Gait Optimization of a Rolling Knee Biped at Low Walking Speeds

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Abstract: This paper addresses an optimization problem of trajectories for a biped robot with a new modelled structure of knees which is called rolling knee (RK). The first part of article is to present the new kinematic knee on a biped robot and the different models used to know the dynamic of the robot during a walking step. The gait is cyclic and simplified by a Single Support Phase (SSP) followed by an impact. The second part is a comparison of the influence of the gait trajectory on the control, using cubic spline functions as well as the *Bézier* functions. The energetic criterion is minimized through optimization while using the simplex algorithm and the Lagrange penalty functions to meet the constraints of stability and deflection of mobile foot. The main result is the using of *Bézier* functions permit to improve the energy gain in slow walking speeds. These trajectories permit to the biped robot to walk progressively without energy disturbance unlike those with cubic spline functions.

1 INTRODUCTION

On a biped robot, the knee joints are classically realized by revolute joints. Biomechanical studies talk about the human knees that relate the movement of this articulation which is a combination of a rotation and a translation. (Hamon and Aoustin, 2010) propose knee structures combining these movements with a cross four-bar linkage. The simulations show less energy usage through this solution comparing to classical revolute joint knees. Another knee mechanisms designed by (Van Oort et al., 2011) uses a singularity of the mechanism to save the energy. When the mobile leg is stretched at the end of the step, the knee is locked by the singularity and does not consume energy during the next stance phase. The energy consumption also decreases during the gait. The design of the knee mechanism in the LARP project in (Gini et al., 2007) coming from studies on prosthetic knees and consist of a structure with two cylinder surfaces in contact.

The walk is described by a succession of contacts between feet and the ground. The important issue is to keep the equilibrium of the biped during this progression. Some control strategies need stable reference trajectories to ensure the gait of the robot. (Chevallereau et al., 2009), (Grizzle et al., 2001), (Westervelt et al., 2001) introduce the stability of the gait by studying the zero dynamics. The biped uses reference trajectories to stabilize the zero dynamics. The proposed method replaces the time variable by a monotonic non-actuated variable depending on the state X_e of the robot. This variable is used to replace time parametrizing the periodic motion of the biped. Now, the problem is a constrained nonlinear optimization problem where, the parameters of the desired trajectories are found in order to minimize a criterion defined by the integral-squared torque per step length. Another method proposed by (Kajita et al., 2003) uses reference trajectories of zero moment point (ZMP) to control the stability of the biped during the walk. The method defines a predictive control of the center of mass and of the desired ZMP trajectories.

For a given kinematic structure of robot, the previous control laws have proven their viability when the robot performs the walking gaits at speeds of the order of 0.5 to 1 m/s. Furthermore the robot is also used in very different operating conditions and not only with a constant gait speed. It is indeed necessary:

- To control the acceleration or deceleration phases (see (Sabourin and Bruneau, 2005)).
- To planning feet trajectories, at slow walking speeds, in an environment with moving obstacles (Chestnutt et al., 2005).
- To generate the walking gaits at very low speed

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for collaborative tasks like in (Evrard et al., 2009).

- To slow down speed when the robot starts changing direction like in (Wang et al., 2012).

The objective of our work is also to propose gait support trajectories at low speeds. These trajectories are based on a mathematical parametrized function. We used *Bézier* functions and cubic spline functions. The optimization process of sthenic criterion provides the vector of parameters for each function. The results clearly show the advantages of *Bézier* functions to express support trajectories at low speed. These results are better than those proposed with spline functions in (Hamon and Aoustin, 2010) and (Hobon et al., 2011) for higher speed range.

In this paper, the simplex algorithm will be used to demonstrate the advantage to utilize a rolling knee structure in the design of biped robot.

The outline of the paper is following. In section 2, the biped with the rolling knee structure is discussed. The parametric gait is formulated in the section 3. The section 4 introduces the optimization problem. Simulations and results are presented in section 5. Finally, section 6 presents the conclusion and perspectives.

