

# Watermarking Images in the Frequency Domain by Exploiting Self-inverting Permutations

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**Abstract:** In this work we propose efficient codec algorithms for watermarking images that are intended for uploading on the web under intellectual property protection. Headed to this direction, we recently suggested a way in which an integer number  $w$  which being transformed into a self-inverting permutation, can be represented in a two dimensional (2D) object and thus, since images are 2D structures, we have proposed a watermarking algorithm that embeds marks on them using the 2D representation of  $w$  in the spatial domain. Based on the idea behind this technique, we now expand the usage of this concept by marking the image in the frequency domain. In particular, we propose a watermarking technique that also uses the 2D representation of self-inverting permutations and utilizes marking at specific areas thanks to partial modifications of the image's Discrete Fourier Transform (DFT). Those modifications are made on the magnitude of specific frequency bands and they are the least possible additive information ensuring robustness and imperceptiveness. We have experimentally evaluated our algorithms using various images of different characteristics under JPEG compression. The experimental results show an improvement in comparison to the previously obtained results and they also depict the validity of our proposed codec algorithms.

## 1 INTRODUCTION

Internet technology, in modern communities, becomes day by day an indispensable tool for everyday life since most people use it on a regular basis and do many daily activities online (Garfinkel, 2001). This frequent use of the internet means that measures taken for internet security are indispensable since the web is not risk-free (Chun-Shien et al., 2000; Davis, 1997). One of those risks is the fact that the web is an environment where intellectual property is under threat since a huge amount of public personal data is continuously transferred, and thus such data may end up on a user who falsely claims ownership.

It is without any doubt that images, apart from text, are the most frequent type of data that can be found on the internet. As digital images are a characteristic kind of intellectual material, people hesitate to upload and transfer them via the internet because of the ease of intercepting, copying and redistributing in their exact original form (O'Ruanaidh et al., 1996). Encryption is not the problem's solution in most cases, as most people that upload images in a website want them to be visible by everyone, but safe

and theft protected as well. Watermarks are a solution to this problem, since thanks to them someone can claim the property of an image if he previously inserted one in it. Image watermarks can be visible or not, but if we don't want any cosmetic changes in an image then an invisible watermark should be used and that's what our work suggests, a technique according to which invisible watermarks are embedded into images using features of the image's frequency domain and graph theory as well.

We next briefly describe the main idea behind the watermarking technique, the motivation of our work, and our contribution.

**Watermarking.** In general, watermarks are symbols which are placed into physical objects such as documents, photos, etc. and their purpose is to carry information about objects' authenticity (Cox et al., 2008).

A digital watermark is a kind of marker embedded in a digital object such as image, audio, video, or software and, like a typical watermark, it is used to identify ownership of the copyright of such an object. Digital watermarking (or, hereafter, watermarking) is a technique for protecting the intellectual property of

a digital object; the idea is simple: a unique marker, which is called *watermark*, is embedded into a digital object which may be used to verify its authenticity or the identity of its owners (Grover, 1997; Collberg and Nagra, 2010). More precisely, watermarking can be described as the problem of embedding a watermark  $w$  into an object  $I$  and, thus, producing a new object  $I_w$ , such that  $w$  can be reliably located and extracted from  $I_w$  even after  $I_w$  has been subjected to transformations (Collberg and Nagra, 2010); for example, compression, scaling or rotation in case where the object is an image.

In the image watermarking process the digital information, i.e., the watermark, is hidden in image data. The watermark is embedded into image's data through the introduction of errors not detectable by human perception (Cox et al., 1996); note that, if the image is copied or transferred through the internet then the watermark is also carried with the copy into the image's new location.

**Motivation.** Intellectual property protection is one of the greatest concerns of internet users today. Digital images are considered a representative part of such properties so we consider important, the development of methods that deter malicious users from claiming others' ownership, motivating internet users to feel more safe to publish their work online.

Image Watermarking, is a technique that serves the purpose of image intellectual property protection ideally as in contrast with other techniques it allows images to be available to third internet users but simultaneously carry an "identity" that is actually the proof of ownership with them. This way image watermarking achieves its target of deterring copy and usage without permission of the owner. What is more by saying watermarking we don't necessarily mean that we put a logo or a sign on the image as research is also done towards watermarks that are both invisible and robust.

Our work suggests a method of embedding a numerical watermark into the image's structure in an invisible and robust way to specific transformations, such as JPEG compression.

**Contribution.** In this work we present an efficient and easily implemented technique for watermarking images that we are interested in uploading in the web and making them public online; this way web users are enabled to claim the ownership of their images.

What is important for our idea is the fact that it suggests a way in which an integer number can be represented with a two dimensional representation (or, for short, 2D representation). Thus, since images are two dimensional objects that representation can be ef-

ficiently marked on them resulting the watermarked images. In a similar way, such a 2D representation can be extracted for a watermarked image and converted back to the integer  $w$ .

Having designed an efficient method for encoding integers as self-inverting permutations, we propose an efficient algorithm for encoding a self-inverting permutation  $\pi^*$  into an image  $I$  by first mapping the elements of  $\pi^*$  into an  $n^* \times n^*$  matrix  $A^*$  and then using the information stored in  $A^*$  to mark specific areas of image  $I$  in the frequency domain resulting the watermarked image  $I_w$ . We also propose an efficient algorithm for extracting the embedded self-inverting permutation  $\pi^*$  from the watermarked image  $I_w$  by locating the positions of the marks in  $I_w$ ; it enables us to reconstruct the 2D representation of the self-inverting permutation  $\pi^*$ .

