Innovation Cycles Control through Markov Decision Processes

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Abstract: Innovations are introduced in several cycles, or steps which are of stochastic character. Successful completion of each cycle results in the beginning of the next one. Initial stages are connected with expenses of risk (venture) capital and the investments are returned in the final stages, usually with quite big profit. A helpful approach for control of the innovation process is the use of Markov decision processes which have proved to be an efficient tool for control of multi state stochastic processes. Those stages may be summarizes as: 1 – prestart stage; 2 – start stage; 3 – initial expansion stage; 4 – quick expansion stage; 5 – stage of reaching liquidity of venture investments; 6 – stage of project failure and its cancelling. The transition from state to state may be controlled through control techniques of Markov Decision Processes so that maximum profit is achieved in shortest time. The stages are conditional and some of them may be united, e.g. 1 and 2, or 3 and 4.

1 INTRODUCTION

It is known that the innovations' introduction through the respective innovation cycles as a rule is accompanied with considerable uncertainty and it is of definitely expressed stochastic character. As the successful completion of each innovation project very often results in considerable profit this stimulates the investment of considerable venture (risk) means. A very important task arises for preliminary careful considering and calculating the stochastic character of the on going processes.

A multi step discrete Markov decision process with mixed policies is proposed in the present work, for the innovation risks interpretation. The innovation process is accomplished, and probably finished, as a rule, in a cycle of the following 6 stages: 1 – prestart and start stage; 2 – initial expansion stage; 3 – quick expansion stage; 4 – preparatory stage; 5 – stage of reaching liquidity of the venture investment; 6 – stage of project failure and its liquidation (Grossi, 1990, Cormican, 2004, Bernsteina, 2006). Besides, the process at each stage may be in different states where the decision maker may undertake different actions which result in the transition to a new state with respective profits and losses. The first three stages are connected with initial investments and respective losses. The objective is they to be minimized. The last three stages may generate profit and ensure full return of the investments and considerable gains, but it may also result in considerable loss if the innovation product is a failure. It is to be clearly noticed that the innovation introduction is a risky enterprise and not each attempt is successful and winning.

It should be explicitly noticed that the innovation process may only pass from a given stage to the next one and can never return to a previous stage. No other stages except the last ones – success or failure, are absorbing - i.e. the innovation process may not stay for ever in any of the initial stages or it fails. The process may stay in a given stage for some time. It is a responsibility of the decision maker to undertake such control actions that the process leaves as soon as possible the first three stages, which generate expenses, with min losses and reaches the final stage, which generate profit.

It is to be also noted, that depending on the decision makers actions a stage may be omitted, e.g. to pass directly from stage r to stage $r+2$. I.e. stages so described are to some degree conditional but nonetheless the process may develop in only forward direction.

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2 MARKOV DECISION MODEL FOR THE INNOVATION PROCESS

We consider an innovation process, which might be at any of the six stages of implementation of a new product. Of course this is for purposes of methodology. In fact one should begin from the first stage and reach the last one.

We introduce the following denotation: $N_j = \Gamma_j^{-1}$ where the right hand part of the upper equation is a reverse mapping of node *j* of the graph from Figure 1. P_{ij}^k denotes the transition probability of the innovation process to pass from state $i \in N$ to state $j \in N$ when using control $k \in K_i$, where K_i is the set of possible policies from state *i*. As leaping across or going back to stages of the innovation process is impossible, then:

$$P_{ij}^k = \begin{cases} \geq 0, i \in N_j; j \in N; k \in K_i; \\ = 0, otherwise; \end{cases}$$

$$0 \leq P_{ij}^k \leq 1; \sum_{j \in N} P_{ij}^k = 1; i \in N; k \in K_i.$$

By x_i^k will be denoted the probability the innovation process to fall in state i, at using control $k \in K_i$ from this state.

An important feature of the innovation process is that at transition from one stage to the next one in the first three stages resources are spent, and the transition from stage to the other in the last three stages increasing profit is gained, i.e.:

$$r_i^k = \begin{cases} \leq 0; i \in \{1,2,3\}; k \in K_i; \\ \geq 0; i \in \{4,5,6\}; k \in K_i. \end{cases} \quad (1)$$

Then the maximum restoration of the venture funds initially invested will be obtained at optimal choice of control actions from each possible state of the process, i.e.:

$$\{k^* \in K_i / i \in N_r\}.$$

This optimal control selection from the separate states corresponds to maximization of the objective function:

$$\sum_{j \in N} \sum_{k \in K_j} r_j^k x_j^k \to \max \quad (2)$$

Different methods of linear and dynamic programming (Mine, 1975) may be used for finding the optimal solution of the objective function above with the existing linear probability constraints.

