Phase-frequency Domain Model of Costas Loop with Mixer Discriminator

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Abstract: Problem of rigorous mathematical analysis of classical Costas Loop for non-sinusoidal signals is considered. The analytical method for phase detector characteristics computation is proposed and new classes of phase detector characteristics are computed for the first time. Effective methods for nonlinear analysis of Costas Loop are discussed.

1 INTRODUCTION

The Costas loop is a classical phase-locked loop (PLL) based circuit for carrier recovery (Best, 2007; Costas, 1956; Djordjevic et al., 1998; Waters, 1982). Nowadays various modifications of Costas loop circuit are used in many communication devices, e.g. Global Positioning Systems (GPS) (Kaplan and Hegarty, 2006; Nowsheen et al., 2010; Hasegawa et al., 2001).

Although PLL-based circuits are essentially a nonlinear control systems, in modern engineering literature, devoted to the analysis of PLL-based circuits, the main direction is the use of simplified linear models, the methods of linear analysis, empirical rules, and numerical simulation (see a plenary lecture of D. Abramovich at American Control Conference 2002 (Abramovitch, 2002)). While the linearization and analysis of linearized models of control systems may lead to incorrect conclusions[1], the attempts to justify the reliability of conclusions, based on the application of such simplified approaches, are quite rare (see, e.g., (Suarez and Quere, 2003; Margaris, 2004; Feely, 2007; Banerjee and Sarkar, 2008; Feely et al., 2012; Suarez et al., 2012)). Rigorous nonlinear analysis of PLL-based circuit models is often very difficult task, so for analysis of nonlinear PLL models, numerical simulation is widely used (Troedsson, 2009; Best, 2007; Bouaricha et al., 2012). However for high-frequency signals, complete numerical simulation of *physical model of PLL-based circuit in signals/time space*, described by nonlinear non-autonomous system of differential equations, is highly complicated (Abramovitch, 2008a; Abramovitch, 2008b) since it is necessary to simultaneously observe "*very fast time scale of the input signals*" and "*slow time scale of signal's phases*". Here a relatively small discretization step in numerical procedure does not allow one to consider transition processes for high-frequency signals in a reasonable time.

To overcome these difficulties, it is possible to construct *mathematical model of PLL-based circuit in phase-frequency/time space*, described by nonlinear dynamical system of differential equations. In this case only slow time scale of signal's phases is investigated. That, in turn, requires (Leonov et al., 2012b) the computation of phase detector characteristic (a nonlinear element used to match reference and controllable signals), which depends on waveforms of considered signals (Kuznetsov et al., 2011c; Kuznetsov et al., 2011b; Kuznetsov et al., 2010a). However, the use of results of this mathematical model analysis for conclusions, concerning the be-

[1]see, e.g. counterexamples to filter hypothesis and to Aizerman's and Kalman's conjectures on absolute stability (Leonov and Kuznetsov, 2011; Leonov et al., 2011b; Kuznetsov et al., 2011a; Bragin et al., 2011; Leonov and Kuznetsov, 2013) and Perron effects of Lyapunov exponents sign inversions for time varying linearizations (Leonov and Kuznetsov, 2007) etc.

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havior of considered physical model, requires rigorous justification (Leonov et al., 2011b; Leonov et al., 2012b).

All in all discovery of hidden oscillations[2] in nonlinear dynamical models of PLL (Leonov and Kuznetsov, 2013) shows that for investigation of possible behavior of nonlinear systems, simple simulation is an unreliable tool and can lead to wrong conclusions. For numerical computation of possible limit solutions in a dynamic system, all initial conditions need to be evaluated, and in a non-autonomous systems with input, in addition, all possible inputs need to be considered. Here to get reliable results of simulations one need to verify analytically a condition of uniqueness of limit solution (i.e. convergent property of systems (van den Berg et al., 2006)) or to apply special analytical-numerical procedures, which allow to compute hidden oscillations (Leonov and Kuznetsov, 2011; Bragin et al., 2011; Leonov et al., 2011a; Leonov et al., 2012a).

In this paper effective approaches to rigorous nonlinear analysis of classical analog Costas loop are discussed. For computation of phase-detector characteristics, effective analytical method is demonstrated. For various non-sinusoidal waveforms of high-frequency signals, new classes of phase-detector characteristics are obtained, and dynamical model of Costas loop is constructed.

