A Novel Mathematical Formulation for the Strategic Planning of a Reverse Supply Chain Network
Theoretical and Computational Results

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Abstract: In the last decade, literature on strategic planning of a supply chain network grew rapidly. In this paper we address a classical three-layer remanufacturing supply chain network design problem that covers sourcing, reprocessing and remanufacturing activities, in which strategic decisions regarding the number, location of reprocessing units and the flow of returns through the logistics network are made. First, we propose an alternative mixed-integer mathematical programming (MILP) formulation for this problem and provide theoretical proof of equivalence between the classical and the proposed mathematical formulation. Second, the goodness of both formulations is compared by means of a computational study, and the results for large instances of the problem are discussed. We empirically prove that the proposed formulation provides tighter linear relaxation lower bounds and obtains the integer solutions several times faster than the classical formulation.

1 INTRODUCTION

Many authors have studied the relationship that exists between Supply Chain Management (SCM) and Advanced Planning and Scheduling (APS) systems (see for example Stadtler and Kilger, 2005). For some authors, one of the advantages of an APS approach is that it makes possible to include suppliers and customers in the planning process and thereby optimise the entire supply chain on a real-time basis. As a consequence it enables “to extract real-time information from that chain, with which to calculate a feasible schedule, resulting in a fast, reliable response to the customer” (Amstel, 1998). The planning of supply chain is considered a strategic issue. On this regards, “the strategic level deals with decisions that have a long-lasting effect on the firm. These include decisions regarding the number, location and capacities of warehouses and manufacturing plants, or the flow of material through the logistics network.” (Simchi-Levi et al., 2007).

This paper addresses a class of planning problems that arise in the design of a remanufacturing supply chain network. We study a classical three-layer facility location model for designing a reverse supply chain that covers the sourcing, processing and remanufacturing activities. The management of return products and waste stewardship has become major concerns for companies and organisations that are interested in sustainable practices. In this context, remanufacturing activities are recognised as a main option of recovery in terms of their feasibility and benefits.

The problem is an NP-hard combinatorial optimisation problem and it has been previously modeled as a mixed integer linear programming (MILP) problem (Jayaraman et al., 2003).

This paper makes three main contributions. First, it is proposed a novel MILP formulation for the problem. Second, we provide theoretical proof that the proposed MILP formulation is stronger than the existing classical weak and strong formulation of the problem. Third, we conclude that the proposed MILP formulation outperforms the classical formulation in terms of the quality of the linear relaxation lower bound and the computing times.

The remainder of this paper is organized as follows: In the second section, we provide a literature review with a brief introduction to

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sustainable and reverse supply chains and with a special emphasis on the remanufacturing case. In the third section, we propose a new mathematical model and prove that the convex-hull associated with the linear relaxation of this formulation is contained in the convex-hull of the classical formulation. In the fourth section, we present experimental results for large sets of data that were generated randomly. The last section contains conclusions and directions for future research.

2 LITERATURE REVIEW

In the last few years, mathematical modeling and solution methods for the efficient management of return flows (and/or integrated with forward flows) has been studied in the context of reverse logistics, closed-loop supply chains, and sustainable supply chains. The problem of locating facilities and allocating customers is not new to the operations research community and covers the key aspects of supply chain design (Daskin et al., 2005). This problem is one of "the most comprehensive strategic decision problems that need to be optimized for long-term efficient operation of the whole supply chain" (Altiparmak et al., 2006).

Mathematical models proposed in the literature for planning reverse logistics networks have been reviewed by Fleischmann et al., (2000). Fleischmann et al., (2001) proposed a generic recovery network model based on the elementary characteristics of return networks identified in Fleischmann et al. (2000). Zhou and Wang (2008) proposed a generic mixed integer model for the design of a reverse distribution network including repairing and remanufacturing options simultaneously.

Models for reverse logistics networks in connection with location problems have been discussed by Bloemhof-Ruwaard, Salomon, and Van Wassenhove (1996) and Barros, Dekker, and Scholten (1998). For example, the last authors described a network for recycling sand from construction waste and proposed a two-level location model to solve the location problem of two types of intermediate facility.