It is worth noting that although digital watermarking has made considerable progress and became a popular technique for copyright protection of multimedia information (Cox et al., 1996), our work proposes something new. We first point out that our watermarking method incorporates such properties which allow us to successfully extract the watermark  $w$  from the image  $I_w$  even if the input image has been compressed with a lossy method, scaled and/or rotated. In addition, our embedding method can transform a watermark from a numerical form into a two dimensional (2D) representation and, since images are 2D structures, it can efficiently embed the 2D representation of the watermark by marking the high frequency bands of specific areas of an image. The key idea behind our extracting method is that it does not actually extract the embedded information instead it locates the marked areas reconstructing the watermark's numerical value.

We have evaluated the embedding and extracting algorithms by testing them on various and different in characteristics images that were initially in JPEG format and we had positive results as the watermark was successfully extracted even if the image was converted back into JPEG format with various compression ratios. What is more, the method is open to extensions as the same method might be used with a different marking procedure such as the one we used in our previous work. Note that, all the algorithms have been developed and tested in MATLAB (Gonzalez et al., 2003).

## 2 THEORETICAL FRAMEWORK

In this section we first describe discrete structures, namely, permutations and self-inverting permuta-

tions, and briefly discuss a codec system which encodes an integer number  $w$  into a self-inverting permutation  $\pi$ . Then, we present a transformation of a watermark from a numerical form to a 2D form (i.e., 2D representation) through the exploitation of self-inverting permutation properties.

## 2.1 Self-inverting Permutations

Informally, a permutation of a set of objects  $S$  is an arrangement of those objects into a particular order, while in a formal (mathematical) way a permutation of a set of objects  $S$  is defined as a bijection from  $S$  to itself (i.e., a map  $S \rightarrow S$  for which every element of  $S$  occurs exactly once as image value).

Permutations may be represented in many ways. The most straightforward is simply a rearrangement of the elements of the set  $N_n = \{1, 2, \dots, n\}$ ; in this way we think of the permutation  $\pi = (5, 6, 9, 8, 1, 2, 7, 4, 3)$  as a rearrangement of the elements of the set  $N_9$  such that “1 goes to 5”, “2 goes to 6”, “3 goes to 9”, “4 goes to 8”, and so on (Sedgewick and Flajolet, 1996; Golubic, 1980). Hereafter, we shall say that  $\pi$  is a permutation over the set  $N_9$ .

**Definition 2.1.1.** Let  $\pi = (\pi_1, \pi_2, \dots, \pi_n)$  be a permutation over the set  $N_n$ , where  $n > 1$ . The inverse of the permutation  $\pi$  is the permutation  $q = (q_1, q_2, \dots, q_n)$  with  $q_{\pi_i} = \pi_{q_i} = i$ . A *self-inverting permutation* (or, for short, SiP) is a permutation that is its own inverse:  $\pi_{\pi_i} = i$ .

By definition, a permutation is a SiP (self-inverting permutation) if and only if all its cycles are of length 1 or 2; for example, the permutation  $\pi = (5, 6, 9, 8, 1, 2, 7, 4, 3)$  is a SiP with cycles: (1, 5), (2, 6), (3, 9), (4, 8), and (7).

## 2.2 Encoding Numbers as SiPs

There are several systems that correspond integer numbers into permutations or self-inverting permutation (Sedgewick and Flajolet, 1996). Recently, we have proposed algorithms for such a system which efficiently encodes an integer  $w$  into a self-inverting permutations  $\pi$  and efficiently decodes it. The algorithms of our codec system run in  $O(n)$  time, where  $n$  is the length of the binary representation of the integer  $w$ , while the key-idea behind its algorithms is mainly based on mathematical objects, namely, bitonic permutations (Chroni and Nikolopoulos, 2010).

## 2.3 2D Representations

We first define the two-dimensional representation

(2D representation) of a permutation  $\pi$  over the set  $N_n = \{1, 2, \dots, n\}$ , and then its 2DM representation which is more suitable for efficient use in our codec system.

In the 2D representation, the elements of the permutation  $\pi = (\pi_1, \pi_2, \dots, \pi_n)$  are mapped in specific cells of an  $n \times n$  matrix  $A$  as follows:

$$\text{number } \pi_i \longrightarrow \text{entry } A(\pi_i^{-1}, \pi_i)$$

or, equivalently, the cell at row  $i$  and column  $\pi_i$  is labeled by the number  $\pi_i$ , for each  $i = 1, 2, \dots, n$ .

Figure 1(a) shows the 2D representation of the self-inverting permutation  $\pi = (6, 3, 2, 4, 5, 1)$ .

Note that, there is one label in each row and in each column, so each cell in the matrix  $A$  corresponds to a unique pair of labels; see, (Sedgewick and Flajolet, 1996) for a long bibliography on permutation representations and also in (Chroni and Nikolopoulos, 2011) for a DAG representation.

Based on the previously defined 2D representation of a permutation  $\pi$ , we next propose a two-dimensional marked representation (2DM representation) of  $\pi$  which is an efficient tool for watermarking images.

In our 2DM representation, a permutation  $\pi$  over the set  $N_n = \{1, 2, \dots, n\}$  is represented by an  $n \times n$  matrix  $A^*$  as follows:

- the cell at row  $i$  and column  $\pi_i$  is marked by a specific symbol, for each  $i = 1, 2, \dots, n$ ;
- in our implementation, the used symbol is the asterisk, i.e., the character “\*”.

Figure 1(b) shows the 2DM representation of the permutation  $\pi$ . It is easy to see that, since the 2DM representation of  $\pi$  is constructed from the corresponding 2D representation, there is also one symbol in each row and in each column of the matrix  $A^*$ .