2 DESCRIPTION OF THE COSTAS LOOP IN THE SIGNAL SPACE

Consider the Costas loop with harmonic carrier (Fig. 1). Here input signal is Binary Phase Shift Keying (BPSK) signal, which is a product of transferred data $m(t) = \pm 1$ and harmonic carrier $\sin(\omega t)$ with high frequency ω. Input signal is multiplied (block $\otimes$) with the output of voltage-controlled oscillator (VCO) in the upper branch (I branch)

$$I = \frac{1}{2}\big(m(t)\cos(0) - m(t)\cos(2\omega t)\big). \qquad (1)$$

[2]From a computational point of view, in nonlinear dynamical systems, attractors can be regarded as *self-excited* and *hidden attractors* (Kuznetsov et al., 2010b; Leonov and Kuznetsov, 2011; Leonov et al., 2011a; Bragin et al., 2011; Leonov et al., 2012a; Leonov and Kuznetsov, 2013). Self-excited attractors can be localized numerically by a *standard computational procedure, in which after a transient process a trajectory, started from a point of unstable manifold in a neighborhood of equilibrium, reaches a state of oscillation and therefore one can easily identify it*. In contrast, for a *hidden attractor, its basin of attraction does not intersect with small neighborhoods of equilibria*.

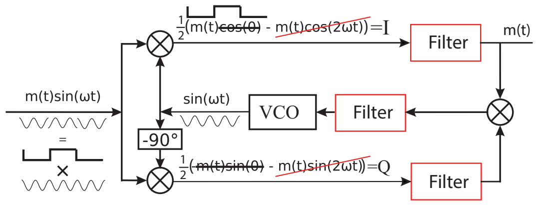

Figure 1: Costas loop: $m(t)$ is a transferred data (± 1); $m(t)\sin(\omega t)$ is an input signal; ω is a frequency of oscillators.

Filtration (blocks Filter) of high frequency component $\cos(2\omega t)$ allows to get demodulated data $m(t)$. In the lower branch (Q branch) the phase of VCO signal is shifted by $-\frac{\pi}{2}$

$$Q = \frac{1}{2}\big(m(t)\sin(0) - m(t)\sin(2\omega t)\big). \qquad (2)$$

This signal after filtration of high frequency component $\sin(2\omega t)$ can be used to detect a lock state of the loop. Then both branches are multiplied together and next low-pass filter forms control signal for VCO in order to adjust its frequency to the frequency of the carrier.

Here to avoid these non-rigorous arguments, and to consider non-sinusoidal (see applications, e.g., in (Henning, 1981; Wang and Emura, 1998; Sutterlin and Downey, 1999; Wang and Emura, 2001; Chang and Chen, 2008; Sarkar and Sengupta, 2010)) mathematical properties of high-frequency oscillations will be considered.

3 COMPUTATION OF PHASE DETECTOR CHARACTERISTIC

Since two arm filters are used for data demodulation and can be applied apart from the loop (see e.g. (Kaplan and Hegarty, 2006)), for investigation of transient processes one can consider simplified block diagram of Costas loop in Fig. 2. Here signals $f^{1,2}(t) =$

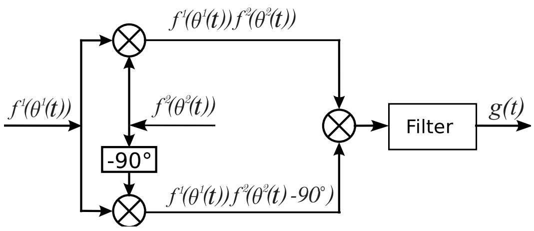

Figure 2: Multiplier and filter.

$f^{1,2}(\theta^{1,2}(t))$ with $\theta^{1,2}(t)$ as phases are carrier and output of VCO. Suppose that the waveforms $f^p(\theta)$ of signals are bounded 2π-periodic piecewise differentiable functions[3] (this is true for the most considered waveforms). Consider Fourier series representation of such functions:

$$
\begin{aligned}
f^p(\theta) &= \sum_{i=1}^{\infty} \left(a_i^p \cos(i\theta) + b_i^p \sin(i\theta)\right),\\
a_i^p &= \frac{1}{\pi}\int_{-\pi}^{\pi} f^p(\theta)\cos(i\theta)d\theta,\\
b_i^p &= \frac{1}{\pi}\int_{-\pi}^{\pi} f^p(\theta)\sin(i\theta)d\theta,\ p=1,2,
\end{aligned}
$$

Assume that relation between the input $\xi(t)$ and the output $\sigma(t)$ of Filter has the form:

$$\sigma(t) = \alpha_0(t) + \int_0^t \gamma(t-\tau)\xi(\tau)\,d\tau, \tag{3}$$

where $\gamma(t)$ is an impulse response function of filter and $\alpha_0(t)$ is an exponentially damped function depending on the initial data of filter at moment $t=0$. By assumption, $\gamma(t)$ is a differentiable function with bounded derivative (this is true for the most considered liner filters (Thede, 2005)).

