

On the Strategy to Follow for Skeleton Pruning

Sequential, Parallel and Hybrid Approaches

Maria Frucci, Gabriella Sanniti di Baja, Carlo Arcelli and Luigi P. Cordella
Institute of Cybernetics "E.Caianello", CNR, Via Campi Flegrei 34, Pozzuoli (Naples), Italy

Keywords: Shape Representation, Skeleton, Pruning.

Abstract: Pruning is an important step in a skeletonization process and a number of pruning criteria have been suggested in the literature. However, the modality to be followed when checking the pruning criterion is not generally described in detail. In our opinion, two main pruning modalities can be envisaged and in this paper we discuss their impact on the performance of pruning. Moreover, we introduce a third modality, which we regard as able to provide a more satisfactory pruning performance.

1 INTRODUCTION

The skeleton of an object is one of the most popular medial representations and is useful for shape analysis. A number of papers exist in the literature proposing skeletonization methodologies or dealing with the use of the skeleton for practical applications, e.g., see (Siddiqi and Pizer, 2008). However, a factor limiting the use of the skeleton for applications is its sensitivity to deformations along the boundary of the object. In fact, even negligible noise along the boundary may cause spurious branches in the skeleton. Thus, proper techniques able to remove scarcely significant skeleton branches, or to prevent their creation, are of interest.

In general, skeleton branches are expected in correspondence with regions of the object that are perceived as individually meaningful. In particular, peripheral branches are expected in correspondence with limbs and smooth boundary convexities. However, the structure of the skeleton may be very complex, especially if continuous skeletonization methodologies, such as those based on the Voronoi diagram (Ogniewicz and Kubler, 1995), are used. Thus, a one-to-one correspondence between skeleton branches and object regions may not be satisfied. Pruning is aimed at removing scarcely significant peripheral branches so that the previous correspondence can be established.

The most commonly employed criteria to evaluate the significance of skeleton branches and accordingly perform pruning were extensively discussed in (Shaked and Bruckstein, 1998) and deal

with propagation velocity, maximal thickness, radius function, axis arc length, and the boundary/axis length ratio. More recently, new pruning methodologies have been suggested in (Bai et al., 2007) and in (Shen et al., 2011) dealing with contour partitioning via discrete curve evolution and with bending potential ratio, where pruning can be accomplished during a post-processing phase, or can be integrated into the skeleton computation process.

Generally, any significance criterion aims at establishing a strict relation between a skeleton branch and the relevance of the object part the branch represents. Branch removal caused by pruning modifies the skeleton in such a way that the object it represents turns out to differ from the original object for a smoother boundary or for the number of protrusions. A pruning process is adequate if these differences are negligible in the framework of the specific application.

An aspect that has not received enough attention in the literature is the modality that is followed when pruning is seen as a post-processing phase, after the skeleton has been computed. Another aspect that is not taken into account is that, due to branch removal, branches that were internal in the original skeleton are likely to be transformed into peripheral branches. Since the new peripheral branches may be not significant, pruning may need to be iterated. Thus, the problem of establishing how many times pruning can be iterated has to be faced to avoid that the structure of the skeleton be excessively simplified.

In this paper we discuss the performance of pruning methods by taking into account both the

above aspects. We do not introduce new criteria to evaluate the significance of skeleton branches, but investigate how the application of a given pruning criterion may condition the obtained results. Reference to a specific criterion will be done only to exemplify the different modalities of pruning.

2 PRELIMINARY NOTIONS

The skeleton S of a digital object P is a subset of P consisting of the union of curves symmetrically placed within P and characterized by the same topology of P . Each point of S (a pixel in 2D and a voxel in 3D) is labeled with the value of its distance from the complement of P . A point p of S is an end point if it has only one neighbor in S , a normal point if it has two neighbors in S , and a branch point if it has more than two neighbors in S .

A skeleton branch is a curve of S entirely consisting of normal points, except for the two extremes of the curve that are end points or branch points. The only case in which a skeleton branch can be delimited by two end points is when the skeleton consists of a single branch. Such a simple case of skeleton structure is not of interest in the framework of pruning. A skeleton branch delimited by one end point and one branch point is a peripheral branch. A skeleton branch delimited by two branch points is an internal branch.

In an ideal skeleton, branches should correspond to perceptually meaningful object regions, while the skeleton S of P includes a generally much larger number of branches, some of which originating from noisy convexities along the boundary of P . Therefore, pruning can be done to remove scarcely significant branches, so favoring the similarity between S and the ideal skeleton.

Pruning consists in removing, partially or totally, skeleton branches from S . It should be accomplished exclusively on peripheral branches to guarantee that the pruned skeleton is characterized by the same topology as the original skeleton and, hence, the same topology as P . Due to the linear structure of S , pruning can be efficiently implemented by resorting to skeleton tracing, starting from the end points.

3 PRUNING APPROACHES

To decide whether the peripheral branch currently traced should be pruned, suitable criteria are necessary to evaluate the perceptual relevance of the

object region mapped into that skeleton branch. The pruning criterion can be checked on the whole peripheral branch, so as to establish whether the entire branch should be removed or should be kept in S . Alternatively, while tracing the branch pruning can remove skeleton points one after the other as far as the pruning criterion is satisfied, leading to a possibly partial skeleton branch removal.

