Parameterised Fuzzy Petri Nets for Knowledge Representation and Reasoning

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Abstract: The paper presents a new methodology for knowledge representation and reasoning based on parameterised fuzzy Petri nets. Recently, this net model has been proposed as a natural extension of generalised fuzzy Petri nets. The new class extends the generalised fuzzy Petri nets by introducing two parameterised families of sums and products, which are supposed to provide the suitable t-norms and s-norms. The nature of the fuzzy reasoning realised by a given net model changes variously depending on t- and/or s-norms to be used. However, it is very difficult to select a suitable t- and/or s-norm function for actual applications. Therefore, we proposed to use in the net model parameterised families of sums and products, which nature change variously depending on the values of the parameters. Taking into account this aspect, we can say that the parameterised fuzzy Petri nets are more flexible than the classical fuzzy Petri nets, because they allow to define the parameterised input/output operators. Moreover, the choice of suitable operators for a given reasoning process and the speed of reasoning process are very important, especially in real-time decision support systems. Some advantages of the proposed methodology are shown in its application in train traffic control decision support.

1 INTRODUCTION

A substantial share of currently developed IT systems is based on knowledge often represented in the form of a decision rule system. Knowledge can be acquired either from experts directly, in the course of an IT system's implementation or automatically from properly processed training data. The knowledge combined with an appropriate inference engine serves as a fundamental component of, among others, decision support systems. The steady growth in the number of IT systems' applications contributes greatly to the development of systems capable of processing larger and larger data amounts with major constraints related to time limits in decision making. In response to the IT system market, numerous scientific initiatives are undertaken whose main aim is to either improve existing or launch new methods supporting development of systems with knowledge base that could fulfill all requirements. Most of all, new methods of knowledge representation are being searched these days, as well as knowledge quality verification or knowledge coding methods during implementation of decision support systems. A key feature of the methods is their ability to verify quality of any rule system at the earliest possible stage of its development. The early fault detection is crucial for economic reasons, such as allowing cost reduction of the system's development. Moreover, it is also for the sake of the final product's quality, as errors identified and excluded at early development stages, are not transferred to the successive ones (Avram, 2005),(Jackson, 1999).

In the last years we can observe growing interest in the design and exploitation of decision support systems built on the basis of uncertain knowledge. In this way various methods of knowledge representation and reasoning have already been proposed. One of the most popular approaches to knowledge representation are the fuzzy production rules. They are often presented in the form of IF-THEN. One of its benefits is that they create a modular and well-structured knowledge base. Moreover, the rule knowledge representation is quite natural and allows for an easy understanding and tracing the reasoning process. Nevertheless, there is much need and interest in improving the existing solutions in this area. Human decision-making power is mainly based on reasoning and decision-making in an uncertain environment, where only imprecise, incomplete, and vague information is available. This functional feature of

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knowledge-based systems should be taken into account in the course of their creation. In this field of research, the representation of uncertain information and the ability to draw conclusions from suppositions contribute to a major research challenge (Dubois and Prade, 1996). The fuzzy set theory has emerged as a powerful means to describe and deal with that kind of uncertainty (Zadeh, 1965),(Zimmermann, 1993).

For further improvement of the implementation of large knowledge bases, a graphical representation of the rule base is desirable. Petri nets are a suitable graphical and mathematical means of description for this purpose. For relatively long time, Petri nets have been very popular among people specialised in Artificial Intelligence due to the nets' adequacy to represent an approximate process as a dynamic discrete event system (Cardoso and Camargo, 1999).

The concept of a fuzzy Petri net has its origin in C.G. Looney's article (Looney, 1988). In the last four decades, many extensions of Petri nets or their modifications have been proposed (Chen et al., 1990),(Fryc et al., 2004a),(Fryc et al., 2004b),(Pedrycz and Gomide, 1994),(Pedrycz and Peters, 1998),(Peters et al., 1998),(Suraj, 2012a),(Suraj, 2012b),(Suraj, 2012c).

The paper presents a new methodology for knowledge representation and reasoning based on parameterised fuzzy Petri nets (in short *PFPNs*) (Suraj, 2012c). Recently, this net model has been proposed as a natural extension of generalised fuzzy Petri nets (Suraj, 2012a). The new class extends the generalised fuzzy Petri nets by introducing two parameterised families of sums and products, which are supposed to provide the suitable t-norms and s-norms.