A high-frequency property of signals can be reformulated in the following condition.

Frequencies $\omega^p(t)$ of signals can be defined (Leonov, 2008) as follows $\theta^{1,2}(t) = \omega^{1,2}(t)t + \psi^{1,2}$. Consider a large fixed time interval $[0,T]$, which can be partitioned into small intervals of the form

$$[\tau,\tau+\delta],\ \tau\in[0,T],$$

where the following relations

$$
\begin{aligned}
&|\gamma(t)-\gamma(\tau)| \le C\delta,\ |\omega^p(t)-\omega^p(\tau)| \le C\delta,\\
&\forall t\in[\tau,\tau+\delta],\ \forall\tau\in[0,T],
\end{aligned} \tag{4}
$$

$$|\omega^1(\tau)-\omega^2(\tau)| \le C_1,\ \forall\tau\in[0,T], \tag{5}$$

$$\omega^p(t) \ge \omega_{min},\ \forall t\in[0,T],\quad p=1,2 \tag{6}$$

are satisfied.

Suppose that δ is sufficiently small as compared with the fixed numbers T, C, C_1 and ω_{min} is sufficiently large as compared with the number $\delta : \omega_{min}^{-1} = O(\delta^2)$.

The latter means that on small intervals $[\tau,\tau+\delta]$ the functions $\gamma(t)$ and $\omega^{1,2}(t)$ are "almost constant" and the functions $f^{1,2}(t)$ are rapidly oscillating. Obviously, such a condition occurs for high-frequency oscillations.

Consider equivalent block diagram of Costas loop (Fig. 3) in phase-frequency space. Here $\theta^{1,2}(t)$ are phases of oscillations $f^{1,2}\left(\theta^{1,2}(t)\right)$, PD is a nonlinear block with the characteristic $\varphi(\theta)$ (being called a

[3]The functions with a finite number of jump discontinuity points differentiable on their continuity intervals.

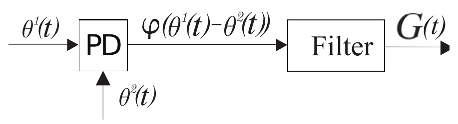

Figure 3: Phase detector and filter.

Phase Detector or discriminator). The phases $\theta^{1,2}(t)$ are the inputs of PD block and the output is the function $\varphi(\theta^1(t)-\theta^2(t))$. A shape of phase detector characteristic is based on a shape of input signals.

In the both diagrams (Figs. 2 and 3) the filters are the same with the same impulse transient function $\gamma(t)$ and the same initial data. The filters outputs are the functions $g(t)$ and $G(t)$, respectively. By (3) one can obtain $g(t)$ and $G(t)$:

$$
\begin{aligned}
&g(t) = \alpha_0(t) + \int_0^t \gamma(t-\tau) f^1\left(\theta^1(\tau)\right) f^2\left(\theta^2(\tau)\right)\\
&f^1\left(\theta^1(\tau)\right) f^2\left(\theta^2(\tau)\right) d\tau, \qquad (7)\\
&G(t) = \alpha_0(t) + \int_0^t \gamma(t-\tau)\varphi\left(\theta^1(\tau)-\theta^2(\tau)\right) d\tau.
\end{aligned}
$$

Using the approaches outlined in (Leonov, 2008; Kuznetsov et al., 2010a; Kuznetsov et al., 2011c; Kuznetsov et al., 2012), the following result can be proved.