Regarding remanufacturing location models, Krikke et al., (1999) described a small reverse logistics network for the returns, processing, and recovery of discarded copiers. They presented a MILP model based on a multi-level uncapacitated warehouse location model. The model was used to determine the locations and capacities of the recovery facilities as well as the transportation links connecting various locations. In Jayaraman et al., (2003), a 0-1 MILP model for a product recall distribution problem is proposed. They analysed a particular case in which the customer returns the product to a retail store and the product is sent to a refurbishing site which will rework the product or dispose it properly. The reverse supply chain is composed of origination, collection, and refurbishing sites. With the objective to minimize fixed and distribution costs, the model has to decide which collection sites and which refurbishing sites to open, subject to a limit on the number of collection sites and refurbishing sites that can be opened.

Several authors have studied different aspects of closed-loop supply chain design problems. See, for example, Jayaraman et al., (1999), Fleischmann (2003), Barbosa-povoa et al., (2007); Guide and Van Wassenhove (2009), and Neto et al., (2010). For example, Sahyouni et al., (2007) presented three generic facility location MIP models for the integrated decision making in the design of forward and reverse logistics networks. The formulations are based on the well-known uncapacitated fixed-charge location model, and they include the location of used product collection centers and the assignment of product return flows to these centers. Lu and Bostel (2007) presented a two-level location problem with three types of facilities to be located in a reverse logistics system. They proposed a 0–1 MILP model which simultaneously considers "forward" and "reverse" flows and their mutual interactions. The model has to decide the number and locations of three different types of facilities: producers, remanufacturing centers, and intermediate centers.

Reverse logistics models are recently discussed by Salema et al., (2010); Gomes et al., (2011) and Alumur et al., (2012). Almost all this research proposed MILP models. The majority of solution methods are based on standard commercial packages.

3 MATHEMATICAL MODEL

In this section a new MILP model for the problem of designing a remanufacturing and sustainable supply chain network is proposed. This problem is a single product, static, three-layer, capacitated location model with known demands. The remanufacturing supply chain network consists of three types of members: sourcing facilities (origination sites such as a retail store), collection sites (reprocessing facilities) and remanufacturing facilities. At the customer layer, there are product demands and used products that are ready to be recovered. It is assumed that customers return the products to origination sites such as a retail store. In the second layer of the supply chain network, there are reprocessing sites that are used only in the reverse channel, and they are responsible for activities such as cleaning, disassembly, checking and sorting before the returned products are sent back to the remanufacturing facilities. In the third layer, remanufacturing facilities accept the checked returns from reprocessing facilities, and they are responsible for the process of remanufacturing. A classical MILP model was proposed by Jayaraman et al., (2003). In such a supply chain network, the reverse flow, from customers through collection sites to remanufacturing facilities, is formed by used products, while the other flow (the "forward" flow) is from remanufacturing facilities directly to the point of sales for the "new" products. The number and location of reprocessing and remanufacturing facilities must be decided to minimise the transportation, distribution and fixed costs. As noted by Jayaraman et al. (2003), the type of remanufacturing problem that is addressed in this paper is more like the type called product recalls, whereby customers return products that have reached the end their useful life or are defective.

3.1 Classical Model

This is a MILP model proposed by Jayaraman et al., (2003). It is assumed that the product demands (new products) and the available quantities of used products from the customers are known and deterministic. All of the returned products are first shipped back to collection facilities, where some of them will be disposed of for various reasons, such as poor quality. The checked return-products will then be sent back to remanufacturing facilities, where some of them could still be disposed of. The product demands from the customers can be met by point of sale facilities, which receive products from the remanufacturing facilities. In this problem, remanufactured products are considered to be the same as the new products coming from "traditional" producers in terms of satisfying the customer demands.

The model proposed by Jayaraman et al., (2003) introduces the triply subscripted flow variable X_{jkl} to represent a fraction of the unit demand at location j that is shipped to l through a reprocessing facility located at k. We introduce a constant M, which represents the cardinality of set J.