Let N be the number of end points (hence of peripheral branches) of S . During the same iteration of pruning, all peripheral branches are analyzed.

Total removal of a peripheral branch may cause a point of S , classified as branch point in the initial skeleton, to be transformed into a normal point. Thus, two different pruning modalities (here called parallel and sequential modalities) can be considered. If the parallel modality is followed, points classified as branch points in the initial skeleton maintain their status until all peripheral branches have been examined and possibly pruned. If the sequential modality is followed, the branch point status is updated as soon as a branch point is reached by pruning. Thus, during an iteration of pruning according to the parallel modality, all peripheral branches can be removed at most up to the branch points delimiting them in the initial S . In the sequential modality, a peripheral branch B delimited by the branch point b_1 in the initial S , may be pruned until a branch point b_2 , more internal than b_1 in the initial S . This happens if the status of b_1 has been transformed from the status of branch point to that of normal point before analyzing the branch B , due to removal of other already examined peripheral branches meeting into b_1 .

Once the N peripheral branches have been examined and those satisfying the pruning criterion have been removed, the skeleton structure results to be modified. Let M be the number of end points characterizing the pruned skeleton. Obviously, it is $M \leq N$. Some of the M peripheral branches can originate from end points already existing in the initial skeleton. The delimiting branch points of these peripheral branches may differ from those delimiting the corresponding branches in the initial skeleton. Some of the M peripheral branches can originate from new end points that in the initial skeleton were classified as branch points. A second iteration of pruning can then be accomplished by considering the M peripheral branches. Pruning can be iterated producing at each iteration a skeleton with a structure simpler than that of the skeleton obtained at the previous iteration.

To our knowledge, no discussion has been done in the literature regarding both the modality

followed when pruning the skeleton, and the number of iterations necessary to get a satisfactory result. In this paper, we are interested in discussing both the above aspects. Do parallel and sequential modalities produce the same results? If not, is one of those modalities preferable? How could we establish the number of iterations of pruning that sufficiently well simplify the structure of S while still producing a satisfactory shape representation?

Obviously, the performance of pruning strongly depends on the goodness of the pruning criterion. However, independently of the selected criterion, we think that the parallel and sequential modalities are likely to produce different results. Since we are not interested in judging the goodness of pruning criteria, but only in showing that the selected modality has an impact on the results, we may use a very simple pruning criterion. To this aim, we note that among the points of S , a crucial role is played by the centers of maximal balls, CMBs, (Pfaltz and Rosenfeld, 1967). In fact, the union of the balls associated to the CMBs of S recovers the object. Then, we use a simple pruning criterion based on the ratio R between the number of CMBs in a peripheral branch and the total number of points in the branch. The rationale is that the larger is the percentage of CMBs in a branch, the higher is the representative power of that skeleton branch. The proper value of the threshold ϑ for R should be fixed depending on the problem at hand. In this paper, we set $\vartheta=0.4$.

Since we are also interested in finding a way to determine the proper number of iterations for pruning, we consider pruning that either removes a whole peripheral branch or keeps it in the skeleton. In fact, in order pruning can be iterated, necessarily some initial branch points have to be transformed into new end points and this is not guaranteed when partial skeleton branch removal is considered.

In our opinion, pruning in sequential modality is likely to be more conservative as far as preserving shape information is concerned. Its main drawback is that the result is conditioned by the order in which branches are examined. By changing the branch inspection order, the delimiting branch point for the currently traced peripheral branch may be more or less internal in S . The order also conditions the number of possible further iterations.

As for the parallel modality, the result is obviously independent of the order in which peripheral branches are examined. The main problem occurs when all peripheral branches meeting in common branch points are pruned and pruning is iterated. In fact, some of the end points in the pruned skeleton were branch points in the initial

skeleton, but the pruning criterion is checked only for the branches that are peripheral at the current iteration. Thus, the relevance of an object region mapped into a subset of the initial skeleton, whose branches are pruned at different iterations, is not correctly evaluated. As a consequence, successive iterations may cause an additive negative impact on the representative power of the skeleton.

For illustrative purposes, let us refer to a 2D case and consider the object in Figure 1 left, where the skeleton is shown superimposed on the object. The result after one iteration of pruning done according to the parallel modality is shown in Figure 1 middle left. The results obtained at the end of the first iteration when following the sequential modality, and by selecting a different order for tracing the peripheral branches are shown in Figure 1 middle right and Figure 1 right, respectively. We observe that the obtained results are different notwithstanding the same pruning criterion based on the ratio R and the same value for the threshold ϑ have been adopted.

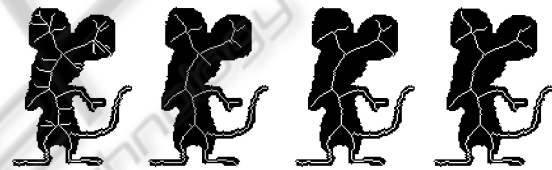


Figure 1: From left to right, the initial skeleton and the pruned skeletons obtained with different modalities.