Theorem 1. *If conditions* (4)–(6) *are satisfied (high-frequency property) and*

$$
\begin{aligned}
&\varphi(\theta) = \frac{A_0^1 A_0^2}{4} + \frac{1}{2}\sum_{l=1}^{\infty}\Big((A_l^1 A_l^2 + B_l^1 B_l^2)\cos(l\theta) +\\
&(A_l^1 B_l^2 - B_l^1 A_l^2)\sin(l\theta)\Big),
\end{aligned} \tag{8}
$$

where

$$
\begin{aligned}
A_0^1 &= \frac{a_0^1 a_0^1}{2} + \sum_{m=1}^{\infty}(a_m^1 a_m^1 + b_m^1 b_m^1),\\
A_k^1 &= \frac{a_0^1 a_k^1}{2} + \frac{1}{2}\sum_{m=1}^{\infty}[a_m^1(a_{m+k}^1 + a_{m-k}^1) + b_m^1(b_{m+k}^1 + b_{m-k}^1)],\\
B_k^1 &= \frac{a_0^1 b_k^1}{2} + \frac{1}{2}\sum_{m=1}^{\infty}[a_m^1(b_{m+k}^1 - b_{m-k}^1) - b_m^1(a_{m+k}^1 - a_{m-k}^1)],\\
A_0^2 &= \frac{a_0^2 a_0^2}{2} + \sum_{m=1}^{\infty}(\alpha_m^2 \alpha_m^2 + \beta_m^2 \beta_m^2),\\
A_k^2 &= \frac{a_0^2 \alpha_k^2}{2} + \frac{1}{2}\sum_{m=1}^{\infty}[\alpha_m^2(\alpha_{m+k}^2 + \alpha_{m-k}^2) + \beta_m^2(\beta_{m+k}^2 + \beta_{m-k}^2)],\\
B_k^2 &= \frac{a_0^2 \beta_k^2}{2} + \frac{1}{2}\sum_{m=1}^{\infty}[\alpha_m^2(\beta_{m+k}^2 - \beta_{m-k}^2 - \beta_m^2(\alpha_{m+k}^2 - \alpha_{m-k}^2)],
\end{aligned} \tag{9}
$$

and

$$\alpha_k^2 = \begin{cases} a_k^2, & k=4p, \\ b_k^2, & k=4p+1, \\ -a_k^2, & k=4p+2, \\ -b_k^2, & k=4p+3, \end{cases} \quad \beta_k^2 = \begin{cases} b_k^2, & k=4p, \\ -a_k^2, & k=4p+1, \\ -b_k^2, & k=4p+2, \\ a_k^2, & k=4p+3, \end{cases} \tag{10}$$

then for the same initial data of filter the following relation

$$G(t) - g(t) = O(\delta), \quad \forall t \in [0,T] \tag{11}$$

is valid.

3.1 Proof of Theorem

Let $t \in [0,T]$. Consider the difference

$$g(t) - G(t) = \int_0^t \gamma(t-s) \Big[f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s)\big) f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s) - \frac{\pi}{2}\big) - \\ - \varphi\big(\theta^1(s) - \theta^2(s)\big) \Big] ds. \tag{12}$$

Denote by $m \in \mathbb{N} \cup \{0\}$ a natural number such that $t \in [m\delta, (m+1)\delta]$. Then

$$m < \frac{T}{\delta} + 1. \tag{13}$$

The function $\gamma(t)$ is continuous and, therefore, it is bounded on $[0,T]$. In addition, $f^1(\theta), f^2(\theta), \varphi(\theta)$ are bounded on $\mathbb{R}$. Then

$$\int_t^{(m+1)\delta} \gamma(t-s) f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s)\big) f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s) - \frac{\pi}{2}\big) ds = O(\delta), \tag{14}$$

$$\int_t^{(m+1)\delta} \gamma(t-s) \varphi\big(\theta^1(s) - \theta^2(s)\big) ds = O(\delta).$$

So (12) can be rewritten as

$$g(t) - G(t) = \sum_{k=0}^{m} \int_{[k\delta,(k+1)\delta]} \gamma(t-s) \Big[f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s)\big) f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s) - \frac{\pi}{2}\big) - \\ - \varphi\big(\theta^1(s) - \theta^2(s)\big) \Big] ds + O(\delta). \tag{15}$$

Since (4), it follows that on any interval

$$[k\delta, (k+1)\delta] \tag{16}$$

one obtains

$$\gamma(t-s) = \gamma(t-k\delta) + O(\delta), \\ t > s,\ s \in [k\delta, (k+1)\delta]. \tag{17}$$