The specific structure of the proposed here multi step discrete Markov decision process corresponds to a sufficient degree to the processes of realization of innovations and provides possibilities for efficient control of venture financing of innovations at their realization.

3 NUMERICAL EXAMPLES

Next Figure 1 illustrates a Markov Decision Process for control of the development of an innovation through the 6 stages. The set of arcs *U* show the possible transition from one stage (state) of the innovation process to another one. The denotations on the arcs of the decision graph should be decoded as follows:

$P_{i,j}^{k_l}$ - the probability for transition from stage *i* to stage *j* using control action k_l in state *i*.

In the final two stages, 5 and 6, which are ergodic there is one only possible action. At the other stages we accept for illustration that there are two possible control actions with the respective transition probabilities.

Formally this means that the possible actions $\{k_j\}$ in each state $j \in N$ are defined in the following way:

$$k_j = \begin{cases} \{1\}, j \in \{5,6\}; \\ \{1,2\}, j \in N \setminus \{5,6\} \end{cases}$$

The initial probability the process to be in state $i \in N$ is equal to:

$$a_i = \begin{cases} 1, i = 1; \\ 0, otherwise. \end{cases}$$

The problem for finding optimal policies for the Markov Decision Process shown in Figure 1 may be reduced to the following linear programming problem with objective function (2) and constraints:

$$\sum_{k \in K_j} x_i^k = 1, \text{ if } i = 1 \quad (3)$$

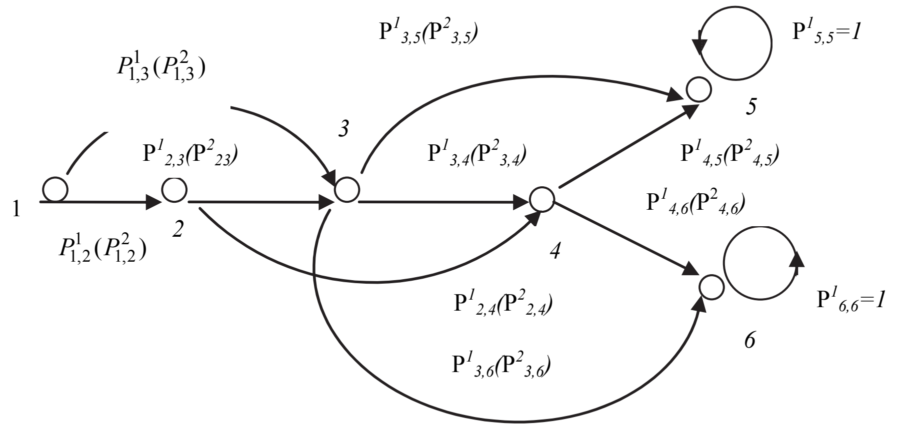

Figure 1: Exemplary transition graph for an innovation process.

$$\sum_{k\in K_j} x_j^k - \sum_{j\in N_j}\sum_{k\in K_i} P_{ij}^k x_i^k = 0 \text{, if } j \in N \quad (4)$$

$$x_j^k \geq 0 \text{, if } k \in K_j; j \in N \quad (5)$$

If we take as a base the graph in Figure 1, then the equations (2) and (3) to (5) will acquire the following form:

$$r_1^1x_1^1 + r_1^2x_1^2 + r_2^1x_2^1 + r_2^2x_2^2 + r_3^1x_3^1 + r_3^2x_3^2 + r_4^1x_4^1 + r_4^2x_4^2 + r_5^1x_5^1 + r_6^1x_6^1 \rightarrow \max \quad (6)$$

under the constraints:

$$x_1^1 + x_1^2 = 1 \quad (7)$$

$$x_2^1 + x_2^2 - P_{1,2}^1x_1^1 - P_{1,2}^1x_1^2 = 0 \quad (8)$$

$$x_3^1 + x_3^2 - P_{1,2}^1x_1^1 - P_{1,3}^1x_1^2 - P_{2,3}^1x_2^1 - P_{2,3}^2x_2^2 = 0 \quad (9)$$

$$x_4^1 + x_4^2 + P_{2,4}^1x_2^1 - P_{2,4}^2x_2^2 - P_{3,4}^1x_3^1 - P_{3,4}^2x_3^2 = 0 \quad (10)$$