4 HYBRID APPROACH

We think that a possible solution to the drawbacks of sequential and parallel pruning modalities can be obtained by following an hybrid approach that mixes the sequential and parallel modalities in such a way to take the benefits of both. We suggest that the branch point status is not updated during the current iteration. We also suggest that if all peripheral branches meeting in a common branch point satisfy the pruning criterion, the peripheral branch characterized by the highest relevance is not removed. By postponing branch point status updating, we exploit the good feature of parallel pruning that the result is not influenced by the order in which branches are examined. By keeping in S the most relevant branch, we exploit the positive performance of sequential pruning. In fact, at the end of each iteration some branches always exist that originate from end points present in the initial skeleton, so that the negative additive impact on the

representative power of the skeleton is prevented. As an example, see Figure 2, where the initial skeleton and the results of pruning after one iteration, when following the parallel approach, the sequential approach (with two different inspection orders) and the hybrid approach are shown from left to right. The pruning criterion involving the ratio R and the value $\vartheta=0.4$ for the threshold have been used.



Figure 2: From left to right, the initial skeleton and the pruned skeletons obtained by following parallel, sequential and hybrid approaches.

Once the current iteration is completed, end points and branch points in the pruned skeleton are identified and a new iteration can be accomplished. Actually, pruning iterations are accomplished until the pruning criterion is not satisfied by any peripheral branch, so leading to a pruned skeleton that, in the limits of the adopted threshold, has simple structure and adequate representative power.

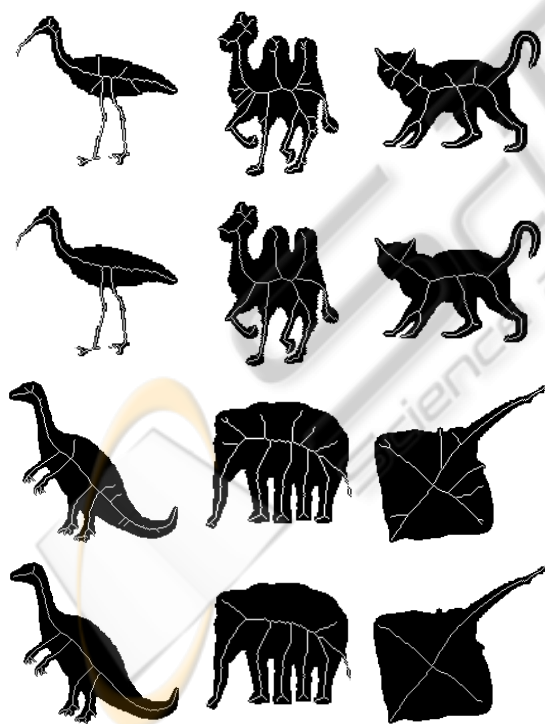


Figure 3: Initial skeletons (odd lines) and skeletons pruned by following the hybrid approach (even lines).

We have checked the above pruning criterion on a number of digital objects by following the parallel,

sequential and hybrid modalities. Sometimes the results obtained by the hybrid approach were equal to those obtained by the sequential method in a given branch inspection order; sometimes the hybrid and the parallel approaches provided the same results. Sometimes the results provided by the three approaches were all different. Though the differences in the results are not so large to clearly show the supremacy of one of the three modalities, we think that the hybrid approach should be preferred since it is less affected by drawbacks.

A few examples of the performance of pruning involving the ratio R with $\vartheta=0.4$, and accomplished by the hybrid approach are shown in Figure 3.

5 CONCLUSIONS

In this paper we have discussed the performance of skeleton pruning accomplished by following parallel and sequential modalities. We have also introduced a hybrid modality that, in our opinion, overcomes the drawbacks of the parallel and sequential approaches.

We have used a simple pruning criterion since we are interested in showing that the selected modality has an impact on the results. Our future work will deal with checking whether the hybrid approach can still be seen as preferable with respect to the sequential and parallel approaches even if using more sophisticated pruning criteria.

REFERENCES

Siddiqi, K., Pizer, S. M., (Eds.), 2008. *Medial Representations: Mathematics, Algorithms and Applications*. Springer, Berlin.

Ogniewicz, R. L., Kubler, O., 1995. Hierarchic Voronoi Skeletons. *Pattern Recognition* 28(3), 343-359.

Shaked, D., Bruckstein, A. M., 1998. Pruning medial axes. *Computer Vision and Image Understanding* 69(2), 156-169.

Bai, X., Latecki, L. J., Liu, W-Y., 2007. Skeleton pruning by contour partitioning with discrete curve evolution. *IEEE Trans. PAMI* 29(3), 449-462.

Shen, W., Bai, X., Hu, R., Wang, H., Latecki, L.J., 2011. Skeleton growing and pruning with bending potential ratio. *Pattern Recognition* 44, 196-209.

Pfaltz, J. L., Rosenfeld, A., 1967. Computer representation of planar regions by their skeletons. *Comm. ACM* 10(2), 119-125.