### Sensitivity Analysis in Bed Capacity Studies including the Medical Staff's Decision Making

Cristina Azcárate<sup>1</sup>, Julio Barado<sup>2</sup> and Fermín Mallor<sup>1</sup>

<sup>1</sup>Department of Statistics and Operations Research, Public University of Navarre, Campus Arrosadia, Pamplona, Spain <sup>2</sup>Intensive Care Unit, Hospital of Navarre, Irunlarrea 3, Pamplona, Spain

Keywords: Simulation, Decision Making Modelling, Sensitivity Analysis, Intensive Care Unit, Bed Capacity.

Abstract: This paper deals with capacity planning studies in intensive care units (ICU). Our aim is to provide a framework in which the discharge policy from an ICU can be modelled and included in a simulation model. This is a very unique contribution of this research. We highlight the influence of the assumed policy in the ICU quality of service. A high quality of service means a low percentage of rejected patients and a length of stay in the ICU as long as necessary for the patient recovery. We introduce a parameterized set of rules to mathematically model the discharging decisions made by the physicians of an ICU. Then we present a sensitivity study carried out for the ICU of the Hospital of Navarra in Spain. The set of discharge policies is represented in the space of the performance measures to distinguish efficient from no efficient policies. Finally, the sensitivity analysis is extended, firstly, by considering variation in the number of beds and, then, by varying the patient arrival ratio.

### **1 INTRODUCTION**

The Intensive Care Unit (ICU) is a key area within hospitals caring for critically ill patients. The beds and the specialized staff of an ICU are costly resources and then the ICU managers should balance the contradictory goals of providing a high quality health service and minimizing the operating costs.

Simulation has been widely used to tackle health-care system management and operation problems. Recent reviews dealing with the application of simulation modeling in health care can be found in (Brailsford et al., 2009), (Eldabi et al., 2007), (Günal and Pidd, 2010), and (Katsaliaki and Mustafee, 2010). Many studies use simulation to analyze hospital capacity and bed allocation, but only a few deal specifically with ICUs. Among these Kim et al., in a series of papers (1999, 2000 and 2002), developed a simulation model of an ICU in Hong Kong to study the unit's capacity utilization and the quality of care provided to its patients. They also considered, (Kim et al., 2000), the conflict between ICU physicians and the operating surgeons when these last ones proposed to reserve some ICU beds exclusively for elective surgery. Authors used the simulation model to explore the implications of these bed-reservation strategies. In (Kim and

Horowitz, 2002) the analysis is extended by using a daily quota system for the elective surgery together with the knowledge of a 1-week or 2-week scheduling window. Similarly, (Kolker, 2009) also developed a simulation model to determine the maximum number of elective surgeries per day that should be scheduled in order to reduce diversion of an ICU to an acceptable low level.

(Litvack et al., 2008) analysed the bed capacity problem in the ICUs of several hospitals in a region on the Netherlands. They proposed a cooperative solution that is found by a mathematical method inspired by overflow models in telecommunication systems. Simulation is only used here to assess the quality of the provided solution.

(Ridge et al., 1998) developed a simulation model for bed planning in an ICU. They studied the relationship between the number of beds and the percentage of patients that have to be transferred because of lack of bed space. The authors performed a sensitivity analysis by varying the number of ICU beds but also by considering different admission rules by varying the planned patients deferral periods and by changing the number of beds reserved for emergency admissions.

They also pointed out that the "early discharge" of the more able patients to alternative wards is a

Azcárate C., Barado J. and Mallor F..

Copyright © 2012 SCITEPRESS (Science and Technology Publications, Lda.)

<sup>548</sup> Sensitivity Analysis in Bed Capacity Studies including the Medical Staff's Decision Making. DOI: 10.5220/0004165305480555

In Proceedings of the 2nd International Conference on Simulation and Modeling Methodologies, Technologies and Applications (HA-2012), pages 548-555 ISBN: 978-989-8565-20-4

solution commonly adopted to cope with insufficient number of ICU beds. Nevertheless, they did not included an "early discharge" operating rule within the model. Costa et al. (2003) extended these models and discuss detailed mathematical models for the number of beds required by an ICU to meet its own individual workload.