Here $s \in [k\delta, (k+1)\delta]$ and $O(\delta)$ is independent on k. Then by (15), (17) and the boundedness of $f^1(\theta), f^2(\theta), \varphi(\theta)$ one gets

$$g(t) - G(t) = \sum_{k=0}^{m} \gamma(t-k\delta) \int_{[k\delta,(k+1)\delta]} \Big[f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s)\big) f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s) - \frac{\pi}{2}\big) - \\ - \varphi\big(\theta^1(s) - \theta^2(s)\big) \Big] ds + O(\delta). \tag{18}$$

Denote

$$\theta_k^p(s) = \theta^p(k\delta) + \omega^p(k\delta)(s - k\delta),\ p = 1,2. \tag{19}$$

Now, by (5) with $s \in [k\delta, (k+1)\delta]$

$$\theta^p(s) = \theta_k^p(s) + O(\delta). \tag{20}$$

Since $\varphi(\theta)$ is bounded and continuous on $\mathbb{R}$, one obtains

$$\int_{[k\delta,(k+1)\delta]} \big| \varphi\big(\theta^1(s) - \theta^2(s)\big) - \varphi\big(\theta_k^1(s) - \theta_k^2(s)\big) \big| ds = \\ = O(\delta^2), \tag{21}$$

The function $f^2(\theta)$ is smooth while the function $f^1(\theta)$ is piecewise differentiable and bounded. If $f^1(\theta)$ is also continuous on $\mathbb{R}$, then

$$\int_{[k\delta,(k+1)\delta]} f^1(\theta^1(s)) f^2(\theta^2(s)) f^1\big(\theta^1(s)\big) f^2\big(\theta^2(s) - \frac{\pi}{2}\big) ds = \\ = \int_{[k\delta,(k+1)\delta]} f^1\big(\theta_k^1(s)\big) f^2\big(\theta_k^2(s)\big) f^1\big(\theta_k^1(s)\big) f^2\big(\theta_k^2(s) - \frac{\pi}{2}\big) ds + O(\delta^2). \tag{22}$$

Considering sets (22) outside of sufficiently small neighbourhoods of discontinuity points and using (4)–(6), the proof of theorem is completed. ■

3.2 Example

Consider triangular and sawtooth signals (Fig. 4)

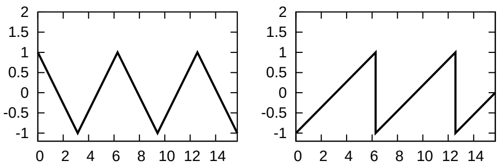
Figure 4: Triangular and sawtooth signal waveforms.

$$f^1(\theta) = \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\big((2n-1)\theta\big)$$
$$f^2(\theta) = -\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\big(n\theta\big).$$

Then by Theorem 1 one gets the following characteristics (Fig. 5)

$$\varphi(\theta) = -\frac{1}{72} + \frac{2}{\pi^2}\sin(2\theta) + \\ + \frac{2}{\pi^2}\sum_{l=1}^{\infty}\begin{cases} \frac{1}{\pi^2 l^4}\cos(2l\theta), & l = 2p, \\ \frac{1}{\pi l^3}\sin(2l\theta), & l = 2p+1, \ p \in \mathbb{N} \end{cases}$$

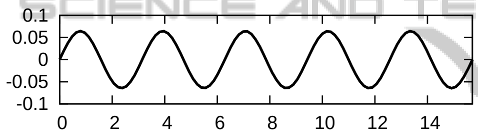
Figure 5: PD characteristics.

4 COSTAS EQUATIONS IN PHASE-FREQUENCY SPACE

From Theorem 1 it follows that physical model of classical Costas loop in signals space (Fig. 1) can be asymptotically changed (for high-frequency generators) to mathematical model in phase-frequency space (Fig. 6).

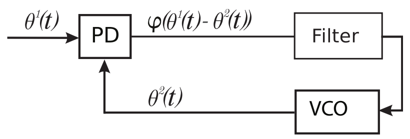

Figure 6: Phase-locked loop with phase detector.

Here PD is a phase detector with corresponding characteristics. Thus, on basis of asymptotic analysis of high-frequency oscillations, characteristics of phase detector can be computed.

The following remark is useful for derivation of differential equations of PLL.