2 MODEL

2.1 Biped Model

This study is focused on the cyclic walk of the biped in the sagittal plane. The considered robot biped is composed of seven rigid bodies with two feet, two shins, two thighs and one trunk. The biped is all actuated by six actuators. The objective is to compare the performance of this robot using two trajectory functions. The robot has revolute joints placed on hips and ankles but the knee joints is composed of a structure called rolling knees and present in (Hobon et al., 2011). Fig. 1 shows the robot studied with this configuration. The rolling knee consists of a movement of two cylindrical surfaces rolling without sliding, the two surfaces are the terminal surface of the femur and the tibia. The reference frame is $\Re_0 = (O_0, \vec{x}_0, \vec{y}_0, \vec{z}_0)$. O_0 is defined by the projection of the point A_1 on the ground. The direction of the walk is according to $\vec{x}_0$ and $\vec{z}_0$ the unit vector perpendicular to the ground. The orientation of the links are defined by the absolute angles $q_i, \{i = 0 \ldots 6\}$ referenced by the vertical, the speed vector $\dot{q}_i, \{i = 0 \ldots 6\}$ and the vector $\Gamma = [\Gamma_1 \ldots \Gamma_6]^{\mathbf{T}}$ which represents the torques placed on the hips, the knees and the ankles (see Fig. 1). Fig. 2 shows the details of the knee configuration. The contact between the femur and the tibia is maintained with a bar on C_1 and C_2 of length $r_1 + r_2$ with r_1 and

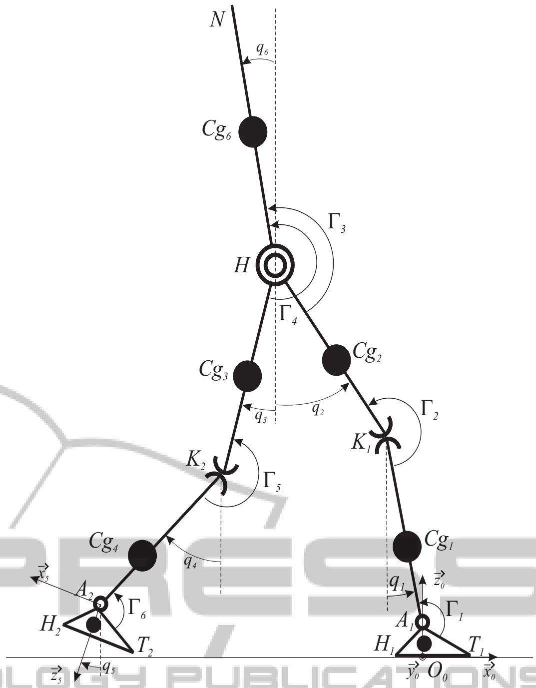

Figure 1: Biped robot with rolling joint knees.

r_2 are respectively the distance B_1C_1 and B_2C_2. With the rotation without sliding, we can write (1) and find the relation of the angle γ_1 which shows the coupling between the angles q_1 and q_2 for the leg support in (2). Similarly for the mobile leg, γ_2 is the coupling angle between q_3 and q_4 in 3.

$$\widehat{B_1K_1} = \widehat{B_2K_1} \tag{1}$$

$$\gamma_1 = \frac{r_1 q_1 + r_2 q_2}{r_1 + r_2} \tag{2}$$

$$\gamma_2 = \frac{r_1 q_4 + r_2 q_3}{r_1 + r_2} \tag{3}$$

The dynamic model's parameters are the length l_i of the links for the robot, $i = \{0 \ldots 6\}$ and we have an assumption gives us $l'_1 = A_1C_1 = l_1 - r_1$ and $l'_2 = HC_2 = l_2 - r_2$, the position of the center of mass C_{gi}, the masses m_i, the moments of inertia I_i of each bodies C_i around the $\vec{y}_0$ axis at C_{gi}.