$$x_5^1 - P_{3,5}^1x_3^1 - P_{3,5}^2x_3^2 - P_{4,5}^1x_4^1 - P_{4,5}^2x_4^2 = 0 \quad (11)$$

$$x_6^1 - P_{3,6}^1x_3^1 - P_{3,6}^2x_3^2 - P_{4,6}^1x_4^1 - P_{4,6}^2x_4^2 = 0 \quad (12)$$

$$x_5^1 + x_6^1 = 1 \quad (13)$$

$$x_j^k \geq 0; k \in K_j; j \in N \quad (14)$$

Let the profits (expenses) $\{r_i^k\}$ have the following values:

$$r_1^1 = -10; r_1^2 = -11; r_2^1 = -5; r_2^2 = -6;$$
$$r_3^1 = 7; r_3^2 = 8; r_4^1 = 10; r_4^2 = 12;$$
$$r_5^1 = 0; r_6^1 = 0.$$

The transition probability values are defined in the following table:

Table 1: Transition probabilities.

STATE 1				STATE 2			
Policy 1		Policy 2		Policy 1		Policy 2	
$P_{1,2}^1$	0,8	$P_{1,2}^2$	0,9	$P_{2,3}^1$	0,7	$P_{2,3}^2$	0,8
$P_{1,3}^1$	0,2	$P_{1,3}^2$	0,1	$P_{2,4}^1$	0,3	$P_{2,4}^1$	0,2
STATE 3				**STATE 4**			
Policy 1		Policy 2		Policy 1		Policy 2	
$P_{3,4}^1$	0,6	$P_{3,4}^2$	0,5	$P_{4,5}^1$	0,8	$P_{4,5}^2$	0,7
$P_{3,5}^1$	0,3	$P_{3,5}^2$	0,3	$P_{4,6}^1$	0,2	$P_{4,6}^2$	0,3
$P_{3,6}^1$	0,1	$P_{3,6}^2$	0,2				
STATE 5				**STATE 6**			
Policy 1		-		Policy 1		-	
$P_{5,5}^1$	1	-		$P_{6,6}^1$	1	-	

The respective transition probabilities are shown above the arcs of the graph shown in Figure 1, when using different possible policies. If only policy 1 or respectively – only policy 2 is used, then the transition probabilities tables will have the following form:

Table 2: Transition probabilities for policy 1.

	1	2	3	4	5	6
1.	0	0,8	0,2	0	0	0
2.	0	0	0,7	0,3	0	0
3.	0	0	0	0,6	0,3	0,1
4.	0	0	0	0	0,8	0,2
5.	0	0	0	0	1	0
6.	0	0	0	0	0	1

Table 3: Transition probabilities for policy 2.

	1	2	3	4	5	6
1.	0	0,9	0,1	0	0	0
2.	0	0	0,8	0,2	0	0
3.	0	0	0	0,5	0,3	0,2
4.	0	0	0	0	0,7	0,3
5.	0	0	0	0	1	0
6.	0	0	0	0	0	1

Table 2 reflects transition probabilities for policy 1 and Table 3 – for policy 2 respectively.

At least two classes may be distinguished in this matrix – one quasi block diagonal ergodic, and one absorbing, corresponding to states 5 and 6. When the process being controlled falls in one of the latter states it remains there for ever.

At defining the optimal control through relations (5) to (13) in the rows of both matrices #3 and #4 will be used, in general with different probabilities, i.e. both pure and mixed policies will be used, as seen in the solving of the particular problem.

The linear programming problem (6) to (14) includes 10 variables $\{x_j^k\}$ and 8 constraints. Its solution results in the following optimal values of the variables:

Table 4: Linear programming problem solution.

Variables	Optimal values
x_1^1	1
x_1^2	0
x_2^1	0,8
x_2^2	0
x_3^1	0,76
x_3^2	0
x_4^1	0
x_4^2	0,696
x_5^1	0,7152
x_6^1	0,248

It is seen from the table above, that in the example considered the optimal solution leads to pure optimal policies of both types – 1 or 2. Next table shows the optimal pure policy and the respective optimal strategy.

Table 5: Optimal pure policy and respective strategy.

State $i \in N$	1	2	3	4	5	6
Optimal policy $k^* \in K_i$	1	1	1	2	1	1
Optimal strategy $\{k^* \in K_i \,/\, i \in N\}$	{1, 1, 1, 2, 1, 1}					

The following matrix of the achieved optimal transition probabilities of the Markov process may be drawn up on the base of the optimal policies.

