

Using the Expanded IWO Algorithm to Solve the Traveling Salesman Problem

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Abstract: The Invasive Weed Optimization algorithm (IWO) is an optimization metaheuristic inspired by dynamic growth of weeds colony. The authors of the present paper have expanded the strategy of the search space exploration of the IWO algorithm introducing a hybrid method along with a concept of the family selection applied in the phase of creating individuals. The goal of the project was to evaluate the expanded IWO version (exIWO) as well as the original IWO by testing their usefulness for solving some test instances of the traveling salesman problem (TSP) taken from the TSPLIB collection which allows to compare the experimental results with outcomes reported in the literature. The results produced by other heuristic algorithms as well as the methods based on the self-organizing maps served as the reference points.

1 INTRODUCTION

The Invasive Weed Optimization (IWO) algorithm is an optimization metaheuristic, in which the exploration strategy of the search space (similarly to the evolutionary algorithm) is based on the transformation of a complete solution into another one. The authors of the original version of the algorithm from University of Tehran were inspired by observation of dynamic spreading of weeds and their quick adaptation to environmental conditions (Mehrabian and Lucas, 2006).

Usefulness of the IWO was confirmed for both continuous and discrete optimization tasks. The research was focused *inter alia* on minimization of the multimodal functions and tuning of a second order compensator (Mehrabian and Lucas, 2006), antenna configurations (Mallahzadeh et al., 2008), electricity market dynamics (Sahraei-Ardakani et al., 2008), a recommender system (Sepehri Rad and Lucas, 2007), and the join ordering problem for database queries (Kostrzewa and Josiński, 2011).

The goal of the present paper is to introduce an expanded version of the IWO (exIWO) distinguished by the hybrid strategy of the search space exploration proposed by the authors. Evaluation of the suggested modification is based on the solution of some test instances of the traveling salesman problem (TSP) taken from the TSPLIB

collection (Reinelt, 1991) of the Research Group Discrete and Combinatorial Optimization at the Ruprecht-Karls-Universität Heidelberg (available at www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/tsp/).

The overview of bibliography describing the methods for solving the TSP would be unusually spacious. Numerous studies related to the usage of exhaustive, greedy, and evolutionary methods were mentioned in (Michalewicz and Fogel, 2004), whereas the IWO algorithm, according to the authors' knowledge, has never been used to this purpose by other researchers.

The organization of this paper is as follows – Section 2 contains a brief description of the modified IWO algorithm taking into serious consideration the proposed hybrid method of the search space exploration. Discussion of the transformations used for creation of a new individual in case of the TSP is presented in Section 3. Section 4 deals with procedure of the experimental research along with its results. The conclusions are formulated in Section 5.

2 DESCRIPTION OF THE EXPANDED IWO ALGORITHM

The simplified pseudocode mentioned below describes the exIWO algorithm by means of

terminological convention consistent with the “natural” inspiration of its idea. Consequently, the words “individual”, “plant”, and “weed” are treated as synonyms.

Create the first population.

For each individual:

 Compute the value of the fitness function.

While the stop criterion is not satisfied:

For each individual from the population:

 Compute the number of seeds.

For each seed:

 Choose the dissemination method.

 Create a new individual.

 Compute the value of its fitness function.

 Create a new population.

The population of initial solutions of the given optimization task is randomly or greedily generated over the search space. Next, the degree of individuals’ adaptation to environmental conditions is estimated by the value of their fitness function, which at the same time determines the number of seeds produced by each plant according to the following formula:

$$S_{ind} = S_{min} + \left[(f_{ind} - f_{min}) \frac{S_{max} - S_{min}}{f_{max} - f_{min}} \right], \quad (1)$$

where S_{max} , S_{min} denote maximum and minimum admissible number of seeds generated, respectively, by the best population member (characterized by fitness f_{max}) and by the worst one (fitness f_{min}).