The coordinates of the hip, the heel and the toes for the rolling knee configuration are:

$$x_H = -l'_2 \sin q_2 - (r_1 + r_2) \sin \gamma_1 - l'_1 \sin q_1 \tag{4}$$

$$z_H = l'_2 \cos q_2 + (r_1 + r_2) \cos \gamma_1 + l'_1 \cos q_1 + h_p \tag{5}$$

$$x_{H_2} = x_H + l'_2 \sin q_3 + l \sin \gamma_2 + l'_1 \sin q_4 - l_p \cos(q_5) + h_p \sin(q_5) \tag{6}$$

$$z_{H_2} = z_H - l'_2 \cos q_3 - l \cos \gamma_2 - l'_1 \cos q_4 - l_p \sin(q_5) - h_p \cos(q_5) \tag{7}$$

$$x_{T_2} = x_H + l'_2 \sin q_3 + l \sin \gamma_2 + l'_1 \sin q_4 - (l_p - L_p) \cos(q_5) + h_p \sin(q_5) \tag{8}$$

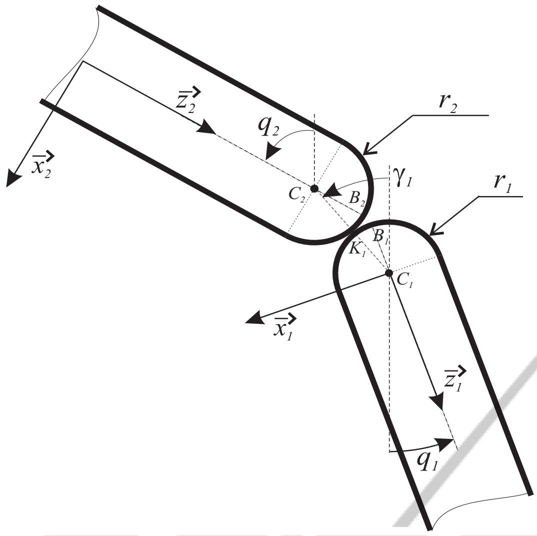

Figure 2: Rolling knee of one leg.

$$\begin{aligned} z_{T_2} &= z_H - l_2' \cos q_3 - l \cos \gamma_2 - l_1' \cos q_4 \\ &\quad - (l_p - L_p)\sin(q_5) - h_p \cos(q_5) \end{aligned} \tag{9}$$

2.2 Dynamic Model

In this work, we consider only the walking gait defined with Simple Support Phases (SSP) followed by an impact between the mobile foot and the ground. The impact produces the instantaneous exchange of supporting leg during the gait. The dynamic model for the SSP is assumed with the left leg on support. Considering the gait like periodic with a permutation of the legs at the impact, the study focuses on one step beginning with the impact. The dynamic and impact models are described as follow:

2.2.1 Dynamic Model During the SSP

The Lagrange equations are used to determine the inverse dynamic model. Details are mentioned in (Spong and Vidyasagar, 1991) and (Khalil and Dombre, 2002). Posing $q = [q_i], i \in [0 \ldots 6]$ and $X_e = [q, x_H, z_H]^{\mathbf{T}}$ the state vector of dimension 9×1. The inverse dynamic equation can be written as:

$$\left[B \;\; A_{c_L}(X_e)^T\right] [\Gamma \;\; F_L]^T = D(X_e)\ddot{X}_e + H(\dot{X}_e, X_e) + Q(X_e) \tag{10}$$

with $D(X_e)$ represents the inertia matrix 9×9, $H(\dot{X}_e, X_e)$ is the vector of Coriolis and centrifugal effects 9×1, $Q(X_e)$ is the vector of torques and forces due to the gravity 9×1 , B is the control matrix 9×6 and $A_{c_L}(X_e)$ is the Jacobian matrix 3×9 of the foot on support. The acceleration of $\ddot{x}_H$ and $\ddot{z}_H$ are calculated with the hypotheses that the support foot remains in contact during the SSP, $x_{A_1} = 0$, $z_{A_1} - h_p = 0$ and $q_0 = 0$. By twice derivation to the state vector, we obtain the constraint dynamic equations:

$$A_{cL}(X_e)^T \ddot{X}_e + H_{cL}(X_e) = 0 \tag{11}$$

With the evolution of X_e satisfying (11), the torques Γ and external forces $F_L = [F_x F_z C_y]^T$ on the stance ankle are calculated with (10) in any instant of the gait.