The new (optimal) transition probabilities matrix thus constructed also consists of a quasi diagonal ergodic class and an absorbing class of two states. In it one row (the fourth) of Table 4 is used, corresponding to policies 2. The remaining rows are from Table 3, corresponding to policies 1. In this sense it is mixed by using both policies – 1 and 2.

Table 6: Optimal transition probabilities matrix.

	1	2	3	4	5	6
1.	0	0,8	0,2	0	0	0
2.	0	0	0,7	0,3	0	0
3.	0	0	0	0,6	0,3	0,1
4.	0	0	0	0	0,7	0,3
5.	0	0	0	0	1	0
6.	0	0	0	0	0	1

The Markov process thus constructed will flow step by step according to the transition probabilities from Table 6. In the next Figure 2 its stochastic parameters are shown for the purpose of clearness – on the arcs the respective probabilities $\{P_{ij}\}$ are shown for falling from the initial state 1 into state $j \in N$ at passing through the previous state $i \in N$, and in squares next to the vertices the final probabilities $\{\pi_{i,j} \,/\, j \in N\}$ are shown for the process to fall from the initial state 1 into the corresponding state $j \in N$.

The final probabilities are also shown in the following table:

Table 7: Final probabilities.

Final prob.	$\pi_{1,1}$	$\pi_{1,2}$	$\pi_{1,3}$	$\pi_{1,4}$	$\pi_{1,5}$	$\pi_{1,6}$
Values	1	0,8	0,76	0,69	0,72	0,28

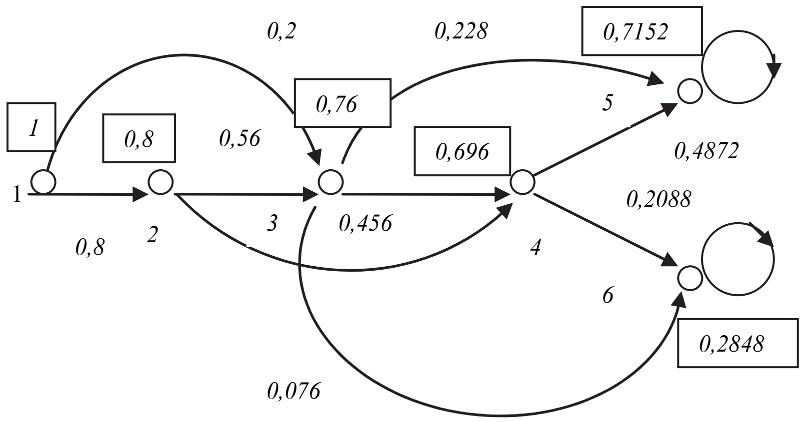

Figure 2: Markov process stochastic parameters from state to state and in the states.

On the base of the optimal values of the variables $\{x_j^k\}$ of Table 4 through (6) the maximum value of the objective function is computed to be - 0,328.

The results obtained provide the possibility some conclusions to be made:

I. When teaching one of the two final states – 5 or 6 the investment made is not paid off in full as 0,328 units remain to be paid off. If the process has fallen in state 5, then the project is successful and in may go on further to pay off the investments made and to produce profit. In case that the process fell in state 6, the project is a failure and it is almost sure it will be cancelled. The amount of 0,328 units should be registered as a loss in this case.

II. Even at optimal decisions for leading the stochastic innovation process, the end the end of the project cannot be certainly predicted – a considerable probability (in the case considered almost 0,3) exists it to end as a failure. This reflects the real conditions in similar class of processes, which are always of explicitly expressed stochastic character.

III. The method proposed for innovation processes control on the base of Markov decision processes has another important advantage - optimal policies and strategies may be recomputed on the base of new and more refined data after each step completed step of the process and the state it falls into. This may result in better final result by improving the strategy initially computed.

IV. It is possible to use more precise classes of Markov decision, e.g. by using profit discount at each step at each step, with constrained capacity or through Markov flows or Markov games (Sgurev, 1993).

4 CONCLUSIONS

In conclusion the following general inferences may be drawn:

1. The innovation processes are highly stochastic and uncertain, which results to highly imprecise prognostication of their completion. And this is connected with a big risk at the venture financing of such processes.

2. The method proposed in the present work for using multistep Markov decision processes for description of the innovation processes provides a possibility their stochastic character to be recognized to a considerable degree and an effective procedure to be proposed for their behavior control.

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