In the medical specialized literature we also find studies reporting that admissions and discharges are triaged when enough beds are not available (e. g., (Snuff et al. 2004) and (Costa et al. 2003)). As a consequence, the number of patients who are rejected from admission increases and the length of stay gets shortened.

The aim of the research presented in this paper is to provide a framework in which the discharge policy from an ICU can be modeled and used in a capacity planning study by using simulation. We will highlight the influence of the assumed policy in the ICU quality of service. We identify two dimensions for this quality of service: the first one -of a social character- is the percentage of population that can benefit from it when needed. The second one -of an individual character- is the degree of recovery reached by a patient when is discharged from the ICU. A high quality of service means a low percentage of rejected patients and a length of stay (LoS) in the ICU as long as necessary.

The paper is organized as follows. Section 2 describes the mathematical modelling of the discharging decisions made by the physicians of the ICU. In section 3 we present a sensitivity study carried out for the ICU of the Hospital of Navarra in Spain. We introduce the performance measures to be considered and represent the set of discharge policies in the space of the performance measures to distinguish efficient policies from no efficient ones. Then the sensitivity analysis is extended, firstly, by considering variation in the number of beds and, secondly, by varying the patient arrival ratio. Finally, we end the paper with a section of conclusions and final remarks about the usefulness of our approach.

### 2 MATHEMATICAL MODELLING OF DISCHARGING DECISIONS

Decisions made by the ICU doctors concerning the discharge of patients have the ultimate purpose of controlling the level of bed occupancy in the ICU, by balancing the full recovery of the current patients

and the bed availability for future ones. Then any ICU mathematical model, in general, and simulation model, in particular, developed to be used in the study of the bed capacity problem should include this doctor's ability to control the number of occupied beds.

In (Mallor and Azcárate, 2012) it is showed that it is crucial to incorporate the management decisions made by the clinical staff to obtain valid ICU simulation models. Discharge decisions are made in order to keep the number of occupied beds in levels neither too high (to compromise the incoming of new patients) nor too low (to "waste" expensive resources). Depending on the clinical situation of the patient and on the bed occupation rate at that moment, the discharge of a patient may be slightly advanced or delayed if considered safe to do it. Furthermore, there is no written protocol to automatically determine the patient discharge time; these decisions are subject to the judgment of the intensive care consultant. From a mathematical point of view, this means that the state of "enough recovery" of a patient to safely leave the ICU is not a discrete event that can be easily identified on the time axis but an ambiguous time interval subject to the intensive care physician assessment.

We took into account these considerations to model the discharge decision of a patient depending on the bed occupancy level. Specifically, we consider two kinds of discharge rules. These rules are based on the idea that the recovery of a patient is a continuous process that leads to its ICU discharge in a time that take values in an interval of admissible values (see Figure 1). The main idea is to compare the time already spent in the ICU, *TS*, with regard to the length of stay, *LoS*, which was simulated for each one of the patients occupying an ICU bed. We define the *LoSR* (LoS ratio) as *TS/LoS*. Then,

if bed occupancy level *i* is *high* and the time already spent in the ICU is sufficiently high (*specifically, if (1- LoSR) is less than a value PRi% and (LoS-TS) less than DRi days)*, a patient leaves the ICU in advance (the one in the best health condition, which means the one with the greatest *LoSR*).



Figure 1: Patient recovery and ICU discharge.

• if bed occupancy level *i* is *low* then the LoS of a patient is increased in one day with certain probability, *Pli*. There is a maximum of days *DEi* that the LoS of a patient can be extended.

These rules can be identically defined for all type of patients or they can be different for different groups of patients. For example, we could distinguish the group of programmed-surgery patients -usually whit a short LoS- and consider that it can only be shortened in one day with certain probability  $PC_i$  when bed occupancy level is *i*.

These set of rules defines infinite management policies for the ICU: one for each set of parameter values. To choose the rule that better fit the decision making process in the ICU, in (Mallor and Azcárate 2012), we formulated an optimization problem with the aim of matching as much as possible the output of the simulation with the ICU historical data (see Figure 2).