In particular, this paper provides a method for knowledge representation as well as an algorithm for construction of a *PFPN* model on the base of a given set of fuzzy production rules. The proposed methodology is not only more convenient in terms of knowledge representation, but most of all it is more effective in the modelling of reasoning process as in the new class of fuzzy Petri nets the user has the chance to define the parameterised input/output operators. The choice of suitable operators for a given reasoning process and the speed of reasoning process are very important, especially in real-time decision support systems.

The structure of this paper is as follows. In Sect. 2 we give a brief introduction to triangular norms, parameterised families of sums and products, and *PFPNs*. Sect. 3 describes transformations of production rules into *PFPNs*. In Sect. 4 we present two algorithms. The first algorithm allows to construct the *PFPN* model based on a set of production rules in an automatic way. The second one describes a reasoning process modelled by means of a given *PFPN*. An example illustrating the performance of this algorithm is given in Sect. 5. Sect. 6 includes remarks on directions for further research related to the presented methodology.

2 PRELIMINARIES

In this section, basic notions and notation (especially concerning *PFPNs* (Suraj, 2012c)) used in the production rule representation and reasoning based on *PFPNs* are recalled.

2.1 Triangular Norms and their Families

Basic operations in the fuzzy set theory such as the intersection and the union, are defined by using the minimum and maximum functions. However, some other definitions of these operations are often employed, too. In particular, for the intersection and the union t-norms and s-norms are often used. As the t-norms and s-norms are also used for defining *PFPNs*, we recall their definitions (Fedrizzi and Kacprzyk, 1999),(Klement et al., 2000).

Let $[0,1]$ be the closed interval of all real numbers from 0 to 1 (0 and 1 are included).

A t-norm is defined as $t : [0,1] \times [0,1] \to [0,1]$ such that, for each $a,b,c \in [0,1]$: (1) it has 1 as the unit element, i.e., $t(a,1) = a$; (2) it is monotone, i.e., if $a \leq b$ then $t(a,c) \leq t(b,c)$; (3) it is commutative, i.e., $t(a,b) = t(b,a)$; (4) it is associative, i.e., $t(t(a,b),c) = t(a,t(b,c))$.

More relevant examples of t-norms are: the minimum $t(a,b) = min(a,b)$ used most widely, the algebraic product $t(a,b) = a * b$, the Łukasiewicz t-norm $t(a,b) = max(0,a+b-1)$.

An s-norm (or a t-conorm) is defined as $s : [0,1] \times [0,1] \to [0,1]$ such that, for each $a,b,c \in [0,1]$: (1) it has 0 as the unit element, i.e., $s(a,0) = a$, (2) it is monotone, i.e., if $a \leq b$ then $s(a,c) \leq s(b,c)$, (3) it is commutative, i.e., $s(a,b) = s(b,a)$, and (4) it is associative, i.e., $s(s(a,b),c) = s(a,s(b,c))$.

More relevant examples of s-norms are: the maximum $s(a,b) = max(a,b)$ used most widely, the probabilistic sum $s(a,b) = a + b - a * b$, the Łukasiewicz s-norm $s(a,b) = min(a+b,1)$.

There has been some intensive research in the field of logical operators carried out for the last three decades, which involves the development of parameterised families of sums and products. In Tables 1-2 exemplary lists of parameterised families of sums and

Table 1: An exemplary list of parameterised families of sums.

Sum S(a,b,v)	*Range v*
$\frac{a+b-(2-v)*a*b}{1-(1-v)*a*b}$	$(0,\infty)$
$1-[max(0,(1-a)^{-v}+(1-b)^{-v}-1)]^{\frac{-1}{v}}$	$(-\infty,\infty)$
$\frac{a+b-a*b-min(a,b,1-v)}{max(1-a,1-b,v)}$	$(0,1)$
$1-\log_v[1+\frac{(v^{1-a}-1)(v^{1-b}-1)}{v-1}$	$(0,\infty)$
$min[1,(a^v+b^v)^{\frac{1}{v}}$	$(0,\infty)$
$\frac{1}{1+[(\frac{1}{a}-1)^{-v}+(\frac{1}{b}-1)^{-v}]^{\frac{-1}{v}}}$	$(0,\infty)$

Table 2: An exemplary list of parameterised families of products.