2.2.2 Impact Model

During the gait, the left foot and then the right foot alternatively touch the ground with a non zero speed. This is the impact phase. The impact phase separates two SSP. The contact phase between two rigid bodies, the foot and the ground, produces a mechanical energy dissipation phenomena (Pfeiffer and Glocker, 1996). We suppose that the restitution coefficient is equal to zero. This assumption ensures that we have no rebounds of the foot after the impact. The proposed model is:

$$D(X_e)\left(\dot{X_e}^+ - \dot{X_e}^-\right) = A_{c_L}^T I_R \tag{12}$$

$$A_{c_L}(X_e)\,\dot{X_e}^+ = 0 \tag{13}$$

$$\Delta^T = D^{-1} A_{c_L}^T I_R \tag{14}$$

This model is used to find the speed vector after the impact $\dot{X_e}^+$ from the configuration X_e and the speed vector before impact $\dot{X_e}^-$. Δ_i is a vector (9×1) and is the difference between the speed after and before the impact for each axis i, $i \in [0 \cdots 6]$. It will be used to define the conditions of the trajectories in the following section. This model also gives the impact forces on the foot $I_R = [I_{Fx}\ I_{Fz}\ C_y]^T$ and the torque applied on the ankle.

3 GAIT REFERENCE PARAMETRIC TRAJECTORIES

Now, the gait is defined by the evolution of angular coordinates of the bodies with respect to the time. The goal is to find the best parametric trajectories to improve the fluidity of the movement and to reduce the energy consumption. The angular coordinates q_i with $i = \{0 \ldots 6\}$ can be parametrized by a cubic spline function also, used in (Hamon and Aoustin, 2010), (Banno et al., 2009) or order 3 *Bézier* function of (Westervelt et al., 2001), (Scheint et al., 2008). To simplify the definition of the trajectories, the time t is normalized to the dimensionless time variable $t_n = t/T$ with T the step period. The gait can be described by: at $t_n = 0$, the left foot is fixed on the ground and the right foot is behind the trunk. At $t_n = 1$, the right foot has an advance of a distance d and it is in front of the trunk.

3.1 Cubic Spline Function

In this case, the trajectories are defined by two cubic spline functions. Each function is parametrized for a half-period. The knot vector has three knots so we define $t_k = [0, 0.5, 1]$. In neighbourhood of $t_i \in t_k, i = [0,2]$, the spline function has the smoothness $\mathcal{C}^1$. We suppose that at the time $t_1 = 0.5$, the second derivative is continuous. Also, for $t_0 = 0$ and $t_2 = 1$, the impact imposes a discontinuity on the velocities.

The expression of the cubic spline function is:

$$0 \leq t_n \leq \tfrac{1}{2} \rightarrow f_{q_i}(t_n) = \sum_{j=0}^{3} a_{i_j} t_n^j \tag{15}$$

$$\tfrac{1}{2} \leq t_n \leq 1 \rightarrow f'_{q_i}(t_n) = \sum_{j=0}^{3} b_{i_j} (1 - t_n)^j \tag{16}$$

where a_j and b_j are the eight parameters expressed for each angle. Supposing $k = \frac{1}{T}$, the velocities and accelerations are obtained by derivation:

$$\dot{q}_i(t) = k \frac{d f_{q_i}}{dt_n} \tag{17}$$

$$\ddot{q}_i(t) = k^2 \frac{d^2 f_{q_i}}{dt_n^2} \tag{18}$$

3.2 *Bézier* Function

The trajectories now are defined by *Bézier* function and parametrized for one period from $t_n = 0$ to $t_n = 1$. The function is $\mathcal{C}^2$ on the interval $]0,1[$. The parameters c_{i_j} are homogeneous to angular coordinates:

$$B_{q_i}(t_n) = \sum_{j=0}^{3} \tfrac{3!}{j!(3-j)!} c_{i_j} t_n^j (1 - t_n)^{(3-j)} \tag{19}$$

where c_j are the four parameters to describe each angle $q_i(t) = B_{q_i}(kt)$. The speeds and accelerations are found by derivation.