We introduce the following inputs and sets:

J = the set of sourcing facilities in the first layer, indexed by j

L = the set of candidate remanufacturing facility locations in the third layer, indexed by l

K = the set of candidate reprocessing facility locations in the middle layer, indexed by k

a_j = the supply quantity at the source location $j \in J$

b_l = the demand quantity at the remanufacturing location $l \in L$

f_k = the fixed cost of locating a mid-layer reprocessing facility at candidate site $k \in K$

g_l = the fixed cost of locating a remanufacturing facility at candidate site $l \in L$

c_{jkl} = the unit cost of delivering products at $l \in L$ from a source facility located in $j \in J$ through facility $k \in K$

m_k = the capacity at reprocessing facility location $k \in K$

We consider the following decision variables:

w_k= 1 if we locate a reprocessing facility at candidate site $k \in K$; 0 otherwise

y_l = 1 if we locate a remanufacturing facility at candidate site $l \in L$; 0 otherwise

X_{jkl} = fraction of the unit flow from the source facility $j \in J$ to the remanufacturing facility located at $l \in L$ through facility $k \in K$

RSCLP:

Minimize v(RSCLP) =

$$\sum_{k \in K} f_k w_k + \sum_{l \in L} g_l y_l + \sum_{j \in J}\sum_{k \in K}\sum_{l \in L} c_{jkl} a_j X_{jkl} \tag{1}$$

$$\sum_{k \in K}\sum_{l \in L} X_{jkl} = 1 \qquad \forall\, j \in J \tag{2}$$

$$\sum_{j \in J}\sum_{k \in K} a_j X_{jkl} \le b_l \qquad \forall\, l \in L \tag{3}$$

$$\sum_{j \in J}\sum_{l \in L} a_j X_{jkl} \le m_k \qquad \forall\, k \in K \tag{4}$$

$$\sum_{j \in J}\sum_{l \in L} X_{jkl} \le M w_k \qquad \forall\, k \in K \tag{5}$$

$$\sum_{j \in J}\sum_{l \in K} Xx_{jkl} \le M y_l \qquad \forall\, l \in L \tag{6}$$

$$X_{jkl} \ge 0, w_k, y_l \in \{0,1\} \quad \forall\, j \in J, \forall\, k \in K, \forall\, l \in L \tag{7}$$

In this formulation, constraint (2) ensures that all of the products from the sourcing facility j are transported to remanufacturing facilities l through collection sites k. Constraint (3) ensures that all of the products that arrive at site l must be less than its demand. Constraint (4) ensures that all of the products that arrive to and ship from collection site k must be less than its capacity. Constraint (5) ensures that the products that arrive to and ship from a collection site have already been opened at site *k*. Constraint (6) warrants that all of the products that arrive at the remanufacturing facilities have already been opened at site *l*. Constraint (7) is a positive and binary constraint.

This model has $O(n^3)$ positive variables, where $n=max\{|J|, |K|, |L|\}$ and $\{|K|+|L|\}$ are binary variables. The number of constraints is $O(n)$. In fact, this model has $(nqm+q+m)$ variables and $(n+2q+2m)$ constraints, where $|J| = n$, $|K| = q$ and $|L| = m$.

Following the results from the well-known uncapacitated facility location problem (UFLP), this is a weak formulation for the problem because of constraints (5) and (6). A stronger formulation (*RSCLP-T*) is obtained by replacing constraints (5) and (6), respectively, by the following constraints:

$$X_{jkl} \le w_k \qquad \forall j \in J, \forall k \in K, \forall l \in L \qquad (8)$$

$$X_{jkl} \le y_l \qquad \forall j \in J, \forall k \in K, \forall l \in L \qquad (9)$$

By limiting the number of reprocessing and remanufacturing facilities to open, the following two sets of constraints must be added to the *RSCLP* model:

$$p_{min} \le \sum_{k \in K} w_k \le p_{max} \qquad (10)$$

$$q_{min} \le \sum_{l \in L} y_l \le q_{max} \qquad (11)$$

Parameters p_{min} and p_{max} limit the minimum and maximum quantity of reprocessing facilities to open. The same reasoning applies for parameters q_{min} and q_{max} on the remanufacturing facilities.