We next present an algorithm which extracts the permutation  $\pi$  from its 2DM representation matrix. More precisely, let  $\pi$  be a permutation over  $N_n$  and let  $A^*$  be the 2DM representation matrix of  $\pi$  (see, Figure 1(b)); given the matrix  $A^*$ , we can easily extract  $\pi$  from  $A^*$  in linear time (i.e., linear in the size of matrix  $A^*$ ) by the following algorithm:

**Algorithm.** Extract\_ $\pi$ \_from\_2DM

*Input:* the 2DM representation matrix  $A^*$  of  $\pi$ ;

*Output:* the permutation  $\pi$ ;

1. For each row  $i$  of matrix  $A^*$ ,  $1 \leq i \leq n$ , and for each column  $j$  of matrix  $A^*$ ,  $1 \leq j \leq n$ , if the cell  $(i, j)$  is marked then  $\pi_i \leftarrow j$ ;

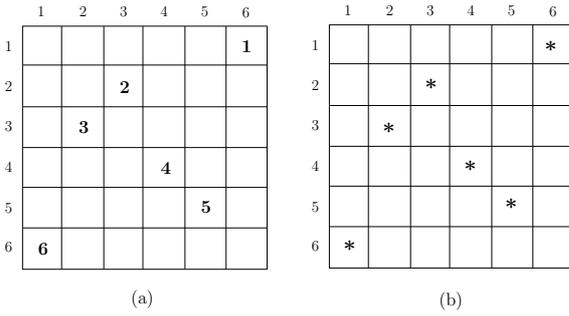


Figure 1: The 2D and 2DM representations of the self-inverting permutation  $\pi = (6, 3, 2, 4, 5, 1)$ .

## 2. Return the permutation $\pi$ ;

**Remark 2.3.1.** It is easy to see that the resulting permutation  $\pi$ , after the execution of Step 1, can be taken by reading the matrix  $A^*$  from top row to bottom row and write down the positions of its marked cells. Since the permutation  $\pi$  is a self-inverting permutation, its 2D matrix  $A$  has the following property:

- $A(i, j) = j$  if  $\pi_i = j$ , and
- $A(i, j) = 0$  otherwise,  $1 \leq i, j \leq n$ .

Thus, the corresponding matrix  $A^*$  is symmetric:

- $A^*(i, j) = A^*(j, i) = \text{“mark”}$  if  $\pi_i = j$ , and
- $A^*(i, j) = A^*(j, i) = 0$  otherwise,  $1 \leq i, j \leq n$ .

Based on this property, it is also easy to see that the resulting permutation  $\pi$  can be also taken by reading the matrix  $A^*$  from left column to right column and write down the positions of its marked cells.

Hereafter, we shall denote by  $\pi^*$  a SiP and by  $n^*$  the number of elements of  $\pi^*$ .

## 2.4 The Discrete Fourier Transform

The Discrete Fourier Transform (DFT) is used to decompose an image into its sine and cosine components. The output of the transformation represents the image in the frequency domain, while the input image is the spatial domain equivalent. In the image's fourier representation, each point represents a particular frequency contained in the image's spatial domain.

If  $f(x, y)$  is an image of size  $N \times M$  we use the following formula for the Discrete Fourier Transform:

$$F(u, v) = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x, y) e^{-j2\pi(\frac{ux}{N} + \frac{vy}{M})} \quad (1)$$

for values of the discrete variables  $u$  and  $v$  in the ranges  $u = 0, 1, \dots, N-1$  and  $v = 0, 1, \dots, M-1$

In a similar manner, if we have the transform  $F(u, v)$  i.e the image's fourier representation we can use the Inverse Fourier Transform to get back the image  $f(x, y)$  using the following formula:

$$f(x, y) = \frac{1}{NM} \sum_{u=0}^{N-1} \sum_{v=0}^{M-1} F(u, v) e^{j2\pi(\frac{ux}{N} + \frac{vy}{M})} \quad (2)$$

for  $x = 0, 1, \dots, N-1$  and  $y = 0, 1, \dots, M-1$

Typically, in our method, we are interested in the magnitudes of DFT coefficients. The magnitude  $|F(u, v)|$  of the Fourier transform at a point is how much frequency content there is and is calculated by Equation (1) (Gonzalez and Woods, 2007).

## 2.5 Previous Results

Recently, we proposed a watermarking technique based on the idea of interfering with the image's pixel values in the spatial domain. In the following paragraphs, we briefly describe the steps of this idea and state main points regarding some of its advantages and disadvantages. Recall that, in the current work we suggest an expansion to this idea by moving from the spatial domain to the image's frequency domain.

The embedding algorithm first computes the 2DM representation of the permutation  $\pi^*$ , that is, the  $n^* \times n^*$  array  $A^*$  (see, Subsection 2.3); the entry  $(i, \pi_i^*)$  of the array  $A^*$  contains the symbol “\*”,  $1 \leq i \leq n^*$ . Next, it takes the  $N \times M$  sized input image  $I$  and covers it with an  $n^* \times n^*$  imaginary grid  $C$ , resulting in  $n^* \times n^*$  grid-cells  $C_{ij}$ ,  $1 \leq i, j \leq n^*$ . Then it goes to each  $C_{ij}$  and locates its central pixel  $p_{ij}^0$  and the four pixels  $p_{ij}^1$ ,  $p_{ij}^2$ ,  $p_{ij}^3$ , and  $p_{ij}^4$  around it,  $1 \leq i, j \leq n^*$  (we shall call them *cross* pixels). It then computes the difference between the value of the central pixel  $p_{ij}^0$  and the average value of the twelve neighboring pixels storing it in the variable  $\text{dif}(p_{ij}^0)$ . Finally, it computes the maximum absolute value of all  $n^* \times n^*$  differences  $\text{dif}(p_{ij}^0)$ ,  $1 \leq i, j \leq n^*$ , and stores it in the variable  $\text{Maxdif}(I)$ . Recall that the embedding algorithm goes to each central pixel  $p_{ij}^0$  of each grid-cell  $C_{ij}$ ,  $1 \leq i, j \leq n^*$ , and if the corresponding entry  $A^*(i, j)$  contains the symbol “\*”, then it increases the value of each one of previously described, cross pixels by  $\text{Maxdif}(I) - \text{dif}(p_{ij}^0) + c$ , where  $c$  is a positive number used to strengthen the marks.