3.3 Smoothness, Cyclicity and Vector of Parameters

The study is focused on a cyclical gait defined by an impact and SSP. We assume that the left foot is on support during the SSP, also $q_0 = 0$. The unknown vector q_i is now for $i = \{1 \ldots 6\}$. Following hypotheses are posed for the gait :

- The mobile foot is flat on the ground at the beginning and at the end of the step
- The angles of the trunk and the mobile foot are T-periodic
- The angles of the tibias and the thighs are $2T$-periodic

For the spline function, we consider the continuity and the smoothness of the function at the half-period. These conditions for the mobile foot, the trunk, the tibias and the thighs lead to :

$$a_{5_0} = b_{5_0} = 0, a_{5_2} = b_{5_2}, a_{5_3} = b_{5_3} = -\frac{4}{3} a_{5_2} \tag{20}$$

$$a_{6_0} = b_{6_0}, a_{6_2} = b_{6_2}, a_{6_3} = b_{6_3} = -\frac{4}{3} a_{6_2} \tag{21}$$

$$\begin{aligned} &a_{1_0} = b_{4_0}, a_{1_2} = b_{4_2} = a_{4_2} + 6(a_{4_0} - a_{1_0}), \\ &b_{1_3} = -6((a_{1_0} - a_{4_0}) + \frac{4}{3}(a_{1_1} - a_{1_2})) \end{aligned} \tag{22}$$

$$\begin{aligned} &a_{4_0} = b_{1_0}, a_{4_2} = b_{1_2} = a_{1_2} + 6(a_{1_0} - a_{4_0}), \\ &b_{4_3} = -6((a_{4_0} - a_{1_0}) + \frac{4}{3}(a_{4_1} - a_{4_2})) \end{aligned} \tag{23}$$

$$\begin{aligned} &a_{2_0} = b_{3_0}, a_{2_2} = b_{3_2} = a_{3_2} + 6(a_{3_0} - a_{2_0}), \\ &b_{2_3} = -6((a_{2_0} - a_{3_0}) + \frac{4}{3}(a_{2_1} - a_{2_2})) \end{aligned} \tag{24}$$

$$\begin{aligned} &a_{3_0} = b_{2_0}, a_{3_2} = b_{2_2} = a_{2_2} + 6(a_{2_0} - a_{3_0}), \\ &b_{3_3} = -6((a_{3_0} - a_{2_0}) + \frac{4}{3}(a_{3_1} - a_{3_2})) \end{aligned} \tag{25}$$

The impact model imposes a relation between the speed before and after the impact time for all mobile bodies. For example, the speed of the right thigh after the impact depend on (14) and on the speed of the left thigh just before the impact. Theses conditions are expressed as:

$$a_{5_1} = b_{5_1} + \Delta_5 \tag{26}$$

$$a_{6_1} = b_{6_1} + \Delta_6 \tag{27}$$

$$a_{2_1} = b_{3_1} + \Delta_2 \tag{28}$$

$$a_{3_1} = b_{2_1} + \Delta_3 \tag{29}$$

$$a_{1_1} = b_{4_1} + \Delta_1 \tag{30}$$

$$a_{4_1} = b_{1_1} + \Delta_4 \tag{31}$$

For the *Bézier* function, the same conditions can be expressed and we obtain:

$$c_{i_0} = c_{i_3},\ i = [5,6] \tag{32}$$

$$c_{i_j} = c_{i_{(3-j)}},\ i = [1,4], j = [0,3] \tag{33}$$

$$c_{i_j} = c_{i_{(3-j)}},\ i = [2,3], j = [0,3] \tag{34}$$

The impact model imposes for the *Bézier* functions :

$$
\begin{aligned}
c_{5_1} &= -3c_{5_2}+\Delta_5 && (35)\\
c_{6_1} &= 3(c_{6_0}-c_{6_2})+\Delta_6 && (36)\\
c_{2_1} &= 3(c_{2_0}-c_{3_2})+\Delta_2 && (37)\\
c_{3_1} &= 3(c_{3_0}-c_{2_2})+\Delta_3 && (38)\\
c_{1_1} &= 3(c_{1_0}-c_{4_2})+\Delta_1 && (39)\\
c_{4_1} &= 3(c_{4_0}-c_{1_2})+\Delta_4 && (40)
\end{aligned}
$$

The remaining parameters of the simplification are equal to 17 for the cubic spline function. For those defined in the *Bézier* function, we identify 11 parameters which are summarized in the table 1.

Table 1: Resume of vector of parameters.