Product T(a,b,v)	*Range v*
$\frac{a*b}{v+(1-v)*(a+b-a*b)}$	$(0,\infty)$
$[max(0,a^{-v}+b^{-v}-1)]^{\frac{-1}{v}}$	$(-\infty,\infty)$
$\frac{a*b}{max(a,b,v)}$	$(0,1)$
$\log_v[1+\frac{(v^a-1)(v^b-1)}{v-1}$	$(0,\infty)$
$1-min[1,((1-a)^v+(1-b)^v)^{\frac{1}{v}}]$	$(0,\infty)$
$\frac{1}{1+[(\frac{1}{a}-1)^v+(\frac{1}{b}-1)^v]^{\frac{1}{v}}}$	$(0,\infty)$

products are presented. For more details one shall refer to (Klement et al., 2000).

It is easy to observe that for the first elements of Tables 1 and 2, and the parameter $v=1$ we obtain the probabilistic sum and the algebraic product, respectively.

2.2 Parameterised Fuzzy Petri Nets

Now we recall the definition of *PFPNs* and their interpretation in the field of decision support systems.

We assume that the reader is familiar with the basic notions of classical Petri nets (Peterson, 1981).

A *parameterised fuzzy Petri net* (*PFP*-net) is a tuple (Suraj, 2012c):

$$N=(P,T,S,I,O,\alpha,\beta,\gamma,Op,\delta,M0),$$

where:

1. $P=\{p_1,\ldots,p_n\}$ is a finite set of *places*, $n>0$; $T=\{t_1,\ldots,t_m\}$ is a finite set of *transitions*, $m>0$; $S=\{s_1,\ldots,s_n\}$ is a finite set of *statements*; the sets P, T, S are pairwise disjoint and card(P) = card(S), where $card(X)$ denotes the number of elements in a set X;
2. $I:T\to 2^P$ is the *input function*, a mapping from a set of transitions to a family of all subsets of the set P; $O:T\to 2^P$ is the *output function*, a mapping from a set of transitions to a family of all subsets of the set P;
3. $\alpha:P\to S$ is the *statement binding function*, a bijective mapping from a set of places to a set of statements; $\beta:T\to[0,1]$ is the *truth degree function*, a mapping from a set of transitions to [0,1]; $\gamma:T\to[0,1]$ is the *threshold function*, a mapping from a set of transitions to [0,1]; Op is a finite set of parameterised families of sums and products called the *set of parameterised operators*; $\delta:T\to Op\times Op\times Op$ is the *operator binding function*, a mapping from a set of transitions to a set of all triples of parameterised operators from the set of parameterised operators Op;
4. $M0:P\to[0,1]$ is the *initial marking*, a mapping from a set of places to [0,1].

As for the graphical interpretation, places are denoted by circles and transitions by rectangles. The function I describes the oriented arcs connecting places with transitions, and the function O describes the oriented arcs connecting transitions with places. If $I(t)=\{p\}$ then a place p is called an *input place* of a transition t, and if $O(t)=\{p'\}$, then a place p' is called an *output place* of t. The initial marking $M0$ is an initial distribution of real numbers in the places. It can be represented by a vector of dimension n of real numbers from $[0,1]$. For $p\in P$, $M0(p)$ can be interpreted as a truth value of the statement s bound with a given place p by means of the binding function α, i.e., $\alpha(p)=s$. We assume that if the truth value of a statement attached to a given place is equal to 0 then the token does not exist in the place. The number $\beta(t)$ is interpreted as the truth degree of an implication (a rule) corresponding to a given transition t (Chen et al., 1990),(Fryc et al., 2004b). The meaning of the threshold function γ is explained below. The operator binding function δ connects transitions with triples of parameterised operators (In^v,Out_1^v,Out_2^v). The first operator in this triple is called the input parameterised operator, and two remaining ones are called the output parameterised operators. The input parameterised operator In^v belongs to one of the parameterised families of sums and products. It concerns the way in which all input places are connected to a given transition t (more precisely, statements corresponding to those places). Moreover, the output parameterised operator Out_1^v belongs to the parameterised families of products and Out_2^v belongs to the parameterised families of sums. Both of them concern the way in which the marking is computed after firing the transition t. This issue is explained more precisely below.