In a similar manner, the extracting algorithm is searching each line  $i$  of the imaginary grid  $C$  to find among the  $n^*$  grid-cells  $C_{i1}, C_{i2}, \dots, C_{in^*}$  the column number  $j$  of the one that has the greatest difference between the neighboring and cross pixels,  $1 \leq i, j \leq n^*$ ; then, the element  $\pi_i^*$  is set equal to  $j$ .

Regarding the main points of the previous technique, we should first mention that for images with general characteristics and relatively large size this method delivers optically good results. By saying “good results” we mean that the modifications made are quite invisible. Also the method’s algorithms run really fast as they simply access a finite number of pixels. Furthermore, both the embedding and extracting algorithms are easy to modify and adjust for various scenarios.

On the other hand, the method fails to deliver good results either for relatively small images or for images that depict something smooth which allows the eye to distinct the modifications on the image. Also we decided to move to a new method as there were also problems due to the fact that the positions of the crosses are centered at strictly specific positions causing difficulties in the extracting methods.

### 3 THE FREQUENCY DOMAIN APPROACH

Having described an efficient method for encoding integers as self-inverting permutations using the 2DM representation of self-inverting permutations, we next describe codec algorithms that efficiently encode and decode a watermark into the image’s frequency domain (Solachidis and Pitas, 2001; Licks and Hordan, 2000; Gonzalez and Woods, 2007).

#### 3.1 Embed Watermark into Image

We next describe the embedding algorithm of our proposed technique which encodes a self-inverting permutation (SiP)  $\pi^*$  into a digital image  $I$ . Recall that, the permutation  $\pi^*$  is obtained over the set  $N_{n^*}$ , where  $n^* = 2n + 1$  and  $n$  is the length of the binary representation of an integer  $w$  which actually is the image’s watermark (Chroni and Nikolopoulos, 2010); see, Subsection 2.2.

The watermark  $w$ , or equivalently the self-inverting permutation  $\pi^*$ , is invisible and it is inserted in the frequency domain of specific areas of the image  $I$ . More precisely, we mark the DFT’s magnitude of an image’s area using two ellipsoidal annuli, denoted hereafter as “Red” and “Blue” (see, Figure 2). The ellipsoidal annuli are specified by the following parameters:

- $P_r$ , the width of the “Red” ellipsoidal annulus,
- $P_b$ , the width of the “Blue” ellipsoidal annulus,
- $R_1$  and  $R_2$ , the radiuses of the “Red” ellipsoidal annulus on  $y$ -axis and  $x$ -axis, respectively.

The algorithm takes as input a SiP  $\pi^*$  and an image  $I$ , in which the user embeds the watermark, and returns the watermarked image  $I_w$ ; it consists of the following steps.

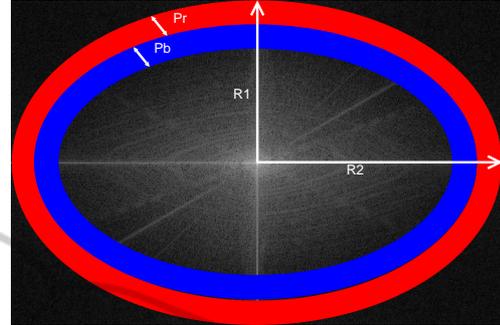


Figure 2: The “Red” and “Blue” ellipsoidal annuli.

#### Algorithm. Embed\_SiP-to-Image

*Input:* the watermark  $\pi^* \equiv w$  and the host image  $I$ ;

*Output:* the watermarked image  $I_w$ ;

**Step 1.** Compute first the 2DM representation of the permutation  $\pi^*$ , i.e., construct an array  $A^*$  of size  $n^* \times n^*$  such that the entry  $A^*(i, \pi_i^*)$  contains the symbol “\*”,  $1 \leq i \leq n^*$ .

**Step 2.** Next, compute the size of the input image  $I$ , say,  $N \times M$ , and cover the image  $I$  with an imaginary grid  $C$  with  $n^* \times n^*$  grid-cells  $C_{ij}$  of size  $\lfloor \frac{N}{n^*} \rfloor \times \lfloor \frac{M}{n^*} \rfloor$ ,  $1 \leq i, j \leq n^*$ .

**Step 3.** For each grid-cell  $C_{ij}$ , compute the Discrete Fourier Transform (DFT) using the Fast Fourier Transform (FFT) algorithm, resulting in a  $n^* \times n^*$  grid of DFT cells  $F_{ij}$ ,  $1 \leq i, j \leq n^*$ .

**Step 4.** For each DFT cell  $F_{ij}$ , compute its magnitude  $M_{ij}$  and phase  $P_{ij}$  matrices which are both of size  $\lfloor \frac{N}{n^*} \rfloor \times \lfloor \frac{M}{n^*} \rfloor$ ,  $1 \leq i, j \leq n^*$ .