The seeds are scattered over the search space. The hybrid strategy of the search space exploration, proposed by the authors of the present paper, makes use of the following component methods: dispersing, spreading and rolling down. Probability values of selection assigned to the particular methods form parameters of the algorithm.

Construction of a new individual according to the *dispersing* method is based on transformations (see Section 3) performed on the copy of the parent individual. The number of transformations equals to the conventional distance between the parent individual and the descendant in the search space. The distance is described by normal distribution with a mean equal to 0 and a standard deviation truncated to nonnegative values. The standard deviation is decreased in each algorithm iteration (i.e. for each population) and computed for the

iteration $iter$, $iter \in [1, iter_{max}]$ according to the following formula:

$$\sigma_{iter} = \left(\frac{iter_{max} - iter}{iter_{max}} \right)^m (\sigma_{init} - \sigma_{fin}) + \sigma_{fin}. \quad (2)$$

The total number of iterations ($iter_{max}$), equivalent to the total number of populations, can be either used with the purpose of determination of the stop criterion or can be estimated based on the stop criterion defined as the execution time limit. The symbols σ_{init} , σ_{fin} represent, respectively, initial and final values of the standard deviation, whereas m is a nonlinear modulation factor. A tendency to gradual reduction of the distance for subsequent populations resulting from the formula (2) accords with intention of the authors of the original IWO algorithm version.

The *spreading* is a random disseminating seeds over the whole of the search space. Therefore, this operation is equivalent to the random construction of new individuals.

The *rolling down* is based on the examination of the neighborhood of the parent individual. In case of discrete optimization task the neighborhood comprises individuals that differ from the parent by exactly one transformation, whereas for the continuous optimization the term “neighbors” stands for individuals located at the same randomly generated distance from the considered one. The best adapted individual is chosen from among the determined number of neighbors, whereupon its neighborhood is analyzed in search of the next best adapted individual. This procedure is repeated k times (k is a parameter of the method) giving the opportunity to select the best adapted individual found in the k -th iteration as a new one.

Creation of the next population is based on the concept of the *family selection*. Each plant from the first population is a protoplast of a separate family. A family consists of a parent weed and its direct descendants. According to the family selection rules only the best individual of each family survives and becomes member of the next population.

3 ADAPTATION OF THE EXPANDED IWO TO THE TSP

The expanded IWO algorithm is a metaheuristic. Hence, its application for solving a given optimization task requires a formulation of a single solution representation as well as a definition of transformations used for creation of a new solution.

From among significant concepts related to the form of a single solution it is worthwhile to mention three vector representations proposed in the literature: *path*, *ordinal*, and *adjacency* as well as a matrix representation (Michalewicz and Fogel, 2004). A plant used by the exIWO was designed according to the simple and natural rule of the path representation – a tour is an ordered list of cities (i.e. expressed as a vector [2 3 9 4 1 5 8 6 7]) and the order of visitation is determined by the order of vector elements (2–3–9–4–1–5–8–6–7–2).

Specific character of weeds reproduction mechanism functioning in the IWO implies rather application of transformations based on a single individual used as a sole parent. In case of the TSP transformations are realized by means of operators. Basic unary operators – *inversion*, *insertion*, *displacement*, and *reciprocal exchange* – are discussed in (Herdy, 1991). The *inver-over* operator is admittedly based on the inversion of a segment of cities, but the inverted segment is most often determined by means of another individual which is randomly selected from the same population. Due to specific character of this operation a fragment of the “second parent” is guaranteed to be a part of the offspring. In this way the *inver-over* operator combines features of unary as well as binary techniques. However, very promising results reported by its inventors (Tao and Michalewicz, 1998) decided on implementation of both – inversion and *inver-over* – operators in the exIWO.

In case of the TSP the individuals from the first population are greedily generated – the nearest city will be visited as the next one.