The *PFPN* dynamics defines how new markings are computed from the current marking when transitions are fired.

Let N be a *PFP*-net. A marking of N is a function $M:\ P\to[0,1]$.

Let $N = (P, T, S, I, O, \alpha, \beta, \gamma, Op, \delta, M0)$ be a *PFP*-net, $t \in T$, $I(t) = \{p_{i1}, \ldots, p_{ik}\}$ be a set of input places for a transition t, $\beta(t)$ be a value of the truth degree function β corresponding to t and $\beta(t) \in (0, 1]$, $\gamma(t)$ be a value of threshold function γ corresponding to t, M be a marking of N, and v be a parameter value for a parameterised family of sums and products. Moreover, let In^v be an input parameterised operator and Out_1^v, Out_2^v be output parameterised operators with a parameter value v corresponding to t.

A transition $t \in T$ is *enabled* for marking M and a parameter value v, if the value of parameterised input operator In^v for the transition t is positive and greater than, or equal to, the value of threshold function γ corresponding to t and the parameter value v, i.e.,

$$In^v(M(p_{i1}), \ldots, M(p_{ik})) \geq \gamma(t) > 0$$

for $p_{ij} \in I(t)$ and $j = 1, \ldots, k$.

In the paper we consider two modes for firing transitions.

(*Mode 1*) If M with a parameter value v is a marking of N enabling a transition t and $M_v^{'}$ the marking derived from M with v by firing t, then for each $p \in P$:

$M_v^{'}(p) =$

1. 0 if $p \in I(t)$,
2. $Out_2^v(Out_1^v(In^v(M(p_{i1}), \ldots, M(p_{ik})), \beta(t)), M(p))$ if $p \in O(t)$,
3. $M(p)$ otherwise.

In this mode, a procedure for computing the marking $M_v^{'}$ is as follows: (1) Numbers from all input places of the transition t are removed (the first condition from $M_v^{'}$ definition). (2) Numbers in all output places of t are modified in the following way: at first the value of a parameter value v is set, then the value of parameterised input operator In^v for all input places of t is computed, next the value of parameterised output operator Out_1^v for the value of In^v and for the value of truth degree function $\beta(t)$ is determined, and finally, the value corresponding to $M_v^{'}(p)$ for each $p \in O(p)$ is obtained as a result of parameterised output operator Out_2^v for the value of Out_1^v and the current marking $M(p)$ (the second condition from $M_v^{'}$ definition). (3) Numbers in the remaining places of net N are not changed (the third condition from $M_v^{'}$ definition).

(*Mode 2*) If M with a parameter value v is a marking of N enabling transition t and $M_v^{'}$ the marking derived from M with v by firing t, then for each $p \in P$:

$M_v^{'}(p) =$

1. $Out_2^v(Out_1^v(In^v(M(p_{i1}), \ldots, M(p_{ik})), \beta(t)), M(p))$ if $p \in O(t)$,
2. $M(p)$ otherwise.

The main difference in the definition of the marking $M_v^{'}$ presented above (*Mode 2*) concerns input places of the fired transition t. In *Mode 1* numbers are removed from all input places of the fired transition t ($cf.$ the first definition condition of *Mode 1*), whereas in *Mode 2* all numbers are copied from input places of the fired transition t (the second definition condition of *Mode 2*).

Let us consider a *PFPN* in Figure 1(a). For the net we have: the set of places $P = \{p_1, p_2, p_3, p_4, p_5\}$, the set of transitions $T = \{t_1, t_2\}$, the set of statements $S = \{s_1, s_2, s_3, s_4, s_5\}$, the input function I and the output function O in the form: $I(t_1) = \{p_1, p_2\}$, $I(t_2) = \{p_2, p_3\}$, $O(t_1) = \{p_4\}$, $O(t_2) = \{p_5\}$. Moreover, there are: the statement binding function $\alpha : \alpha(p_1) = s_1$, $\alpha(p_2) = s_2$, $\alpha(p_3) = s_3$, $\alpha(p_4) = s_4$, $\alpha(p_5) = s_5$, the truth degree function β: $\beta(t_1) = 0.7$, $\beta(t_2) = 0.8$, the threshold function γ: $\gamma(t_1) = 0.4$, $\gamma(t_2) = 0.3$, the set of parameterised operators $Op = \{S(.), T(.)\}$ and the operator binding function δ defined as follows: $\delta(t_1) = (S(.), T(.), S(.))$, $\delta(t_2) = (T(.), T(.), S(.))$ with a relevant parameter value v, and the initial marking $M0 = (0.6, 0.4, 0.7, 0, 0)$.