**Step 5.** Then, the algorithm takes each of the  $n^* \times n^*$  magnitude matrices  $M_{ij}$ ,  $1 \leq i, j \leq n^*$ , and places two imaginary ellipsoidal annuli, denoted as “Red” and “Blue”, in the matrix  $M_{ij}$  (see, Figure 2). In our implementation,

- the “Red” is the outer ellipsoidal annulus while the “Blue” is the inner one. Both are concentric at the center of the  $M_{ij}$  magnitude matrix and have widths  $P_r$  and  $P_b$ , respectively;
- the radiuses of the “Red” ellipsoidal annulus are  $R_1$  ( $y$ -axis) and  $R_2$  ( $x$ -axis), while the “Blue” ellipsoidal annulus radiuses are computed in accordance to the “Red” ellipsoidal annulus and have values  $(R_1 - P_r)$  and  $(R_2 - P_r)$ , respectively;

- the inner perimeter of the “Red” ellipsoidal annulus coincides to the outer perimeter of the “Blue” ellipsoidal annulus;
- the values of the widths of the two ellipsoidal annuli are  $P_r = 2$  and  $P_b = 2$ , while the values of their radiuses are  $R_1 = \lfloor \frac{N}{2n^*} \rfloor$  and  $R_2 = \lfloor \frac{M}{2n^*} \rfloor$ .

The areas covered by the “Red” and the “Blue” ellipsoidal annuli determine two groups of magnitude values on  $M_{ij}$  (see, Figure 2).

**Step 6.** For each magnitude matrix  $M_{ij}$ ,  $1 \leq i, j \leq n^*$ , compute the average of the values that are in the areas covered by the “Red” and the “Blue” ellipsoidal annuli; let  $AvgR_{ij}$  be the average of the magnitude values belonging to the “Red” ellipsoidal annulus and  $AvgB_{ij}$  be the one of the “Blue” ellipsoidal annulus.

**Step 7.** For each magnitude matrix  $M_{ij}$ ,  $1 \leq i, j \leq n^*$ , compute first the variable  $D_{ij}$  as follows:

- $D_{ij} = |AvgB_{ij} - AvgR_{ij}|$ , if  $AvgB_{ij} \leq AvgR_{ij}$
- $D_{ij} = 0$ , otherwise.

Then, for each row  $i$  of the magnitude matrix  $M_{ij}$ ,  $1 \leq i, j \leq n^*$ , compute the maximum value of the variables  $D_{i1}, D_{i2}, \dots, D_{in^*}$  in row  $i$ ; let  $MaxD_i$  be the max value.

**Step 8.** For each cell  $(i, j)$  of the 2DM representation matrix  $A^*$  of the permutation  $\pi^*$  such that  $A_{ij}^* = “*”$  (i.e., marked cell), mark the corresponding grid-cell  $C_{ij}$ ,  $1 \leq i, j \leq n^*$ ; the marking is performed by increasing all the values in magnitude matrix  $M_{ij}$  covered by the “Red” ellipsoidal annulus by the value

$$AvgB_{ij} - AvgR_{ij} + MaxD_i + c, \quad (3)$$

where  $c = c_{opt}$ . The additive value of  $c_{opt}$  is calculated by the function  $f$  (see, Subsection 3.3) which returns the minimum possible value of  $c$  that enables successful extracting.

**Step 9.** Reconstruct the DFT of the corresponding modified magnitude matrices  $M_{ij}$ , using the trigonometric form formula (Gonzalez and Woods, 2007), and then perform the Inverse Fast Fourier Transform (IFFT) for each marked cell  $C_{ij}$ ,  $1 \leq i, j \leq n^*$ , in order to obtain the image  $I_w$ .

**Step 10.** Return the watermarked image  $I_w$ .

In Figure 3 we demonstrate the main operations performed by our embedding algorithm. In particular, we show the marking process of the grid-cell  $C_{44}$  of the Lena image; in this example, we embed in the Lena image the watermark number  $w$  which corresponds to SiP (6, 3, 2, 4, 5, 1).

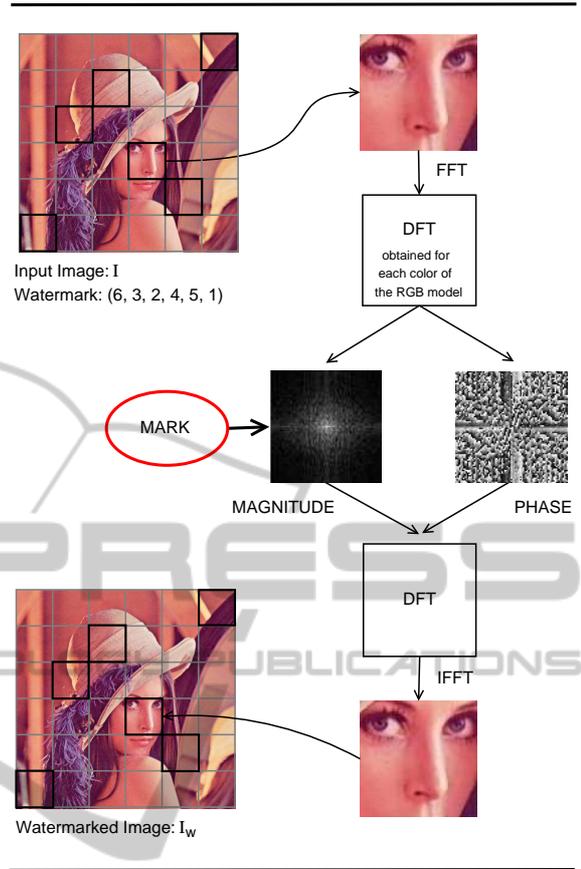


Figure 3: The embedding process.

### 3.2 Extract Watermark from Image

In this section we describe the decoding algorithm of our proposed technique. The algorithm extracts a self-inverting permutation (SiP)  $\pi^*$  from a watermarked digital image  $I_w$ , which can be later represented as an integer  $w$ .