Jayaraman et al., (2003) solved model *RSCLP* and *RSCLP-T* using *AMPL* and *CPLEX* to optimally solve for instances of networks of up to 100 sourcing facilities, 40 candidate sites for locating reprocessing facilities (p_{max} =8) and 30 candidate sites for locating remanufacturing facilities (q_{max} = 6), using a maximum of 50,028.6 seconds of computing time. They also developed some heuristics for solving the problem, but it is not the objective of this paper to discuss them.

3.2 Proposed Model

In this proposed model, we introduce two new variables, x_{jk} and z_{kj}, to break down the flows of return products from the sourcing site *j* to the facility *l* into two parts, i.e., the flow that goes from *j* to *k* and the flow that goes from *k* to *l*. The remaining variables and parameters retain the same values as in the above *RSCLP* model. We also eliminated some constraints, and other constraints are included, as described in the following:

We consider the following parameters and decision variables:

c_{jk} = the unit cost of delivering products at $k \in K$ from a source facility located at $j \in J$

d_{kl} = the unit cost of delivering products at $l \in L$ from a reprocessing facility located at $k \in K$

x_{jk} = the flow from source facility $j \in J$ to the reprocessing facility located at $k \in K$

z_{kl} = the flow from the reprocessing facility located at $k \in K$ to the remanufacturing facility $l \in L$

Note that models such as the *RSCLP-P* do not account for the origin of the products that arrive at the remanufacturing facilities; we lose track of the origins of those products. In some real applications, for example, biomedical waste, we would like to control the origin of the products that arrive at the remanufacturing facilities.

RSCLP-P:

Minimize v(RSCLP-P)

$$\sum_{k \in K} f_k w_k + \sum_{l \in L} g_l y_l + \sum_{j \in J}\sum_{k \in K} c_{jk} x_{jk} + \sum_{k \in K}\sum_{l \in L} d_{kl} z_{kl} \qquad (1a)$$

$$\sum_{k \in K} x_{jk} = a_j \qquad \forall j \in J \qquad (2a)$$

$$\sum_{j \in J} x_{jk} \le m_k w_k \qquad \forall k \in K \qquad (3a)$$

$$\sum_{k \in L} z_{kl} \le b_l y_l \qquad \forall l \in L \qquad (4a)$$

$$\sum_{j \in K} x_{jk} = \sum_{l \in L} z_{kl} \qquad \forall k \in K \qquad (5a)$$

$$x_{jk}, z_{kl} \ge 0, w_k, y_l \in \{0,1\} \; \forall i \in I, \forall j \in J, \forall k \in K \qquad (7)$$

In this formulation, the first two terms of the objective function (*1a*) sum up the installation costs, and its two second terms sum up the transportation and delivery costs. Constraint (*2a*) guarantees that all of the return products available at the origination site *j* must be shipped. Constraint (*3a*) ensures that all of the products that arrive at reprocessing site *k* must be lesser than its capacity and that this facility must be opened. Constraint (*4a*) guarantees that all of the products that are delivered at the remanufacturing facility *l* must be less than its capacity and that this facility must be opened. Constraint (*5a*) is a type of flow conservation, which ensures that all of the products that arrive at reprocessing facility *k* must also leave it. Note that in this model, the number of positive variables is $O(n^2)$, in contrast to $O(n^3)$ of the classical model. The number of integer variables remains the same for both models. Compared with the weak formulation, both of the models have $O(n)$ number of constraints. However, this *RSCLP-P* model has $O(n)$ compared with $O(n^3)$ constraints of the strong formulation of *RSCLP*. In fact, this model has ($2nq+q+m$) variables and ($n+2q+m$) constraints versus ($nqm+q+m$) variables and ($n+2q+2m$) constraints in the weak formulation of *RSCLP*.

3.3 Proof of Equivalency and Strength between the MILP Models

For space reasons, theoretical proof of equivalency between the proposed *RSCLP-P* model and the weak and strong classical models are not presented here. For the same reason, it is only summarized the proof of strength between the proposed *RSCLP-P* model and weak classical model.