Consider a quantity

$$\dot{\theta}^p(t) = \omega^p(t) + \dot{\omega}^p(t)t, \quad p = 1,2.$$

For correctly designed PLL such that it possesses the property of global stability, there occurs exponential damping of $\dot{\omega}^j(t)$:

$$|\dot{\omega}^p(t)| \le Ce^{-\alpha t}, \quad p = 1,2.$$

Here C and α are certain positive numbers independent of t. Therefore, $\dot{\omega}^p(t)t$ is, as a rule, sufficiently small as compared with the number ω_{min} (see conditions (5)– (6)). From the above one can conclude that the following approximate relation $\dot{\theta}^p(t) \approx \omega^p(t)$ is valid. In deriving the differential equations of this PLL, it can be used a block diagram in Fig. 6 and the following relation

$$\dot{\theta}^p(t) = \omega^p(t), \quad p = 1,2. \tag{23}$$

Note that usually the control law of tunable oscillators is considered linear:

$$\dot{\theta}^2(t) = \omega^2(0) + LG(t). \tag{24}$$

Here ω^2 is a free-running frequency of tunable oscillator, L is a certain number, and $G(t)$ is a control signal, which is a filter output (Fig. 3). Thus, the equation of Costas loop is as follows

$$\dot{\theta}^2(t) = \omega^2(0) + \\ + L\left(\alpha_0(t) + \int_0^t \gamma(t-\tau)\varphi\big(\theta^1(\tau) - \theta^2(\tau)\big)\mathrm{d}\tau\right). \tag{25}$$

Assuming that the master oscillator is such that $\dot{\theta}^1(t) \equiv \omega^1(0)$, one can obtain the following equation for Costas loop

$$\big(\theta^1(t) - \theta^2(t)\big)' + \\ + L\left(\alpha_0(t) + \int_0^t \gamma(t-\tau)\varphi\big(\theta^1(\tau) - \theta^2(\tau)\big)\mathrm{d}\tau\right) = \\ = \omega^1(0) - \omega^2(0). \tag{26}$$

This is an equation of classical PLL.

The above theoretical results is justified by simulation of Costas loop model in phase-frequency space and signal space (Fig. 7). Unlike the filter output for mathematical model in phase-frequency space, the output of filter for physical model in signals space contains additional high-frequency oscillations. These high-frequency oscillations interfere with qualitative analysis and efficient simulation of Costas loop.

The analysis of mathematical model of Costas loop is based on the theory of phase synchronization. This theory was developed in the second half of the last century on the basis of three applied theories: the theory of synchronous and induction electrical motors, the theory of auto-synchronization of unbalanced rotors, and the theory of phase-locked

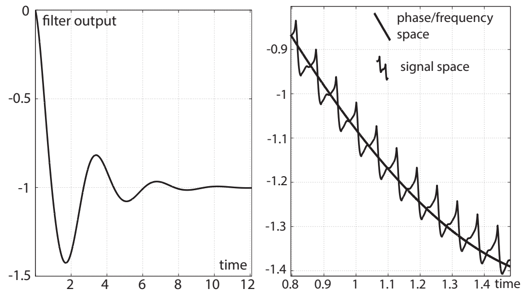

Figure 7: $\omega^2(0) = 101$ Hz, $\omega^1(0) = 100$ Hz, $L = 30$, filter transfer functions $\frac{1}{s+1}$, triangular (input) and sawtooth (VCO) waveforms.

loops. Modification of direct Lyapunov method with the construction of periodic Lyapunov-like functions, the method of positively invariant cone grids, and the method of nonlocal reduction turned out to be most effective (Leonov et al., 1996; Leonov, 2006; Leonov et al., 2009). The latter method, which combines the elements of direct Lyapunov method and bifurcation theory, allows one to extend the classical results of F. Tricomi (Tricomi, 1993) and his progenies (Kudrewicz and Wasowicz, 2007) to the multidimensional dynamical systems.

5 CONCLUSIONS

The approach, proposed in this paper, allows one to compute analytically phase-detector characteristics of costas loop in the case of non-sinusoidal signal waveforms and to proceed from analysis of classical Costas loop in space of signals to analysis and simulation in space of signals phases and, ultimately, to simulate effectively classical Costas loop circuits.

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REFERENCES

Abramovitch, D. (2002). Phase-locked loops: A control centric tutorial. In *Proceedings of the American Control Conference*, volume 1, pages 1–15.

Abramovitch, D. (2008a). Efficient and flexible simulation of phase locked loops, part I: simulator design. In *American Control Conference*, pages 4672–4677, Seattle, WA.

Abramovitch, D. (2008b). Efficient and flexible simulation of phase locked loops, part II: post processing and a design example. In *American Control Conference*, pages 4678–4683, Seattle, WA.