The self-inverting permutation  $\pi^*$  is obtained from the frequency domain of specific areas of the watermarked image  $I_w$ . More precisely, using the same two “Red” and “Blue” ellipsoidal annuli, we detect certain areas of the watermarked image  $I_w$  that are marked by our embedding algorithm and these marked areas enable us to obtain the 2D representation of the permutation  $\pi^*$ . The extracting algorithm works as follows:

**Algorithm.** Extract\_SiP-from-Image

*Input:* the watermarked image  $I_w$  marked with  $\pi^*$ ;

*Output:* the watermark  $\pi^* = w$ ;

**Step 1.** Take the input watermarked image  $I_w$  and compute its  $N \times M$  size. Then, cover  $I_w$  with the same imaginary grid  $C$ , as described in the embedding method, having  $n^* \times n^*$  grid-cells  $C_{ij}$  of size

$$\lfloor \frac{N}{n^*} \rfloor \times \lfloor \frac{M}{n^*} \rfloor.$$

**Step 2.** Then, again for each grid-cell  $C_{ij}$ ,  $1 \leq i, j \leq n^*$ , using the Fast Fourier Transform (FFT) get the Discrete Fourier Transform (DFT) resulting a  $n^* \times n^*$  grid of DFT cells.

**Step 3.** For each DFT cell, compute its magnitude matrix  $M_{ij}$  and phase matrix  $P_{ij}$  which are both of size  $\lfloor \frac{N}{n^*} \rfloor \times \lfloor \frac{M}{n^*} \rfloor$ .

**Step 4.** For each magnitude matrix  $M_{ij}$ , place the same imaginary ‘‘Red’’ and ‘‘Blue’’ ellipsoidal annuli, as described in the embedding method, and compute as before the average values that coincide in the area covered by the ‘‘Red’’ and the ‘‘Blue’’ ellipsoidal annuli; let  $AvgR_{ij}$  and  $AvgB_{ij}$  be these values.

**Step 5.** For each row  $i$  of  $C_{ij}$ ,  $1 \leq i \leq n^*$ , search for the  $j_{th}$  column where  $AvgB_{ij} - AvgR_{ij}$  is minimized and set  $\pi_i^* = j$ ,  $1 \leq j \leq n^*$ .

**Step 6.** Return the self-inverting permutation  $\pi^*$ .

Having presented the embedding and extracting algorithms, let us next describe the function  $f$  which returns the additive value  $c = c_{opt}$  (see, Step 8 of the embedding algorithm).

### 3.3 Function $f$

Based on our marking model, the embedding algorithm amplifies the marks in the ‘‘Red’’ ellipsoidal annulus by adding the output of the function  $f$ . What exactly  $f$  does is returning the optimal value that allows the extracting algorithm under the current requirements, such as JPEG compression, to still be able to extract the watermark from the image.

The function  $f$  takes as an input the characteristics of the image and the parameters  $R_1$ ,  $R_2$ ,  $P_b$ , and  $P_r$  of our proposed mark model (see, Step 5 of embedding algorithm and Figure 2), and returns the minimum possible  $c_{opt}$  that added as  $c$  to the values of the ‘‘Red’’ ellipsoidal annulus enables extracting (see, Step 8 of the embedding algorithm). More precisely, the function  $f$  initially takes the interval  $[0, c_{max}]$ , where  $c_{max}$  is a relatively great value such that if  $c_{max}$  is taken as  $c$  for marking the ‘‘Red’’ ellipsoidal annulus it allows extracting, and computes the  $c_{opt}$  in  $[0, c_{max}]$ .

Note that,  $c_{max}$  allows extracting but because of being great damages the quality of the image (see, Figure 4). We mentioned relatively great because it depends on the characteristics of each image. For a specific image it is useless to use a  $c_{max}$  greater than a specific value, we only need a value that definitely enables the extracting algorithm to successfully extract the watermark.

We next describe the computation of the value  $c_{opt}$  returned by the function  $f$ ; note that, the parameters  $P_b$  and  $P_r$  of our implementation are fixed with the values 2 and 2, respectively. The main steps of this computation are the following:

- (i) Check if the extracting algorithm for  $c = 0$  validly obtains the watermark  $\pi^* = w$  from the image  $I_w$ ; if yes, then the function  $f$  returns  $c_{opt} = 0$ ;
- (ii) If not, that means,  $c = 0$  doesn’t allow extracting; then, the function  $f$  uses binary search on  $[0, c_{max}]$  and computes the interval  $[c_1, c_2]$  such that:
  - $c = c_1$  doesn’t allow extracting,
  - $c = c_2$  do allow extracting, and
  - $|c_1 - c_2| < 0.2$ ;
- (iii) The function  $f$  returns  $c_{opt} = c_2$ ;

As mentioned before, the function  $f$  returns the optimal value  $c_{opt}$ . Recall that, optimal means that it is the smallest possible value which enables extracting  $\pi^* = w$  from the image  $I_w$ . It is important to be the smallest one as that minimizes the additive information to the image and, thus, assures minimum drop to the image quality.

## 4 EXPERIMENTAL RESULTS

In this section we present the experimental results of the proposed watermarking codec algorithms which we have implemented using the general-purpose mathematical software package Matlab (version 7.7.0) (Gonzalez et al., 2003). We tested our codec algorithms on various 24-bit digital color images of various sizes (from  $200 \times 130$  up to  $4600 \times 3700$ ) and quality characteristics. Many of the images in our image repository were taken from a web image gallery (Petitcolas, 2012) and enriched by some other images different in characteristics.

There are three main requirements of digital watermarking: *fidelity*, *robustness*, and *capacity* (Cox et al., 2008). Our watermarking method appears to have high fidelity and robustness against JPEG compression.