The decision variables are the parameters PR, DR, PI, DE and PC above defined. Constraints represent realistic monotonous relationships into each set of parameters and upper bounds for their values (uPR, uDR, uPI, uDE and uPC). The number of ICU beds is denoted by n and by k1 and k2 the boundaries for low and high occupancy levels, respectively. We set as objective function to minimize the squared differences of both occupancy bed frequency distributions: the one observed in the real ICU,  $real_freq(i)$ , and the one obtained from the simulation output, *simul\_freq(i)*. The proposed optimization problem can be solved by combining simulation and optimization techniques. The optimizer produces a sequence of solutions whose performance is tested in the simulator.

$$\begin{array}{ll} Min & \sum_{i=0}^{n} \left( real \_ freq(i) - simul \_ freq(i) \right)^{2} \\ subject & to \\ \left\{ \begin{array}{l} uDR \ge DR_{n} \ge DR_{n-1} \ge \ldots \ge DR_{n-k2} \ge 0 \\ uPR \ge PR_{n} \ge PR_{n-1} \ge \ldots \ge PR_{n-k2} \ge 0 \\ uDE \ge DE_{1} \ge \ldots \ldots \ge DE_{k1} \ge 0 \\ uPI \ge PI_{1} \ge \ldots \ldots \ge PI_{k1} \ge 0 \\ uPC \ge PC_{n} \ge PC_{n-1} \ge \ldots \ge PC_{n-k2} \ge 0 \\ DR_{n}DI_{i} & \text{integer } i = n - k2, \ldots, n \ j = 1, \ldots, kl, \ kl \le k2 \end{array} \right.$$

Figure 2: Optimization problem to determine the parameter values of the medical management rules.

### **3** SENSITIVITY ANALYSIS

The discharge policy is influenced by the occupancy level which also depends on the bed availability and the input rate of patients. Then a bed capacity study should take into account variability in management policies. We show in this section that main performance measures greatly depend on the adopted policy.

### 3.1 Case Study. Parameterization of The Rules

In a first step of our research we developed a simulation model that included the representation of the medical decisions made at the ICU of the Hospital of Navarre, in Spain, following the methodology presented in Section 2.

The Hospital of Navarra is a general public hospital with reference specialties in the Community of Navarra (Neurosurgery, cardiac surgery, vascular surgery, oncology, infectious diseases, etc.). It has 483 beds, 2015 members of staff and 10 surgery rooms. The ICU of this hospital has 20 beds and 86 physicians and nurses. It receives patients from 3 sources (emergency, operating theatre and ward).

A thorough data analysis was conducted to obtain good statistical models for the arrival pattern and LoS of each of the 8 groups of patients considered. The necessary data for the statistical estimation were recorded and provided by the Hospital administration. We used two files: a patient file and a bed occupancy file, containing 9 years of data. The patient file includes all records of patients attended in the ICU. For each patient the following variables are known: age, arrival date, illness group (8 groups were considered), output date, APACHE (illness severity), infections in the ICU, and exitus (recovered or died). The bed occupancy file records the number of occupied beds at 4 p.m., each day. It was used to validate the simulation model.

Cardiac surgery patients are special patients in the performance of this ICU and represent 1/3 of the total amount of patient arrivals.

We simulate the ICU model under different rules to analyse their influence in a set of performance measures. In order to make an easier comparison of the results we simplify the structure of values that can take the decision rule's parameters. We distinguish two levels for the state of high bed occupancy:

- moderate high occupancy (75%-85%): when there are a number of occupied beds from 15 to 17. It is denoted by level *h1*, and
- very high occupancy (+85%): when there are a number of occupied beds from 18 to 20. It is denoted by level h2.

Low occupation levels are also reduced: level *l1* from 1 to 8 beds, and level *l2*, from 9 to 13 beds.

In the presentation of the results we focus on the decisions concerning the situation of high occupancy, because they are more difficult to manage and more important for the patient health. We consider the following parameters related with early discharges:

- *PC<sub>hj</sub>*: the probability of reduction in 1 day the LoS of cardiac surgery patients, with normal post-surgery evolution, when bed occupancy level is *hj*, for *j*=1,2.
- *PR<sub>hj</sub>*: the percentage of reduction in LoS when bed occupancy level is *hj*, for *j*=1,2.
- DR<sub>hj</sub>: upper limit for the reduction in the number of LoS days when bed occupancy level is hj, for j=1,2.