Parametric function	parameters
Cubic spline function	a_{i_0} for $i=\{1,2,3,4,6\}$, b_{j_1} and a_{j_2} for $j=\{1\dots6\}$
Bézier articular function	c_{i_0} for $i=\{1,2,3,4,6\}$, c_{j_2} for $j=\{1\dots6\}$

Finally, the parameters a_{i_0} and c_{i_0} which give the initial angular position can be found with the inverse geometric model of the Cartesian position of the hip $(x_H(0), z_H(0))$. The period T is added as parameter for the gait determination. The parameters vectors $\mathbf{p}_s$=$[x_H(0), z_H(0), a_{6_0}, b_{j_1}, a_{j_2}, T]$ and $\mathbf{p}_b$=$[x_H(0), z_H(0), c_{6_0}, c_{j_2}, T]$ are used in the following optimization process.

In this section, we introduce two types of trajectory functions to describe the gait. The cubic spline function are parametrized by absolute angles, absolute velocities and accelerations. The choice of parameters is sensitive to the walking speed and the acceleration parameters introduces some convergence problems. This motivates us to choose *Bézier* functions that are more homogeneous parameters corresponding to absolute angles. Thus the smoothness of the generated trajectories advantages the criterion convergence.

4 OPTIMIZATION PROBLEM

The research of optimal parameters to produce the best gait trajectories is challenging. The goal is to search trajectories minimizing a criterion representing the energetic consumption of the robots while respecting the constraints due to the biped environment. A gait is considered optimal if the gait is physically feasible and with the minimal of power supply.

We have also to solve a nonlinear minimization problem under constraints that can be expressed:

$$
\begin{gathered}
\min_{\mathbf{p}} C_\Gamma(\mathbf{p})\\
\text{under } \Psi(\mathbf{p}) \geq 0 \text{ with } \mathbf{p}=\mathbf{p}_s \text{ or } \mathbf{p}=\mathbf{p}_b
\end{gathered} \tag{41}
$$

with $\Psi=[\Psi_1\Psi_2\Psi_3\Psi_4\Psi_5\Psi_6\Psi_7]^T$ are the constraints imposed by the initial configuration of the robot and the physical constraints considered before. The constraints are:

- $\Psi_1 = F_z$ define the vector of the force along the z-axis and must be positive,
- $\Psi_2 = x_{ZMP}+l_p$ and $\Psi_3 = -x_{ZMP}+(L_p-l_p)$ represent the limit position of the ZMP on x-axis (ZMP must stay in the foot support) to guaranty the stability of the robot,
- $\Psi_4 = q_2-q_1$ and $\Psi_5 = q_3-q_4$ are the choice made to keep a gait human-like with no bend backward of knees,
- $\Psi_6 = z_{H2}$ and $\Psi_7 = z_{T2}$ are used to have the z-coordinates of heel and toes of the mobile foot above the ground during the SSP.

The x-coordinates of ZMP is calculated by:

$$x_{ZMP} = \frac{\Gamma_1 - h_pF_x - m_ps_xg}{F_z} \tag{42}$$

The criterion used is the estimation of Joules losses. It also named sthenic criterion (Tlalolini et al., 2011) and defined by (43).

$$C_\Gamma(\mathbf{p}) = \frac{2}{d}\int_0^T \Gamma^T\Gamma\, dt \tag{43}$$

We propose to solve this problem by using the Nelder-Mead simplex algorithm see (Lagarias et al., 1998). This algorithm solves nonlinear problem without constraints. The constraints are also added in the criterion (43) as Lagrange multiplier. One of advantages to use the simplex algorithm is the possibility to explore the space around the initial vector. The reloading of the obtained solution allows to avoid local minima and also to converge to the optimal solution. The equation becomes:

$$C_\Gamma(\mathbf{p}) = \frac{2}{d}\int_0^T \left(\Gamma^{\mathrm{T}}\Gamma + K\sum_{i=1}^{7}(e^{(|\Psi_i|-\Psi_i)}-1)\right)dt + Err \tag{44}$$

with Γ calculated from (10), Ψ_i represents the constraints already defined, K is the multiplier of Lagrange equal to 10^6 in our calculation and Err can handle the errors due to the inverse geometric model. At the end of the optimization, we verify that the constraints are positive and Err are equal to zero in all cases.