4 EXPERIMENTAL RESEARCH

The goal of the experiments was to compare the quality of solutions found by the exIWO with the outcomes generated by other methods using 17 test instances of the TSP taken from the TSPLIB collection. As reference points served the results presented in (DePuy et al., 2005), (DePuy et al., 2002) and (Bai et al., 2006). The former study presents application of the metaheuristic for randomized priority search (Meta-RaPS), whereas the latter discusses results achieved by means of the self-organizing maps (SOM). Both papers include comparison of the outcomes reported by other researchers, who investigated methods from different areas of artificial intelligence. Therefore the exIWO algorithm operated on the same test instances as those used in the aforementioned works.

It is worthwhile to mention that a number of cities is included in the name of each test set (e.g., the route described as “bier127” consists of 127 cities).

The workstation used for experiments is described by the following parameters: 2×Intel Xeon E5620 2.40 GHz, RAM 16 GB, MS Windows Server 2008 R2 Datacenter 64-bit SP1.

Evaluation of solutions generated by particular methods was based on 2 criteria – minimum and average difference from optimal or best known solution. Average values for the exIWO were computed based on results of 100 experiments for each test set.

Table 1 shows the minimum difference between the solutions produced by the IWO variants (exIWO and the original version) using inversion or the *inver-over* operator and optimal or best known solution for particular test instances expressed as a percentage.

Table 1: Minimum difference between the solution produced by one of the tested variants of the IWO algorithm (exIWO and the original version) and optimal or best known solution [%].

Test set	exIWO		IWO	
	Inversion	Inver-over	Inversion	Inver-over
kroA100	0	0	0	0
kroB100	0	0	0	0
kroC100	0	0.096	0.096	0
kroD100	0.169	0	0.169	0.169
kroE100	0	0	0	0
bier127	0	0	0	0
eil51	0	0	0	0
eil76	0	0	0	0
kroA200	0.200	0.449	0.446	0.446
lin105	0	0	0	0
pcb442	1.554	1.829	1.544	1.916
pr107	0	0	0	0
pr136	1.588	2.265	1.130	2.585
pr152	0.294	0.455	0.277	0.563
rat195	0.689	0.861	0.732	0.646
rd100	0	0	0	0
st70	0	0	0.148	0.148

The optimal or best known solution was found by the exIWO in case of 12 from among 17 test instances.

Average run times of the exIWO implementation in Java for 10000 iterations (equivalent to the number of populations) are included in Table 2. The number of individuals in a single population depends on the number of cities in a test set – for routes with less than 150 cities a single population contains 200 individuals, in other cases – 50. The best exIWO results reported in all tables were achieved for

particular test instances using different values of other algorithm parameters (a nonlinear modulation factor m , probabilities of selection of dispersing, spreading and rolling down, maximum and minimum admissible number of seeds S_{max} , S_{min} , initial and final values of the standard deviation σ_{init} , σ_{fin}). Only the number k of neighborhoods examined during the rolling down remained equal to 2 in the most successful exIWO experiments.

Table 2: Average run times of the exIWO variants [s].

Test set	Inversion	Inver-over	Number of individuals
kroA100	58.8	59.9	200
kroB100	62.2	62.7	200
kroC100	67.1	67.6	200
kroD100	70.9	68.8	200
kroE100	74.1	73.0	200
bier127	89.4	89.5	200
eil51	24.4	28.9	200
eil76	38.3	39.6	200
kroA200	35.5	25.6	50
lin105	62.8	61.6	200
pcb442	69.6	51.9	50
pr107	70.2	68.8	200
pr136	79.6	90.6	200
pr152	27.1	20.4	50
rat195	37.1	23.2	50
rd100	58.1	59.6	200
st70	47.8	43.1	200

The average run times of both exIWO variants are similar in most cases.

