosen objective gene we make N_3 iterations of the local optimization with the step h_l to determine the better solution. It is the random coordinate-wise optimization. Local optimization is executed until fitness function increases.

4 TESTING THE ALGORITHMS WITH DIFFERENT SETTINGS

To make an investigation 50 systems were generated. It means that for every order of the differential equation from the first to the ninth we have 5 different systems. Parameters of the systems were randomly generated: $\bar{a}_k^i = U(-5,5)$, $\hat{b}_k = U(-5,5)$, $i = \overline{2,10}$, $k = \overline{1,i}$, where $U(-5,5)$ is the uniform distribution. The solution of every system was found with the Runge-Kutta integration method with the step $h_i = 0.05$. The time of the process was set to 5. The control function was the step excitation and we know what was the control for every system, so $u(t) = 1$. Let $\{x_i, t_i\}$, $i = \overline{1, T/h_i}$ be the numerical solution for the system. We take $s < T/h_i$, $s = 100$ points randomly. For every system 10 runs of the algorithm were executed with every combination of its parameters. Now, to estimate the efficiency of different approaches we consider the identification without any noise.

Having different types of the selection and the crossover, we would also vary the $p_m \in \left\{\frac{1}{11}, \frac{5}{11}, \frac{1}{5}, 1\right\}$ to find out the most effective combination of the algorithm settings. As a preset we use population size in 50, number of populations in 50, $N_1 = 50$, $N_2 = 50$ and $N_3 = 1$ with $h_l = 0.05$.

Now we can compare the efficiency of following algorithms: 1 – the evolutionary strategies (ES) algorithm; 2 – ES with the local optimization, hybrid evolutionary strategies (HES); 3 – HES with modified mutation; 4 – HES with turning real numbers into integer numbers; 5 - HES with modified mutation and turning real numbers to integer ones.

After testing the algorithms on different samples of the systems, the efficient presets were found: modified HES algorithm with turning the real numbers to integer ones, 50 individuals for 50 populations, $N_1 = 50$, $N_2 = 50$ and $N_3 = 1$ with $h_l = 0.05$, the tournament selection with the tournament size 25%, the discrete crossover and the mutation with the probability $p_m = \frac{5}{11}$.

Table 1: Mean criterion values for different algorithms and system orders.

	Algorithm				
Order	1	2	3	4	5
1	0,63	0,72	0,93	0,92	0,93
2	0,69	0,73	0,74	0,79	0,85
3	0,74	0,76	0,90	0,88	0,91
4	0,69	0,79	0,99	0,98	0,99
5	0,89	0,96	0,99	0,99	0,99
6	0,76	0,80	0,82	0,83	0,86
7	0,89	0,96	0,96	0,98	0,99
8	0,85	0,89	0,93	0,91	0,93
9	0,99	0,99	0,99	0,99	0,99
10	0,99	0,99	0,99	0,99	0,99

It is important to notice that even if criterion (6) is equal to 0, it does not mean that the model has the same structure and parameters as the real system structure and parameters are. For the proper structure and parameters determination we need an adequate sample that reflects all the transient process. Let us take some stable systems that come into the steady state in time $T = 5$. In Table 2 we would make an efficiency investigation for the modified HES algorithm. 20 runs of the algorithm were made for every system. We will say that the algorithm determines the structure and parameters if $\max(\hat{a} - a) < 0.05$.

Table 2: The efficiency of "true" parameters estimation.

Order	$\hat{p}(\max(\hat{a} - a) < 0.05)$	Fitness
1	0,65	0,959344
2	0,95	0,99795
3	0,9	0,997798
4	0,95	1
5	0,8	0,996173

As we can see from Table 2, the high fitness is not the sufficient condition for the solution found to be true one. Let us highlight that for every solution found from this study for stable systems, the order was found correctly.

Now let us consider an example of the identification task solving for the system of the third order to show that even with $M = 10$ the satisfying solution can be found. Let the differential equation coefficients be $a = (0, \ldots, 0, 1, 2, 1, 2)$. With the recommended settings of the algorithm, the absolute error mean for 20 runs is 0.063. The model output, the sample and the real system output are shown on the Figure 1, found parameters are $\hat{a} = (0, \ldots, 0, 1, 2.05, 1, 2.05)$.

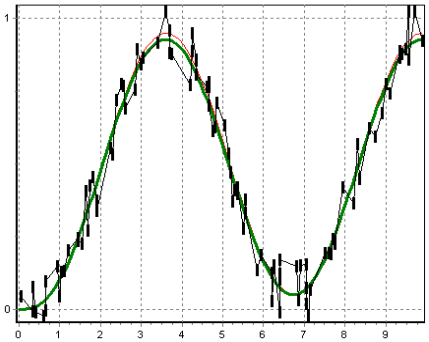

Figure 1: Measurements, model and the real object (thin line).

5 CONCLUSIONS

In this paper, the method of ordinary differential equation structure and parameters identification was described. With the proposed approach, the structure and parameters are automatically determined. Modifications of evolutionary strategies algorithm increase the accuracy of model and allow solving two tasks at the same time. It is important to note that proposed modifications allow the algorithm to find, in general, the right system order. The efficiency of the algorithm for reduced identification problem depends mostly on the sample. The better sample represents the transient process, the better it would be estimated. The further work with the approach proposed will be concentrated on investigation algorithm performance on the problems with different noise levels, sizes of the sample and different input functions.

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