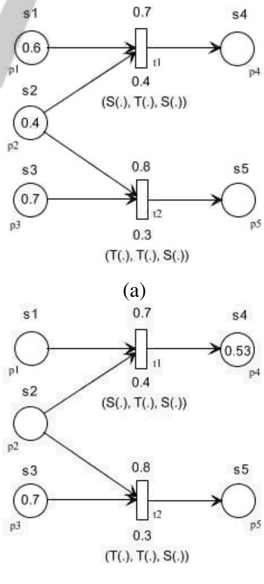


Figure 1: (a) A *PFPN* with the initial marking before firing the enabled transitions t_1 and t_2. (b) An illustration of a firing rule: the marking after firing t_1, where t_2 is disabled (*Mode 1*).

If we take, for instance, the first parameterised family of sums $S(a, b, v)$ from Table 1, the first parameterised family of products $T(a, b, v)$ from Table 2 and a parameter value $v = 1$, then $S(a, b, 1) = a + b - a * b$ and $T(a, b, 1) = a * b$. For the transition

t_1 we have $S(0.6, 0.4, 1) = 0.6 + 0.4 - 0.6 * 0.4 = 0.76$ and $T(0.76, 0.7, 1) = 0.76 * 0.7 = 0.53$ by the initial marking $M0$ and $v = 1$. As the global value for all input places of t_1 and $v = 1$ equals 0.76, it is positive and greater than $\gamma(t_1)$ = 0.4. Thus, the transition t_1 is enabled by $M0$ and $v = 1$. Firing transition t_1 by the marking $M0$ and $v = 1$ according to *Mode 1* transforms $M0$ to the marking $M_1^{'} = (0, 0, 0.7, 0.53, 0)$ (Figure 1(b)). It is easy to check that by the initial marking $M0$ and $v = 1$ the transition t_2 is not enabled.

For more detailed information about *PFPNs* the reader is referred to (Suraj, 2012c).

3 NET REPRESENTATION OF PRODUCTION RULES

Now, we present a method of transforming rules into a *PFPN* depending on the form of a transformed rule.

By using a *PFPN*, a basic rule (**Type 0**) of the form:

IF s_j THEN s_k $(CF = q_i)$

can be modelled as shown in Figure 2(a). The value of a certainty factor (CF) is $q_i \in [0, 1]$. It represents the reliability degree of the rule. The larger value q_i, the more credible the rule. Similarly, the value $r_i \in [0, 1]$ and represents the threshold value. Larger value of r_i requires greater truth degree of the rule precedence, i.e., s_j. However, the operator In^v, and the operators Out_1^v, Out_2^v represent the parameterised input operator and the parameterised output operators, respectively. They play an important role in a description of rule firing.

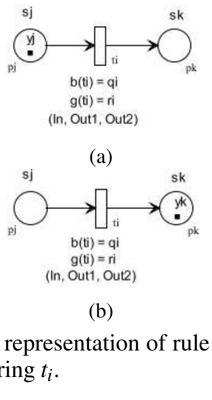


Figure 2: A *PFPN* representation of rule type 0: (a) before firing t_i, (b) after firing t_i.

According to Figure 2(b) the value in an output place p_k of a transition t_i corresponding to the rule is calculated as

$$y_k = Out_1^v(y_j, q_i).$$

If the premise or the conclusion of a production rule contains AND or OR (classical propositional connectives), it is called a *composite production rule*.