Banerjee, T. and Sarkar, B. (2008). Chaos and bifurcation in a third-order digital phase-locked loop. *International Journal of Electronics and Communications*, (62):86–91.

Best, R. E. (2007). *Phase-Lock Loops: Design, Simulation and Application*. McGraw-Hill.

Bouaricha, A. et al. (2012). Hybrid time and frequency solution for PLL sub-block simulation. US Patent 8,209,154.

Bragin, V. O., Vagaitsev, V. I., Kuznetsov, N. V., and Leonov, G. A. (2011). Algorithms for finding hidden oscillations in nonlinear systems. The Aizerman and Kalman conjectures and Chua's circuits. *Journal of Computer and Systems Sciences International*, 50(4):511–543. 10.1134/S106423071104006X.

Chang, G. and Chen, C. (2008). A comparative study of voltage flicker envelope estimation methods. In *Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century*, pages 1–6.

Costas, J. (1956). Synchoronous communications. In *Proc. IRE*, volume 44, pages 1713–1718.

Djordjevic, I. B., Stefanovic, M. C., Ilic, S. S., and Djordjevic, G. T. (1998). An example of a hybrid system: Coherent optical system with costas loop in receiver-system for transmission in baseband. *J. Lightwave Technol.*, 16(2):177.

Feely, O. (2007). Nonlinear dynamics of discrete-time circuits: A survey. *International Journal of Circuit Theory and Applications*, (35):515–531.

Feely, O., Curran, P. F., and Bi, C. (2012). Dynamics of charge-pump phase-locked loops. *International Journal of Circuit Theory and Applications*. 10.1002/cta.

Hasegawa, K., Kanetsuna, H., and Wakamori, M. (2001). GPS positioning method and GPS reception apparatus. EP1092987 A2.

Henning, F. H. (1981). *Nonsinusoidal Waves for Radar and Radio Communication*. Academic Pr, first edition.

Kaplan, E. and Hegarty, C. (2006). *Understanding GPS: Principles and Applications*. Artech House.

Kudrewicz, J. and Wasowicz, S. (2007). *Equations of Phase-Locked Loops: Dynamics on the Circle, Torus and Cylinder*, volume 59 of *A*. World Scientific.

Kuznetsov, N. V., Leonov, G. A., Neittaanmäki, P., Seledzhi, S. M., Yuldashev, M. V., and Yuldashev, R. V. (2010a). Nonlinear analysis of phase-locked loop. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 4(1):34–38. 10.3182/20100826-3-TR-4016.00010.

Kuznetsov, N. V., Leonov, G. A., and Seledzhi, S. M. (2011a). Hidden oscillations in nonlinear control systems. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 18(1):2506–2510. 10.3182/20110828-6-IT-1002.03316.

Kuznetsov, N. V., Leonov, G. A., and Vagaitsev, V. I. (2010b). Analytical-numerical method for attractor localization of generalized Chua's system. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 4(1):29–33. 10.3182/20100826-3-TR-4016.00009.

Kuznetsov, N. V., Leonov, G. A., Yuldashev, M. V., and Yuldashev, R. V. (2011b). Analytical methods for computation of phase-detector characteristics and PLL design. In *ISSCS 2011 - International Symposium on Signals, Circuits and Systems, Proceedings*, pages 7–10. 10.1109/ISSCS.2011.5978639.

Kuznetsov, N. V., Leonov, G. A., Yuldashev, M. V., and Yuldashev, R. V. (2012). Nonlinear analysis of Costas loop circuit. *ICINCO 2012 - Proceedings of the 9th International Conference on Informatics in Control, Automation and Robotics*, 1:557–560. 10.5220/0003976705570560.

Kuznetsov, N. V., Neittaanmäki, P., Leonov, G. A., Seledzhi, S. M., Yuldashev, M. V., and Yuldashev, R. V. (2011c). High-frequency analysis of phase-locked loop and phase detector characteristic computation. *ICINCO 2011 - Proceedings of the 8th International Conference on Informatics in Control, Automation and Robotics*, 1:272–278. 10.5220/0003522502720278.

Leonov, G. A. (2006). Families of transverse curves for two-dimensional systems of differential equations. *Vestnik St.Petersburg University*, (4):48–78.

Leonov, G. A. (2008). Computation of phase detector characteristics in phase-locked loops for clock synchronization. *Doklady Mathematics*, 78(1):643–645.