Observe that the last four parameters do not affect to cardiac surgery patients, with normal post-surgery evolution.

Consequently, an early discharge policy is described by a vector of 6 values: two for the cardiac surgery patients and four for the other patient groups.

We also consider the following four parameters related with extended discharges:

- *PI<sub>lj</sub>*: the probability of one-day increase in LoS when bed occupancy level is *lj*, for *j*=1,2.
- *ER<sub>lj</sub>*: upper limit number of LoS days increased when bed occupancy level is *lj*, for *j*=1,2.

#### 3.2 Efficient Discharge Policies

Two are the main objectives of an ICU: it should provide service to all patients that can benefit from it and it should provide a full service to all admitted patients. The first objective means that no patient should be rejected because the ICU is full and the second one means that no LoS should be reduced, risking a full recovery, because the occupied bed is needed. Based in these two objectives, two performance measures can be defined:

- *percentage of rejected patients.* Emergency patients arrive at random and they are transferred to other hospitals -which is no desirable- if they cannot be immediately admitted. In the past, the lack of beds caused to postpone surgeries. Nowadays, no surgery is canceled due to the lack of operating rooms, and the patients are also transferred to other health facilities in the region, if necessary.
- percentage of shortened days. To calculate this measure the truncated LoS and the "ideal" LoS for each patient should be known. This

information is not included in ICU databases. At least this is not reported in the literature neither the ICU of the Hospital of Navarra records it. Nevertheless, this information is collected from the simulation model, because a LoS -which is considered as the "ideal" LoS - is simulated from the estimated statistical model, when a patient is created. The performance measure is obtained as the ratio of the sum of the shortened days for all patients to the sum of the LoSs of all patients.

To conduct the sensitivity analysis, we vary the value of some parameters into different ranges while other parameters have a fixed value. Varying and fixed parameters are included in tables 1 and 2, respectively.

Table 1: Set of variation for parameters  $PR_{hi}$  and  $PC_{hi}$ .

Parameters	Set of values	
$PR_{hj}$ for $j=1,2$	0, 5, 10, 15, 20	
$PC_{hj}$ for $j=1,2$	0, 10, 20, , 90, 100	

Table 2: Parameters with fixed values.

Parameters	Fixed values	
$DR_{h1}$	2	
$DR_{h2}$	3	
$PI_{l1}$	0	
$PI_{l2}$	15	
$ER_{lj}$ for $j=1,2$	2	

Observe that monotonicity conditions imply that  $PC_{h2} \ge PC_{h1}$  and  $PR_{h2} \ge PR_{h1}$ . This leads to 15x66=990 different combinations of values for parameters  $PR_{h1}$ ,  $PR_{h2}$ ,  $PC_{h1}$  and  $PC_{h2}$ . Each combination denoted by  $(PR_{h1}, PR_{h2}, PC_{h1}, PC_{h2})$  defines a different discharge policy. For example, (5, 10, 30, 60) means that in case of occupancy level from 15 to 17, with probability 0.3, the LoS of a cardiac surgery patient is shortened in one day while, for the rest of patients, in a maximum of 5%. In case of bed occupancy level from 18 to 20, the probability of one-day reduction increases to 0.6 and the maximum of LoS reduction increases to 10%.

For each scenario, 100 replications of 50-year simulation experiments were run, with a 3-year warm-up period.

Figure 3 shows the representation of the 990 different discharge policies in the objective space. Policies are grouped by the values of the  $PR_{hj}$  parameters defining the discharge for a general patient (that is, a patient of any other group different of cardiac surgery). We see that the set of all

SIMULTECH 2012 - 2nd International Conference on Simulation and Modeling Methodologies, Technologies and Applications



Figure 3: Discharge policies for the present situation: 20 beds.

policies approximately fills a triangle and also the policies within each group. We can analyse the trade-off between both objectives. The policy (0, 20, 0, 100) located in one of the corners provides a percentage of 1.5% of rejected patients and a percentage of shortened stay of 1.8%. Any movement from this policy to other policy to get a reduction of the rejected patients implies a sharp increase in the shortened stay: from the point (1.8, 1.5) (associated to policy (0, 20, 0, 100)) to the point (4.9, 1.4) (associated to policy (20, 20, 100, 100) in the right corner) there is a line of slope -0.03, over which is located a piece of the Pareto frontier. Then for each unit added to the percentage of shortened stay the percentage of rejected patients is reduced only by 0.03 units. Reasoning in the same way, by using the policy (0, 0, 0, 0), located in the upper corner and with associated point (3.3, 0), we get that reducing by one unit the percentage of shortened stay implies to increase, approximately (and in the best of the cases) by one unit the percentage of rejected patients. Figure 4 shows the same graphical representation of Figure 3 but now distinguishing among efficient and no efficient policies. There are 98 efficient policies, which are included in table 3 (see appendix).