Below, there types of composite production rules are discussed:

Type 1: IF s_{j1} AND(OR) ... AND(OR) s_{jn} THEN s_k $(CF = q_i)$. This rule type can be modelled by a *PFPN* as shown in Figure 3(a). The value is calculated in the output place as follows (Figure 3(b)):

$$y_k = Out_1^v(In^v(y_{j1}, \dots, y_{jn}), q_i).$$

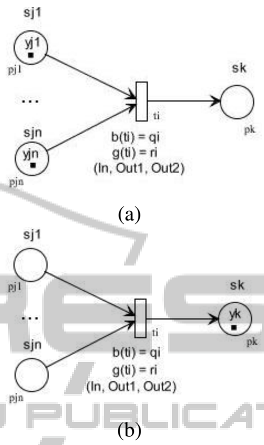


Figure 3: A *PFPN* representation of rule type 1: (a) before firing t_i, (b) after firing t_i.

Type 2: IF s_j THEN s_{k1} AND ... AND s_{kn} $(CF = q_i)$. This rule type can be modelled by a *PFPN* as shown in Figure 4(a). The value is calculated in each output place as follows (Figure 4(b)):

$$y_k = Out_1^v(y_j, q_i).$$

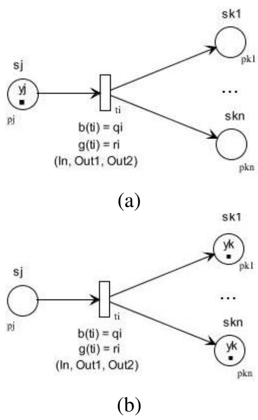


Figure 4: A *PFPN* representation of rule type 2: (a) before firing t_i, (b) after firing t_i.

Type 3: IF s_j THEN s_{k1} OR ... OR s_{kn} $(CF = q_i)$. Due to the fact that this rule type does not describe any specific implication, we do not consider it in the paper.

Remarks:

1. Due to technical reasons the names of functions β, γ in Figures 2-4 are represented by b and g, respectively.
2. Similarly, the parameter v appearing in the operators In^v, Out_1^v and Out_2^v in Figures 2-4 is omitted.
3. As rule Types 0 and 2 have only one statement in their premises, we may omit the input parameterised operator In^v in Figures 2 and 4. Nevertheless, for better readability of these figures we leave the operator where it is.
4. We assume that the initial markings of output places are equal to 0 in all net models corresponding to the considered rule types. Therefore, in the descriptions of the values in output places we do not regard the parameterised output operator Out_2^v. In the opposite case, i.e., for non-zero markings of output places, we should take into account this parameterised output operator. Thus, in each formula presented above the final value $y_k^{'}$ should be computed as follows:

$$y_k^{'} = Out_2^v(y_k, M(p)),$$

where y_k denotes the values computed for suitable rule types by means of formulas presented above, and $M(p)$ is a marking of output place p. Intuitively, a final value corresponding to $M_v^{'}$ for each $p \in O(p)$ is obtained as a result of Out_2^v operation for the computed Out_1^v operation value and the current marking $M(p)$ (the first/second condition depending on the firing mode from $M_v^{'}$ definition, Subsect. 2.2).

4 ALGORITHMS

4.1 Knowledge Representation

This section presents an algorithm for constructing a *PFPN* on the base of a given set of production rules.

Algorithm 1: A construction of *PFPN*.

Input: A finite set R of production rules.
Output: A *PFP*-net N.
begin
  $F := \emptyset$;
  foreach $r \in R$ **do**
   begin
    if *r is a rule of Type 0* **then** *Construct a subnet N_r as shown in Figure* 2(a)
    else if *r is a rule of Type 1* **then** *Construct a subnet N_r as shown in Figure* 3(a)
    else if *r is a rule of Type 2* **then** *Construct a subnet N_r as shown in Figure* 4(a);
    $F := F \cup \{N_r\}$;
   end;
  Integrate all subnets from a family F on joint places and create a result net N;
  return N;
end.

Notice: The symbol := denotes the assignment operator.

The input for this algorithm is a set of given production rules R, whereas the output is a *PFP*-net N. We can divide operating of the algorithm into two phases. In the first phase the algorithm constructs a suitable subnet N_r depending on the rule type described in Sect. 3. In the second one it creates a structure of a result net N integrating the constructed subnets on joint places, i.e., places with which the same statements appearing in different rules are associated. In order to obtain a *PFPN* serving as the net model of the approximate process described by production rules from the set R, an initial marking corresponding to the given statement truth values should be added to the result net.