5 SIMULATIONS AND RESULTS

The simulations were done with the geometrical and dynamic parameters of HYDROID Robot. The height of this biped robot is 1.39m with a total mass of 45.36 kg. The table 2 in appendix gives the physical parameters of each body part of this robot. For these simulations, the radii r_1 and r_2 are chosen equal to 5 cm. The objective of the simulation is to acquire the best trajectory function for the configuration defined in 2 and to find the best solution. Two series of optimization were done following the mathematical expressions defined in 3. We will observe the evolution of criteria and compare the angles, the velocities and joint torques. We will examine the evolution of the convergence criterion during optimization phases using the cubic spline functions and the *Bézier* functions.

In Fig. 3, the evolution of optimal criteria versus the walking speed is presented between 0.1 m/s to 0.3 m/s. The criteria with the *Bézier* functions are lower than the one with the cubic spline functions and the criterion gain is about 96% for the walking speed at 0.1 m/s, it is about 68% at 0.2 m/s. The progression of the criterion using the *Bézier* functions increases slightly. It is an advantage to starting up of the walk of a biped robot compared to the use of the cubic spline functions. To generate the acceleration phase to the robot, it is also interesting to use trajectories obtained by *Bézier* functions for speed lower at 0.25 m/s. After this speed, the cubic spline trajectories are more suited.

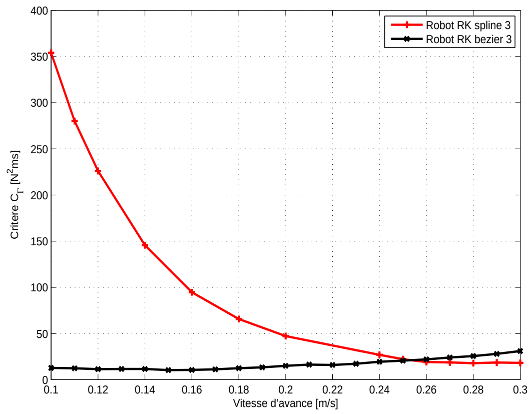

Figure 3: Evolution of optimal criterion in function of the walking speed.

Fig. 4 presents the convergence of the criterion for both functions at the walking speed at 0.2 m/s. We disturb the initial conditions of 1% on all the parameters. The obtained vector of parameters results is restarted for a new optimization and it is done until the solution is performed to the precision of 10^{-6}. We observe the criterion, using *Bézier* functions, decreasing faster than the criterion using cubic spline functions. The gait solutions are different for each case and present in the stick diagram figure 5. The walking gait using *Bézier* functions advantage a strategy of quickly little step.

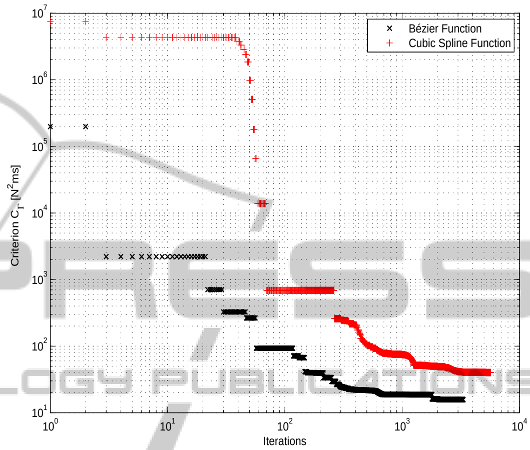

Figure 4: Convergence of minimal criterion at 0.2 m/s for robot with RK.

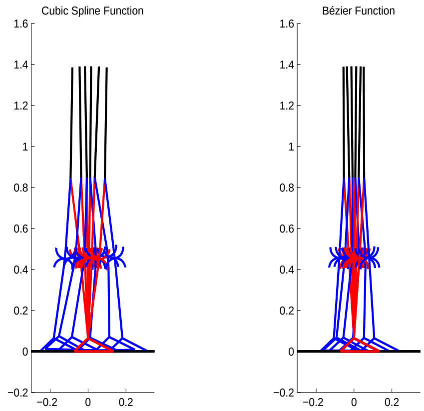

Figure 5: Stick diagrams of the biped for the walking speed at 0.2 m/s. On the left, the robot using cubic spline function and on the right, the robot using *Bézier* functions.