Initially, we had to choose the appropriate values for the parameters of the quality function  $f$ . In our implementation we set both of the parameters  $P_r$  and  $P_b$  equal to 2 (see, Section 3.1). Recall that, the value 2 is a relatively small value which allows us to modify a satisfactory number of pixels in order to embed the watermark and successfully extract it, without affecting images’ quality. Note that, for great in size images, a smaller width reduces the strength of the

watermark. There isn't a distance between the two ellipsoidal annuli as that enables the algorithm to apply a small additive information to the values of the "Red" annulus. The two ellipsoidal annuli are inscribed to the rectangle magnitude matrix, as we want to mark images' cells on the high frequency bands.

We mark the high frequencies by increasing their values using mainly the additive parameter  $c = c_{opt}$  because alterations in the high frequencies are less detectable by human eye (Kaur et al., 2012). What is more, in high frequencies most images contain less information.

In this work we used JPEG images due to their great importance on the web, since they are small in size, while storing full color information (24 bit/pixel), and can be easily and efficiently transmitted. Moreover, robustness to lossy compression is an important issue when dealing with image authentication. It should be observed that the design goal of lossy compression systems is opposed to that of watermark embedding systems. The Human Visual System model of the compression system attempts to identify and discard perceptually insignificant information of the image, whereas the goal of the watermarking system is to embed the watermark information without altering the visual perception of the image (Zain, 2011).

The quality factor (or, for short,  $Q$ -factor) is a number that determines the degree of loss in the compression process when saving an image. In general, JPEG recommends a quality factor of 75–95 for visually indistinguishable quality difference, and a quality factor of 50–75 for merely acceptable quality. We compressed the images with Matlab JPEG compressor from `imwrite` with different quality factors; we present results for  $Q = 85$ ,  $Q = 75$  and  $Q = 65$ .

The quality function  $f$  returns the factor  $c$ , which has the minimum value  $c_{opt}$  that allows the extracting algorithm to successfully extract the watermark. In fact, this value  $c_{opt}$  (see, Formula 3) is the main additive information embedded into the image. Depending on the images and the amount of compression, we need to increase the watermark strength by increasing the factor  $c$ . The value of  $c$  increases as the quality factor of JPEG compression decreases. It is obvious that the embedding algorithm is image dependent. It is worth noting that, the  $c_{opt}$  values are small for images of relatively small size while these values increase as we move to images of greater size.

To demonstrate the differences on watermarked image quality, with respect to the values of the additive factor  $c$ , we watermarked the original image `lena.jpg` and we embedded a watermark with  $c = c_{max}$  and  $c = c_{opt}$ , where  $c_{max} \gg c_{opt}$  (see, Figure 4);

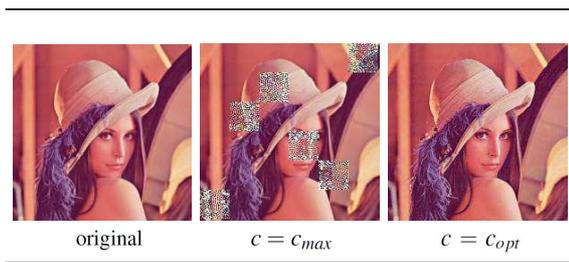


Figure 4: The original image of Lena and its two watermarked images with  $c = c_{max}$  and  $c = c_{opt}$ ; the watermark corresponds to SiP (6,3,2,4,5,1).

in the watermarked image in the middle we used  $c = c_{max}$  for illustrative purposes.

In order to evaluate the watermarked image quality obtained from our proposed watermarking method we used two objective image quality assessment metrics, the Peak Signal to Noise Ratio (PSNR) and the Structural Similarity Index Metric (SSIM). Our aim was to prove that the watermarked image is closely related to the original (image fidelity), because watermarking should not introduce visible distortions in the original image as that would reduce images' commercial value.

The PSNR metric is the ratio between the reference signal and the distortion signal, i.e., watermark, in an image given in decibels (dB). It is well known that, PSNR is most commonly used as a measure of quality of reconstruction of lossy compression codecs (e.g., for image compression). The higher the PSNR value the closer the distorted image is to the original or the better the watermark conceals. It is a popular metric due to its simplicity, although it is well known that this distortion metric is not absolutely correlated with human vision.

For an initial image  $I$  of size  $N \times M$  and its watermarked image  $I_w$ , PSNR is defined by the formula:

$$\text{PSNR}(I, I_w) = 10 \log_{10} \frac{N_{max}^2}{MSE}, \quad (4)$$

where  $N_{max}$  is the maximum signal value that exists in the original image and MSE is the Mean Square Error which is represented by the formula as follows:

$$\text{MSE}(I, I_w) = \frac{1}{N \times M} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} (I(i, j) - I_w(i, j))^2. \quad (5)$$

The SSIM image quality metric, developed by (Wang et al., 2004), is considered to be correlated with the quality perception of the HVS (Hore and Ziou, 2010). The SSIM metric is defined as:

$$\text{SSIM}(I, I_w) = \frac{(2\mu_w + C_1)(2\sigma(I, I_w) + C_2)}{(\mu^2 + \mu_w^2 + C_1)(\sigma(I)^2 + \sigma(I_w)^2 + C_2)}, \quad (6)$$

Table 1: The PSNR values of the original and watermarked images, for compression of qualities  $Q = 85, 75$  and  $65$ .

| Filename      | Q = 85 | Q = 75 | Q = 65 |
|---------------|--------|--------|--------|
| lena.jpg      | 54.0   | 50.0   | 46.8   |
| baboon.jpg    | 49.3   | 46.2   | 42.5   |
| trattoria.jpg | 67.8   | 60.6   | 53.5   |
| dome.jpg      | 64.6   | 59.8   | 54.9   |
| aquarium.jpg  | 65.2   | 61.2   | 58.3   |