4.2 Approximate Reasoning

In many situations we may want to determine the antecedent-consequence relationships between two groups of statements: the starting (given) statements $s_{i1}, \ldots, s_{ik}$, and goal (computed) statements $s_{o1}, \ldots, s_{ol}$. In the Petri net representation, the places associated with the first group of statements are called *starting places*, whereas places associated with the second one are called *goal places*. Furthermore, if the truth degrees of starting statements $s_{i1}, \ldots, s_{ik}$ are given, we may want to know what the truth degrees of the goal statements $s_{o1}, \ldots, s_{ol}$ are. These problems can be solved by using an approximate reasoning algorithm based on *PFPNs*.

We assume that the truth degrees of starting statements are given by the expert or they are identified by sensors in finite time units. The goal of reasoning is to determine the truth degrees of output (goal) statements.

Algorithm 2: A description of approximate reasoning process.

Input: The initial marking of starting places.
Output: The final marking of goal places.
begin
  while *It is not the end of the simulation* **do**

begin
Determine transitions enabled for firing;
while *There is a transition enabled for firing* **do**
begin
Compute a new marking of all places after firing the transition;
Determine a new transition enabled for firing;
end;
Read the final marking of goal places;
Reset the final marking of all places;
end;
end.

Algorithm 2 can be performed in two different modes corresponding to *Mode 1* or *Mode 2* described in section 2.2. After firing a given transition (rule) in *Mode 1*, tokens from its input places are removed. Firing a given transition (rule) and removing tokens can be intuitively interpreted as an execution of reasoning by using this rule in a given reasoning process. Hence in the next steps, markings of input places of a fired rule are already unnecessary. Such reasoning can be understood as a kind of forward reasoning. However, in *Mode 2* after firing a given transition (rule), tokens remain in its input places. Contrary to *Mode 1*, this way of firing rules can be applied to the systems for which some statements appear in antecedents of several rules fired in different reasoning steps. In the following section we present an example of these two algorithms' use.

5 ILLUSTRATING EXAMPLE

In order to illustrate the simplicity and the power of our methodology, let us show an application of the algorithms described in Sect. 4 in the domain of train traffic control.

We consider the following situation: a train B waits at a certain station for a train A to arrive in order to allow some passengers to change train A to train B. Now, a conflict arises when the train A is late. In this situation, the following alternatives can be taken into consideration:

1. Train B waits for train A to arrive. In this case, train B will depart with delay.
2. Train B departs in time. In this case, passengers disembarking train A have to wait for a later train.
3. Train B departs in time, and an additional train is employed for the train $A's$ passengers.

To make a decision, several inner conditions have to be taken into account, for example: the delay period, the number of passengers changing trains, etc. The discussion regarding an optimal solution to the problem of divergent aims such as: minimisation of delays throughout the traffic network, warranty of connections for the customer satisfaction, efficient use of expensive resources, etc. is disregarded at this point.

In order to describe the traffic conflict, we propose to consider the following set R of three rules:

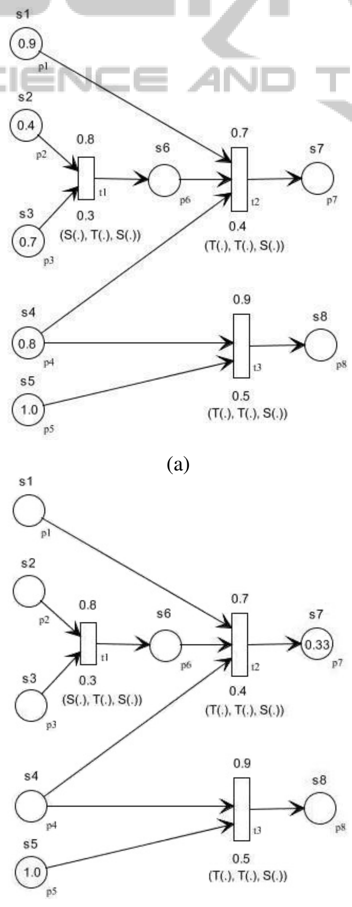
1. IF s_2 OR s_3 THEN s_6 ($CF = 0.8$)
2. IF s_1 AND s_4 AND s_6 THEN s_7 ($CF = 0.7$)
3. IF s_4 AND s_5 THEN s_8 ($CF = 0.9$)

where: s_1: 'Train B is the last train in this direction today', s_2: 'The delay of train A is huge', s_3: 'There is an urgent need for the track of train B', s_4: 'Many passengers would like to change for train B', s_5: 'The delay of train A is short', s_6: '(Let) train B depart according to schedule', s_7: 'Employ an additional train C (in the same direction as train B)', and s_8: 'Let train B wait for train A'.