Lemma 1: RSCP-P Model is stronger than RSCP Model. Let $S_{RSCP} = \{(X,w,y) \in R^{|J|x|K|x|L|+|k|+|L|} \mid X_{jkl}$, w_k and y_l satisfy (2)-(7), (10)-(11)$\}$ be the set of feasible solutions of *RSCLP*, $S_{RSCP\text{-}P} = \{(x,z,w,y) \in R^{|J|x|K|+|K|x|L|+|k|+|L|} \mid x_{jk}, z_{kl}, w_k$ and y_l satisfy (2a)-(5a), (7), (10)-(11)$\}$ the set of feasible solutions of *RSCLP-P*, $\overline{S}_{RSCP} = \{(X,w,y) \in R^{|J|x|K|x|L|+|k|+|L|} \mid X_{jkl} \geq 0$, $0 \leq w_k \leq 1$, $0 \leq y_l \leq 1$ and satisfying (2)-(6), (10)-(11)$\}$ the set of solutions to the LP relaxation of *RSCLP*, and $\overline{S}_{RSCP\text{-}P} = \{(x,z,w,y) \in R^{|J|x|K|+|K|x|L|+|k|+|L|} \mid x_{jk} \geq 0$, $z_{kl} \geq 0$, $0 \leq w_k \leq 1$, $0 \leq y_l \leq 1$ and satisfying (2a)-(5a), (10)-(11)$\}$ the set of solutions to the LP relaxation of *RSCLP-P*. Then, $\overline{S}_{RSCP\text{-}P} \subseteq \overline{S}_{RSCP}$.

Proof: Consider an instance of *RSCP* with $J=\{1,2,3\}$, $K=\{1,2\}$, $L=\{1,2\}$; $C_{jkl}=1$, $\forall j \in J, k \in K, l \in L$; $f_k=1$, $\forall k \in K$; $g_l=1$, $\forall l \in L$; $a_j=1$, $\forall j \in J$; $u_k=2$, $\forall k \in K$; $b_l=3$, $\forall l \in L$; $p_{min}=q_{min}=0$; $p_{max}=q_{max}=2$.

Let $(X,w,y) \in \overline{S}_{RSCP}$ with

$X_{111} = X_{121} = 0.5$ and $X_{211} = X_{321} = 1$; $w_1 = w_2 = 0.5$; $y_1 = 1$ and $y_2 = 0$, and the remaining values equal to zero.

We can verify that (X,w,y) *satisfy (1)-(7), (10) and (11).*

From (12) and (13), we obtain for (x,z)

$x_{11} = x_{12} = 0.5$; $x_{21} = x_{32} = 1$; $z_{11} = z_{21} = 1.5$ and, $w_1 = w_2 = 0.5$; $y_1 = 1$ and $y_2 = 0$. All of the other values are equal to zero.

However, constraint (3a) is violated $\forall j \in \{1,2,3\}$ and $\forall k \in \{1,2\}$.

Thus model, *RSCLP-P* provides a tighter formulation than model *RSCLP* for the same problem.

4 COMPUTATIONAL EXPERIMENTS

The goodness of both formulations is compared by means of a computational study, and the results for large instances of the problem are discussed here. Our interest was to benchmark the computation times and the quality of the lower bound provided by the linear programming relaxation. The models were implemented in GAMS, and it was used CPLEX (version 12.2) with default settings to solve all of the test instances that are presented in this section. All of the computational tests were performed on a PC with 1 GB of RAM memory and a 2.3 GHz processor.

We randomly generated 10 test problems following similar methodologies used for well-known related supply chain problems (for example: (Fleischmann et al., 2001); (Lu and Bostel, 2007)). These test problems correspond to networks of up to 600 origination sites, 100 candidate sites for locating reprocessing facilities and 40 candidate sites for locating remanufacturing facilities. The data sets for the test problems are given in Table 1, and they are available from the authors; because of the restriction on the size of papers, the authors have not provided the full data in this paper. All of the transportation costs were generated randomly using a uniform distribution with parameters [1,40]. The fixed costs for the remanufacturing facilities were obtained by multiplying by 5 the fixed costs of the reprocessing facilities, following Jayaraman et al., (2003). Sourcing units (a_j), the capacity of reprocessing facilities (m_k) and the capacity of remanufacturing facilities (b_l) are shown in Table 1.