Table 2: The SSIM values of the original and watermarked images, for compression of qualities  $Q = 85, 75$  and  $65$ .

| Filename      | Q = 85 | Q = 75 | Q = 65 |
|---------------|--------|--------|--------|
| lena.jpg      | 0.997  | 0.993  | 0.986  |
| baboon.jpg    | 0.995  | 0.989  | 0.980  |
| trattoria.jpg | 0.999  | 0.999  | 0.996  |
| dome.jpg      | 0.999  | 0.999  | 0.997  |
| aquarium.jpg  | 0.999  | 0.999  | 0.998  |

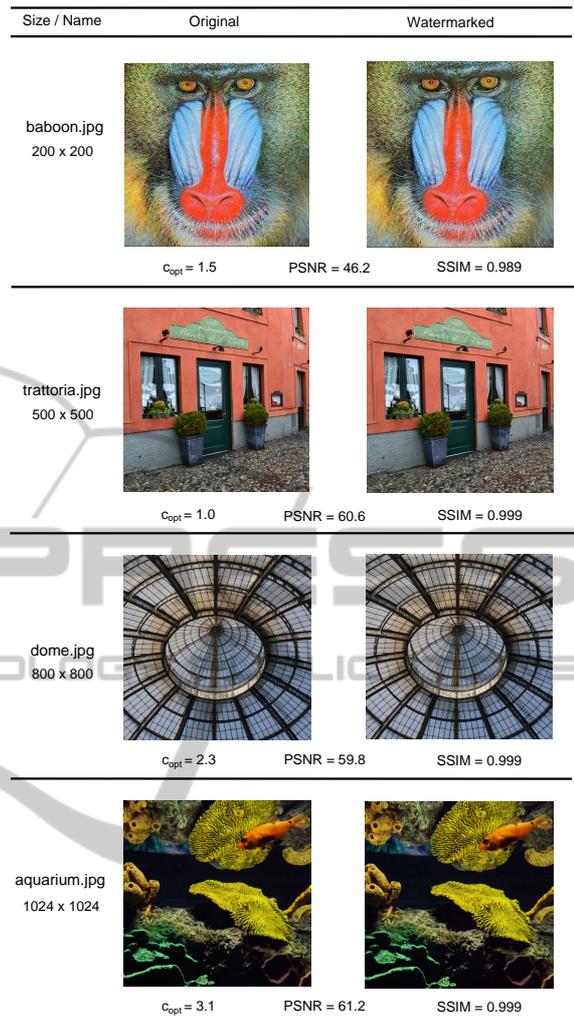
where  $\mu$  and  $\mu_w$  are the mean luminances of the original and watermarked image  $I$  respectively,  $\sigma(I)$  is the standard deviation of  $I$ ,  $\sigma(I_w)$  is the standard deviation of  $I_w$ , whereas  $C_1$  and  $C_2$  are constants to avoid null denominator. We use a mean SSIM (MSSIM) index to evaluate the overall image quality over the  $M$  sliding windows,

$$\text{MSSIM}(I, I_w) = \frac{1}{M} \sum_{i=0}^M \text{SSIM}(I, I_w). \quad (7)$$

The highest value of SSIM is 1, and it is achieved when the original and watermarked images ( $I, I_w$  respectively) are identical.

Our watermarked images have excellent PSNR and SSIM values. In Figure 5 we present four images of different sizes, along with their corresponding PSNR and SSIM values. Typical values for the PSNR in lossy image compression are between 40 and 70 dB, where higher is better. In our experiments, the PSNR values of 90% of the watermarked images were greater than 40 dB. The SSIM values are almost equal to 1, which means that the watermarked image is quite similar to the original one, which explains the method's high fidelity.

In Table 1 and 2 we demonstrate the PSNR and SSIM values of some images that are used in this work. These values are decreasing on smaller quality factors. Also, as the additive value  $c = c_{opt}$  increases for each quality factor, the quality decreases. Moreover, the additive value  $c$  that embeds robust marks for qualities  $Q = 85, Q = 75$  and  $Q = 65$ , does not result in a significant image distortion as the tables suggest.

Figure 5: Some original images and their corresponding watermarked ones; for each image, its size and its  $c_{opt}$ , and PSNR and SSIM values are also shown, for  $Q=75$ .

## 5 CONCLUDING REMARKS

In this paper we propose a method for embedding invisible watermarks into images and their intention is to prove the authenticity of an image.

We experimentally tested our embedding and extracting algorithms on color JPEG images with various and different characteristics; our testing procedure includes the phases of embedding a numerical watermark  $w = \pi^*$  into a colored JPEG image  $I$ , storing the watermarked image  $I_w$  in JPEG format with various compression ratios, and extracting the watermark  $w = \pi^*$  from the image  $I_w$ ; in our method, the watermark  $w$  is a self-inverting permutation  $\pi^*$  over the set  $N_n$ .

We obtained positive results as the watermarks were invisible, they didn't affect the images' quality and they were extractable despite the JPEG compression. In addition, the experimental results show an improvement in comparison to the previously obtained results and they also depict the validity of our proposed codec algorithms.

It is worth noting that the proposed algorithms are robust against cropping or rotation attacks since the watermarks are in SiP form, meaning that they determine the embedding positions in specific image areas, and thus if a part is being cropped or the image is rotated, SiP's symmetry property may allow us to reconstruct the watermark. Furthermore, our codec algorithms can also be modified in the future to get robust against scaling attacks. That can be achieved by selecting multiple widths concerning the ellipsoidal annuli depending on the size of the input image.

Finally, we should point out that the study of our quality function  $f$  remains an open problem; indeed,  $f$  could incorporate learning algorithms (Russell and Norvig, 2010) so that to be able to return the  $c_{opt}$  accurately and in a very short computational time.

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