Tables 3-6 present average difference from optimal or best known solution for the chosen methods expressed as a percentage. Test instances considered in Table 3 belong to the Kro?100 collection (the question mark substitutes for one of the following symbols {A, B, C, D, E}), whereas results of the optimization methods other than exIWO were taken from (DePuy et al., 2005) and (DePuy et al., 2002). Particular names represent the following approaches: *Meta-RaPS TSP* – a Meta-heuristic for Randomized Priority Search, *Priority rule* based on the simple TSP heuristics – cheapest insertion and node insertion, *GRASP* – Greedy Randomized Adaptive Search Procedure – all algorithms were described in (DePuy et al., 2005), *Christofides & 2opt 3opt*, *Convex hull & 3opt*, *NN & 2opt 3opt* (“NN” denotes “Nearest Neighbor”), *2opt 3opt* – algorithms using local search methods (Lawler et al. 1985), *Lin-Kernighan* algorithm (Padberg and Rinaldi, 1991), *Modified Lin-Kernighan* algorithm (Mak and Morton, 1993), *CCAO* – Convex hull, Cheapest insertion, Angle selection and Or-opt – a

heuristic which exploits geometrical properties of symmetric Euclidean TSP (Golden and Stewart, 1985), *Triangul.* – a Delaunay triangulation-based heuristic (Krasnogor et al., 1995), *I³* – a composite heuristic consisting of 3 phases: construction of an Initial envelope, Insertion of the remaining vertices, and Improvement procedure (Renaud et al., 1996), *P-SEC*, *F-SEC* – a Preliminary and a Full Subpath Ejection Chain method, respectively (Rego, 1998). Abbreviations related to the IWO variants have the following meanings – “inv” denotes inversion, whereas “i-o” represents the inver-over operator. The underlined values outperform results produced by the exIWO.

Table 3: Average difference from optimal or best known solution for the chosen methods mentioned in (DePuy et al., 2005), (DePuy et al., 2002) and variants of the IWO based on the Kro?100 test sets [%].

Method	A	B	C	D	E
Meta-RaPS TSP	0	0.25	<u>0</u>	<u>0</u>	<u>0.17</u>
Priority rule	0.5	2.46	0.82	1.43	1.1
GRASP	0	0.55	0.31	<u>0.42</u>	0.37
Christofides & 2opt 3opt	2.51	1.4	1.53	<u>0.17</u>	3.03
Convex hull & 3opt	0.37	1.46	1.06	<u>0.04</u>	2.46
NN & 2opt 3opt	0.14	1.46	1.06	0.73	2.46
2opt 3opt	0.81	1.44	0.53	1.74	<u>0.18</u>
Lin-Kernighan	0.26	<u>0</u>	0.7	<u>0.17</u>	<u>0.16</u>
Modif. Lin-Kernighan	0	0.17	<u>0</u>	<u>0</u>	0.21
CCAO	0	0.97	0.5	0.97	2.54
Triangul.	0.51	2.13	2.79	3.81	2
I ³	0	0.9	0.5	2	2.6
P-SEC	0	0.32	<u>0.02</u>	0.75	0.33
F-SEC	0	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>
exIWO inv	0	0.01	0.24	0.49	0.19
exIWO i-o	0	0.01	0.25	0.51	0.20
IWO inv	0	0.01	0.25	0.54	0.20
IWO i-o	0	0.01	0.24	0.54	0.21

Results presented in Tables 4-6 are related to the SOM-based methods and were taken from (Bai et al., 2006) and compared with outcomes produced by the IWO variants. Particular acronyms represent the following approaches: *PKN* – Pure Kohonen Network (Hueter, 1988) and (Fort, 1988), *GN* – Guilty Net (Burke and Damany, 1992), *AVL* – the procedure of Angéniol, de la Croix Vaubois and Le Texier (Angéniol et al., 1988), *KL*, *KG* – 2 variants of the Kohonen Network Incorporating Explicit Statistics (KNIES) – Local and Global, respectively

(Aras et al., 1999), *SETSP* – SOM Efficiently applied in the TSP (Vieira et al., 2003), *MGSOM* – the Modified Growing ring SOM approach for TSP (Bai et al., 2006).