Table 1: Data set.

#	J	K	L	Fixed costs f_k	Fixed costs g_l	a_j	m_k	b_l
1	40	20	15	3000	15000	150	400	2000
2	70	30	20	5000	25000	150	500	2500
3	100	40	20	5000	25000	150	500	2500
4	150	40	20	10000	50000	200	800	4000
5	200	80	20	10000	50000	300	800	4000
6	300	80	40	20000	100000	200	800	4000
7	350	100	40	20000	100000	200	800	4000
8	400	100	40	20000	100000	200	1500	7500
9	500	100	40	20000	100000	200	1500	7500
10	600	100	40	25000	125000	300	3000	15000

Table 2 illustrates for some instances, the size differences between the *RSCLP* model and proposed formulation. For example, for instance #7 the total number of continuous variables is 1,400,000 and

39,000 for the weak classical and proposed model respectively. For instance #10 the number of continuous variables is 2,400,000 and 64,000 for the weak classical and proposed model respectively. Although they share the same number of integer variables, the classical model has up to 50 times the number of continuous variables.

Table 2: Size differences between the classical model and the proposed formulation.

	Classical (weak) model			Proposed formulation		
#	X_{jkl}	y_l; w_k	*No. cstr*	x_{jk}; z_{kl}	y_l; w_k	*No. cstr*
7	1,400,000	140	632	39,000	140	592
8	1,600,000	140	682	44,000	140	642
9	2,000,000	140	782	54,000	140	742
10	2,400,000	140	882	64,000	140	842

4.1 Results Analysis

Table 3 displays the linear programming (LP) relaxation lower bound values (v_L) and the gap between the integer optimal values of the objective function (v^*) and the LP values for all of the models. They also show the computing times in seconds (secs). Remember that v_L is obtained by solving the LP relaxation of the respective model. The computing time was limited to 5,200 seconds. Columns 2 to 5 give the results that correspond to the classical weak *RSCLP* model; columns 6 to 9 display the results that correspond to the classical strong *RSCLP-T* model, and columns 10 to 13 show the results that correspond to the *RSCLP-P* model (the proposed formulation). To illustrate, for problem instance 4, the LP relaxation lower bounds (v_L) are 168,200.0, 282,916.0, and 858,200.0 for the *RSCLP*, *RSCLP-T,* and *RSCLP-P* models, respectively; while for instance 6, these values are 280,800.0 and 3,160,800.0 for the *RSCLP* and *RSCLP-P* models, respectively. For this instance, the *RSCLP*-T model was unable to provide the LP value because it exceeded the computing time limit. The results indicate that, for all of the test instances, the proposed *RSCLP-P* model outperforms the classical weak (*RSCLP*) and strong formulation (*RSCLP-T*) in terms of the quality of the lower bound and the computing times. In summary, we observed the following:

4.1.1 Lower Bounds (LP)

- For instances with 800 and 1000 sourcing nodes, the weak classical formulation is unable to provide the LP relaxation solution because it runs out of memory.
- For the strong classical formulation, this scenario occurs with instances that have more than 300 sourcing sites
- The lower bound provided by the proposed RSCLP-P model is significantly better than that provided by the weak (RSCLP) and strong (RSCLP-T) classical models

4.1.2 Optimal Integer Solutions

- For test instances with more than 300 origination sites (marked by *), the weak (RSCLP) classical formulation is unable to provide the optimal integer solution because it runs out of memory or the defined time limit for execution is reached;
- For the case of RSCLP-T model, this scenario occurs for networks with more than 100 origination sites;

4.1.3 Computing Time

- The same performance can be observed in terms of the computing times. To illustrate, for instance 4, the computing times for obtaining the lower bounds are 5.16 and 647.0 for the weak and strong classical models, respectively, while the corresponding time is 0.67 for the proposed model. For instance 5, those times are 15.45, 4013.0 and 0.84 seconds for the weak and strong classical models and for the proposed formulation, respectively.

4.1.4 Gaps (%)

Table 3: Lower bound (v_L) and computing times (*seconds*).