Applying the algorithm 1 from Sect. 4 to the set R of rules together with their parameters we obtain the *PFP*-net N corresponding to these rules shown in Figure 5(a), where the logical operators OR, AND are interpreted as the probabilistic sum $S(\cdot)$ and the algebraic product $T(\cdot)$, respectively. Moreover, we assume that the threshold value of these rules is equal to 0.3, 0.4 and 0.5, respectively. Note that the places p_1, p_2, p_3 and p_4 include the fuzzy values 0.9, 0.4, 0.7 and 0.8 corresponding to the statements s_1, s_2, s_3 and s_4, respectively. In this example, the statement s_5 attached to the place p_5 is the only crisp one and its value is equal to 1. Moreover, we assume that the set of parameterised operators $Op = \{S(.), T(.)\}$ and the operator binding function δ is defined as in Figure 5(a). Assessing the statements attached to the places from p_1 up to p_5, we observe that the transitions t_1 and t_3 can be fired. Firing these transitions according to the firing rules for the *PFPN* model allows computation of the support for the alternatives in question. In this way, the possible alternatives are ordered with regard to the preference they achieve from the knowledge base. This order forms the basis for further examinations and simulations and, ultimately, for the dispatching proposal. If one chooses a sequence of transitions $t_1 t_2$ they obtain the final value, corresponding to the statement s_7, equal to 0.33 (Figure 5(b)). In the other case (i.e., for the transition t_3 only), the final value, this time corresponding to the statement s_8, equals 0.72.

If we interpret the logical operators OR, AND as the Łukasiewicz s-norm and Łukasiewicz t-norm, respectively, and if we choose a sequence of transitions $t_1 t_2$ then the final value is not possible to obtain. After firing the transition t_1 by the initial marking $M0$ we obtain the result marking by which the transition t_2 is not able to fire. In the other case, i.e., for the transition t_3, we obtain the final value for the statement s_8 equal to 0.7.

This example shows that different interpretations for the logical operators OR and AND may lead to quite different decision results. Therefore, we propose a new fuzzy net model in the paper, which is more flexible than the classical one as in the former class the user has the chance to define both the parameterised input/output operators. Chosing a suitable interpretation for logical operators OR and AND we may apply the mathematical relationships between t-norms and s-norms or their families presented e.g. in (Klement et al., 2000). The rest in this case certainly depends on the experience of the model designer to a significant degree. We omit this aspect of considerations with respect to a limited space of the paper.



(b)

Figure 5: (a) An example of the *PFPN* model of train traffic control. (b) An illustration of a firing rule: the marking after firing a sequence of transitions $t_1 t_2$ (*Mode 1*).

6 CONCLUSIONS

In this paper, we have proposed a new methodology for knowledge representation and reasoning based on *PFPNs*. In particular, we have also provided two algorithms supporting the proposed methodology. These algorithms have been implemented in an experimental application called *PNES* which supports our methodology. This application has been developed on *IBM PC*, in *Java*. *PNES* consists of an editor and a simulator. The editor allows inputting, editing and selecting the values of the parameters for *PFPNs* while the simulator starts with a given initial marking and executes enabled transitions visualising reached markings. Thanks to parallel firing rules corresponding to transitions in a net representation we can speed up an approximate process.

The proposed methodology has proved to be very suitable for the design and implementation of a decision support system in an exemplary problem with relatively high practical relevance.

In our further investigations we will consider the use of new classes of fuzzy Petri nets for inhibitory rule representation (Delimata et al., 2009). Another interesting problem arises while choosing relevant parameterised families of sums and products for actual applications. These are examples of problems we wish to examine exploring the approach presented in the paper.

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