Figure 4: Discharge policies: efficient vs. no efficient.

## 3.3 Bed Capacity Analysis: Increase in the Number of Beds

Simulation models in health care are frequently used to assist decision makers in capacity analysis, that is, the optimal number of resource units to achieve a determined objective. We have seen in the previous subsection that the management policy is critical to reach the desired levels for each objective when there are conflicting objectives. We know that the answer of a *what if* question of type, "*what is the performance of the ICU when one bed is added?*", will depend on the discharge policy applied by the medical staff. To answer this question we have adapted the decision rules to the case of 21 beds. The moderate high occupancy level h1 is now defined when there are a number of occupied beds from 16 to 18 and the very high occupancy level, h2, when there are a number of occupied beds from 19 to 21. The set of parameters for the rules does not change but we reduce the number of possible values for the percentage of shortened stay to 3: {0, 10, 15}. The number of policies tested is 396.

We obtain a structure for the representation of the policies in the objective space similar to the case of 20 beds. Figure 5 compares both sets of efficient solutions. We observe that the efficient frontier for the 21- bed case is almost a translation of the efficient frontier for the 20-bed case. Each policy with 21 beds dominates the equivalent with 20 beds. That is, when the resource is increased and the same policy is applied then both objectives are improved at the same time. In both cases, in the corner, is the policy (0, 20, 0, 100). Its associated point in the 21bed case, is (1.24, 0.95) and the objective values reached by the policies (0,0,0,0) and (20, 20, 100, 100) are (0, 2.23) and (3.73, 0.83), respectively. Then when we moved out of (0, 20, 0, 100), per each unit of improvement in the shortened stay, we need to increase approximately by one unit the percentage of rejected patients. And conversely, if we wish to

improve the 0.95% rejected patients we would worsen the shortened stay in one unit to get a decrease of 0.05 in the rejected percentage.

# 3.4 Bed Capacity Analysis: Increase in the Arrivals Rate and the Number of Beds

When the capacity analysis is done to determine the resources needed to provide a service in the future it is necessary to include in the model a prediction of the future demand.

In this subsection we study the performance of the ICU under the hypothesis that the patient's arrival ratio is increased due to the increase and the aging process of the population. We consider three different scenarios: increments of 5%, 10% and 15% in the arrivals rate. These three scenarios are studied with both the current capacity and the increased capacity to 21 beds. Figure 6 shows the 6 Pareto frontiers. We observe a shape for these frontiers similar to the one we found in the analysis of the present arrival rate case. Again in the corner is the policy (0, 20, 0, 100) and the trade-offs between objectives keep similar proportions to those found in section 3.2 and 3.3. The efficient frontier corresponding to a stress of 1.05 and 21 beds is similar to the results found for 20 beds in the present arrival rate. This is because both the service capacity and the service demand are increased by a 5%. This



Figure 5: Efficient policies for the ICU with 20 and 21 beds.

SIMULTECH 2012 - 2nd International Conference on Simulation and Modeling Methodologies, Technologies and Applications



Figure 6: Efficient policies for the ICU with different levels of stress in the arrival rates (20 and 21 beds).

argument also justifies the closeness of frontiers corresponding to scenario 20 beds and stress 1.05 and scenario 21 beds and stress 1.10.

### 4 CONCLUSIONS AND FINAL REMARKS

In this paper we have shown that a bed capacity analysis in an ICU requires the consideration of the discharge policy that is applied by the medical staff. The main effect of this management policy is a reduction in the LoS of some patients. Then the percentage of total shortened stay is considered as a performance measure to catch the degree of intervention of the medical staff when there is a high pressure due to lack of beds. This new measure is studied together with the traditional performance measure of percentage of rejected patients.