#	RSCLP (weak classical)		RSCLP-T (strong classical)		RSCLP-P (proposed)	
	v_L	*Secs*	v_L	*secs*	v_L	*secs*
1	37200	0.70	50938	3.97	109200	0.59
2	75300	1.63	119025	45.55	255300	0.41
3	74650	2.88	122070	317.61	419650	0.50
4	168200	5.16	282916	647.19	858200	0.67
5	254000	15.45	426674	4013.3	1694000	0.84
6	280800	49.41	*	-	3160800	1.19
7	290400	125.55	*	-	3670400	7.5
8	305700	90.00	*	-	2319033	1.52
9	341200	137.28	*	-	2887867	6.38
10	594900	290.05	*	-	3444900	5.27

- It was observed that gaps (%) obtained from the weak (RSCLP) and the strong (RSCLP-T) classical models are significantly worse than the gaps obtained by the proposed RSCLP-P model.
- To illustrate this point, for problem instance 1, the gaps (%) are 213.84, 129.20 and 6.91 for the RSCLP, RSCLP-T and RSCLP-P model,

respectively, while for problem instance 5, the gaps (%) are 572.32, 300.24 and 0.81 for the RSCLP, RSCLP-T and RSCLP-P model, respectively;

- The average gap for the proposed model is 3.69%, while this number is 210.25% and 625.19% for the strong and weak classical formulations, respectively.

4.2 Model Sensitivity

We investigated the sensitivity of the models generating several scenarios to provide further information regarding the goodness of both formulations. For example, we increased the capacity of the reprocessing facilities by 200% and the capacity of the remanufacturing facilities by 100%. The results are summarized in Table 4.

Table 4: Lower bound (v_L), integer optimal solution (v^*), and Gap [$100(v^* - v_L)/v_L$].

	RSCLP (weak classical)		*RSCLP-P(proposed)*	
	v_L	*Gap%*	v_L	*Gap%*
1	36300	101.24	55800	30.91
2	73800	138.08	131300	33.82
3	67950	219.28	194200	11.71
4	153600	196.35	406100	12.09
5	216300	287.61[b]	781300	7.19
6	262400	467.00[b]	1392400	6.06
7	276400	511.72[c,d]	1614733	4.71[d]
8	305000	304.10[c,d]	1073889	14.77[d]
9	341200	328.02[c,d]	1332311	9.61[d]
10	595200	226.75[c,d]	1695200	14.72[d]
Min		101.24		4.71
Max		511.72		33.82
Ave		277.77		14.41

[a] Algorithm terminated at 5,200 CPU seconds without reaching an optimal solution

[b] Gap is calculated using the integer optimal solution provided by the *RSCLP-P* model

[c] Algorithm ran out of memory

[d] Gap is calculated using the best integer solution provided by the *RSCLP-P* model

In Table 4, for the classical model and for problem instances 1-10, the maximum and minimum gaps are 511.72% and 101.24%, respectively, with an average gap of 277.77%. The maximum and minimum gaps for the proposed model are 33.82% and 4.71%, respectively, with an average gap of 14.41%. The average gap of the classical model is more than 20 times larger than the proposed model average gap. Observe that, for problem instances 7-10, the classical models ran out of memory and for problem instances 5-6 the same model hit the computational limit. For those instances, the gap was obtained using the integer optimal solution provided by the proposed model.

5 CONCLUSIONS

In this paper, we proposed a new formulation for the problem of planning a reverse supply chain network, and it is provided theoretical and empirical proofs that this model is stronger than the classical (weak and strong) formulations of the problem. We analysed the performance of the proposed MILP formulation in terms of the computing times and the quality of the lower bounds provided by the linear relaxation. We showed, for the large-scale instances with up to 600 sourcing sites, 100 candidate sites for locating reprocessing facilities and 40 candidate sites for locating remanufacturing facilities, that the proposed *RSCLP-P* model outperforms the classical weak and strong formulation with a gap that is several times lower than the gap provided by the weak formulation and with significantly less computing time. Furthermore, the weak formulation cannot provide integer optimal solutions for some instance cases, and it is also unable to obtain the linear optimal solution in a reasonable amount of computing time.

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