Fig. 6 introduces the evolution of the angles at the same walking speed at 0.2 m/s. We note that the allures are the same between the two parametric function except for the feet. We observe that the step period is shorter for the robot using *Bézier* functions. The evolution of the angles is included between -0.15 *rad* and 0.1 *rad* and it is multiplied by two for the robot using cubic spline functions. In the last case,

the amplitude of the leg angles is more important that influences the maximum values of the torques so the sthenic criterion is increased.
The fig. 7 presents the evolution of torques for the walking speed of 0.2 m/s and confirms the previous analysis. The torques of the robot are included between $-3N.m$ and $3N.m$ for the *Bézier* functions and more than two for the cubic spline functions.

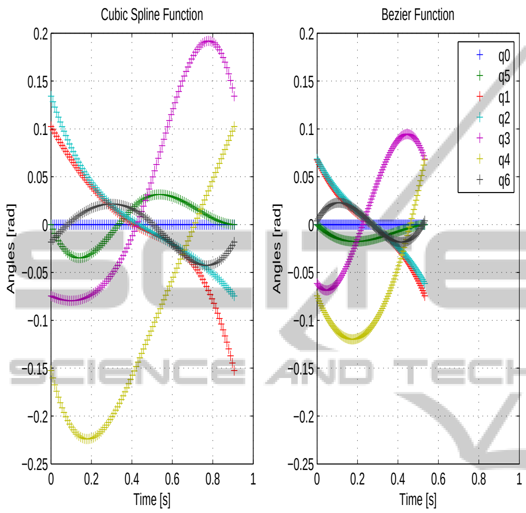

Figure 6: Evolution of absolute angles for the walking speed at 0.2 m/s.

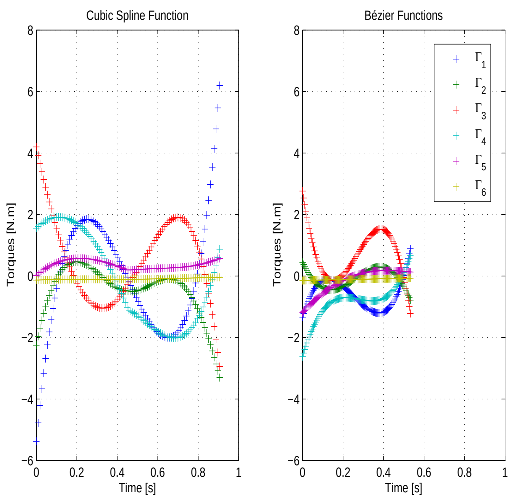

Figure 7: Evolution of torques for the walking speed at 0.2 m/s.

6 DISCUSSION

The improvement of the sthenic criterion at low speeds with *Bézier* functions is the main result in this article The strategy to increase the range of travel without recharging the batteries is to choose at low speeds *Bézier* trajectories and for fast speeds, to use the trajectories defined by cubic spline functions. A study about the sensitivity of the parameters will be necessary to know the parameters influence. From our point of view, the acceleration parameters of the cubic spline functions are too sensitive for walking speeds lower to 0.2 m/s.
To conclude, the model of the robot with rolling knee has been introduced and we have proposed reference trajectories for this type of anthropomorphic robot. Our simulation program computes the joint torques and the forces on the feet for different trajectory functions. These trajectories are then parametrized to allow the resolution of the energetic optimization problem. Two types of trajectories are defined and the optimization process shows that the *Bézier* functions give a significant criterion reduction at slow walking speeds. These trajectories can be used like reference trajectories for the control to starting up the gait of the biped robot. The simulations shows that the cubic spline functions are better adapted for gait speeds greater than 0.25 m/s.
In future, other gaits can be explored such as Double Support Phases (DSP) including foot rotation. The influence of the radii r_1 and r_2 represents another interesting challenge for this type of robot design.

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APPENDIX

Table 2: Parameters of HYDROïD Robot.

Body	Length $[m]$	Masse $[kg]$	Inertia moment $[kg.m^2]$	Position of CoM $[m]$
Feet		0.678	0.001	$sx =$
L_p	0.207			0.013
l_p	0.072			$sz =$
h_p	0.064			0.032
Tibia	0.392	2.188	0.028	0.168
Thigh	0.392	5.025	0.066	0.168
Trunk	0.543	29.27	0.81	0.192