Leonov, G. A. and Kuznetsov, N. V. (2007). Time-varying linearization and the Perron effects. *International Journal of Bifurcation and Chaos*, 17(4):1079–1107. 10.1142/S0218127407017732.

Leonov, G. A. and Kuznetsov, N. V. (2011). Analytical-numerical methods for investigation of hidden oscillations in nonlinear control systems. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 18(1):2494–2505. 10.3182/20110828-6-IT-1002.03315.

Leonov, G. A. and Kuznetsov, N. V. (2013). Hidden attractors in dynamical systems. From hidden oscillations in Hilbert-Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractors in Chua circuits. *International Journal of Bifurcation and Chaos*, 23(1). 10.1142/S0218127413300024, art. no. 1330002.

Leonov, G. A., Kuznetsov, N. V., and Seledzhi, S. M. (2009). *Automation control - Theory and Practice*, chapter Nonlinear Analysis and Design of Phase-Locked Loops, pages 89–114. In-Tech.

Leonov, G. A., Kuznetsov, N. V., and Vagaitsev, V. I. (2011a). Localization of hidden Chua's attractors. *Physics Letters A*, 375(23):2230–2233. 10.1016/j.physleta.2011.04.037.

Leonov, G. A., Kuznetsov, N. V., and Vagaitsev, V. I. (2012a). Hidden attractor in smooth Chua systems. *Physica D: Nonlinear Phenomena*, 241(18):1482–1486. 10.1016/j.physd.2012.05.016.

Leonov, G. A., Kuznetsov, N. V., Yuldahsev, M. V., and Yuldashev, R. V. (2011b). Computation of phase detector characteristics in synchronization systems. *Doklady Mathematics*, 84(1):586–590. 10.1134/S1064562411040223.

Leonov, G. A., Kuznetsov, N. V., Yuldahsev, M. V., and Yuldashev, R. V. (2012b). Analytical method for computation of phase-detector characteristic. *IEEE Transactions on Circuits and Systems – II: Express Briefs*, 59(10):633–647. 10.1109/TCSII.2012.2213362.

Leonov, G. A., Ponomarenko, D. V., and Smirnova, V. B. (1996). *Frequency-Domain Methods for Nonlinear Analysis. Theory and Applications*. World Scientific, Singapore.

Margaris, W. (2004). *Theory of the Non-Linear Analog Phase Locked Loop*. Springer Verlag, New Jersey.

Nowsheen, N., Benson, C., and Frater, M. (2010). Design of a high frequency fpga acoustic modem for underwater communication. In *OCEANS 2010 IEEE-Sydney*, pages 1–6. IEEE.

Sarkar, A. and Sengupta, S. (2010). Second-degree digital differentiator-based power system frequency estimation under non-sinusoidal conditions. *IET Sci. Meas. Technol.*, 4(2):105–114.

Suarez, A., Fernandez, E., Ramirez, F., and Sancho, S. (2012). Stability and bifurcation analysis of self-oscillating quasi-periodic regimes. *IEEE transactions on microwave theory and techniques*, 60(3):528–541.

Suarez, A. and Quere, R. (2003). *Stability Analysis of Nonlinear Microwave Circuits*. Artech House, New Jersey.

Sutterlin, P. and Downey, W. (1999). A power line communication tutorial - challenges and technologies. In *Technical Report*. Echelon Corporation.

Thede, L. (2005). *Practical analog and digital filter design*. Artech House.

Tricomi, F. (1993). Integrazione di differeziale presentasi in electrotechnica. *Annali della Roma Schuola Normale Superiore de Pisa*, 2(2):1–20.

Troedsson, N. (2009). Method and simulator for generating phase noise in system with phase-locked loop. US Patent App. 12/371,828.

van den Berg, R., Pogromsky, A., and Rooda, K. (2006). Convergent systems design: Anti-windup for marginally stable plants. In *45th IEEE Conference on Decision and Control*, pages 5441–5446. Optical Society of America.

Wang, L. and Emura, T. (1998). A high-precision positioning servo-controller using non-sinusoidal two-phase type PLL. In *UK Mechatronics Forum International Conference*, pages 103–108. Elsevier Science Ltd.

Wang, L. and Emura, T. (2001). Servomechanism using traction drive. *JSME International Journal Series C*, 44(1):171–179.

Waters, G. W. (1982). Costas loop QPSK demodulator. US Patent 4,344,178.