Table 4: Average difference from optimal or best known solution for the SOM-based methods and variants of the IWO – part I [%].

Method	bier127	eil51	eil76	kroA200
PKN	3.322	4.202	6.171	5.311
GN	31.181	10.493	14.182	34.058
AVL	3.713	4.108	6.190	5.540
KL	2.762	2.864	4.981	2.836
KG	3.079	2.864	5.483	3.667
SETSP	1.850	2.221	4.234	3.119
MG SOM	1.097	1.398	3.384	1.972
exIWO inv	0.061	0.099	0.338	0.757
exIWO i-o	0.069	0.127	0.292	0.716
IWO inv	0.090	0.131	0.346	0.749
IWO i-o	0.097	0.129	0.323	0.747

Table 5: Average difference from optimal or best known solution for the SOM-based methods and variants of the IWO – part II [%].

Method	lin105	pcb442	pr107	pr136
PKN	6.921	–	0.454	7.343
GN	7.584	–	81.661	–
AVL	6.487	17.472	1.791	6.893
KL	1.985	11.072	0.734	4.531
KG	1.291	10.447	0.425	5.147
SETSP	1.301	10.160	0.409	4.400
MG SOM	<u>0.028</u>	8.577	0.172	<u>2.154</u>
exIWO inv	0.070	2.580	0	3.768
exIWO i-o	0.033	3.011	0	3.834
IWO inv	0.078	2.713	0.001	3.993
IWO i-o	0.106	3.199	0.001	3.895

The column “avg” in Table 6 includes average values computed on all 12 test sets taken into account in Tables 4-6.

The exIWO variants produce results which outperform the most of the outcomes of the SOM-based methods with the exception of the MGSOM in case of 3 test instances.

In the majority of cases solutions obtained by the exIWO are slightly better than those of the original version.

Similarity of both exIWO variants concerns not only the run times but the minimum and average differences from optimal or best known solution as well.

Table 6: Average difference from optimal or best known solution for the SOM-based methods and variants of the IWO – part III [%].

Method	pr152	rat195	rd100	st70	avg
PKN	1.523	–	–	2.637	–
GN	42.817	–	10.382	11.956	–
AVL	1.302	15.420	4.498	2.711	6.344
KL	0.968	12.238	2.095	1.511	4.048
KG	1.285	11.916	2.622	2.326	4.213
SETSP	1.169	11.192	2.601	1.600	3.688
MG SOM	<u>0.741</u>	5.984	1.172	1.183	2.322
exIWO inv	0.863	1.721	0.127	0.533	0.905
exIWO i-o	0.924	1.868	0.105	0.594	0.969
IWO inv	0.943	1.857	0.123	0.624	0.971
IWO i-o	0.944	2.055	0.153	0.636	1.024

The average difference from optimal or best known solution for both exIWO variants computed on all 17 test instances mentioned in this paper amounts to 0.741 % in case of the inver-over operator and 0.693 % for the inversion.

5 CONCLUSIONS

The authors of the present paper have modified the IWO metaheuristic introducing a hybrid strategy of the search space exploration as well as a concept of the family selection. Analysis of the exIWO results presented in this paper enables to expect solutions of good quality in a reasonable amount of time. It is also worth mentioning that each iteration of the exIWO generates a population of individuals representing feasible tours.

The experiments revealed the usefulness of the exIWO algorithm for solving discrete optimization tasks and confirmed the concept of using randomness as a mechanism to avoid local optima. The method can compete with other heuristics, although the influence of the hybrid strategy components (dispersing, spreading, rolling down) on the solution found by the exIWO requires further research – at present the algorithm takes part in the *World TSP Challenge* (www.tsp.gatech.edu/world/index.html) and in the *Mona Lisa TSP Challenge* (www.tsp.gatech.edu/data/ml/monalisa.html).

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