The doctors can find useful the representation of the discharging policies in the space of goals to learn about the trade-off between objectives that can be achieved by modifying the parameters of the rules. These rules can be interpreted and used by the doctors as bench marks for their own decision processes. This constitutes a normative approach to the discharging policies in the sense that it indicates how to proceed to get certain levels of quality of service.

We have also simulated the ICU under the rules estimated according to the methodology exposed in section 2, that is, under the rules that better describe the decision process in the real ICU. The representation of the results in the space of goals showed that the point is very close to the Pareto frontier; specifically, it is located close to the corner point, in the upper part. Thus, we can conclude that the medical staff makes decisions almost efficiently according to both objectives.

#### REFERENCES

- Brailsford, S. C., Harper, P. R., Patel, B., Pitt, M., 2009. An analysis of the academic literature on simulation and modelling in health care, *Journal of Simulation*, Vol. 3, 130-140.
- Costa, A. X., Ridley, S. A., shahani, A. K., Harper, P. R., De Senna, V., Nielsen, M. S. 2003. Mathematical modelling and simulation for planning critical care capacity, *Anaesthesia*, Vol. 58, 320-327.
- Eldabi, T., Paul, R. J., Young, T., 2007. Simulation modelling in healthcare: reviewing legacies and investigating futures, *Journal of the Operational Research Society*, Vol. 58, 262 -270.
- Günal, M. M., Pidd, M., 2010. Discrete event simulation for the performance modelling in health care: a review of the literature, *Journal of Simulation*, Vol. 4, 42-51.

- Katsaliaki, K., Mustafee, N., 2010. Applications of simulation within the healthcare context, *Journal of* the Operational Research Society, Vol. 62, 1431-145.
- Kim, S. C., Horowitz, I., Young, K., T. A. Buckley,1999. Analysis of capacity management of the intensive care unit in a hospital, *European Journal of Operational Research*, Vol. 115, 36-46.
- Kim, S. C., Horowitz, I., Young, K., Buckley, T. A., 2000. Flexible bed allocation and performance in the intensive care unit, *Journal of Operation Management*, Vol. 18, 427-443.
- Kim, S. C., Horowitz, I., 2002. Scheduling hospital services: the efficacy of elective-surgery quotas, *Omega*, Vol. 30, 335-346.
- Kolker, A., 2009. Process modeling of ICU patient flow: effect of daily load levelling of elective surgeries on ICU diversion, *Journal of Medical Systems*, Vol. 33, 27-40.
- Litvack, N., van Rijsbergen, M., Boucherie, R. J., van Houdenhoven, M., 2008. Managing the overflow of intensive care patients, *European Journal of Operational Research*, Vol. 185, 998-1010.
- Mallor, F., Azcárate. C., 2012. Combining optimization with simulation to obtain credible simulation models for Intensive Care Units. *Annals of Operations Research.* DOI: 10.1007/s10479-011-1035-8.
- Ridge, J. C., Jones, S. K., Nielsen, M. S., Shahani, A. K., 1998. Capacity planning for intensive care units, *European Journal of Operational Research*, Vol. 105, 346-355.
- Sinuff T., Kahnamoui K., Cook D. J., Luce J. M., Levy M. M., 2004. Rationing critical care beds: a systematic review, *Critical Care Medicine*, Vol. 32, 1588-1597.

### APPENDIX

In table 3, we include the 98 efficient discharge policies obtained when considering the 20-bed ICU with the present patient arrival ratio. Each policy is represented by a vector with four components:  $(PR_{h1}, PR_{h2}, PC_{h1}, PC_{h2})$ .

Table 3: Efficient discharge policies. The policies are ordered from the top left to the bottom right of the objective space.

0	0	0	0
0	0	0	10
0	0	0	20
0	0	0	30
0	0	0	40
0	0	0	50
0	0	0	60
0	0	0	70
0	0	0	80
0	0	0	90
0	0	0	100
0	0	0	80
0	5	0	0
0	5	0	50
0	5	0	60
0	5	0	70
0	5	0	80
0	5	0	90
Ö	5	Ö	100

Table 3: Efficient discharge policies. The policies are ordered from the top left to the bottom right of the objective space (cont.).

