

Watershed from Propagated Markers based on Morphological Hierarchical Segmentation and Graph Matching

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Abstract: Watershed from propagated markers is a generic method to interactive segmentation of objects in image sequences, given by the combination of classical watershed from markers technique to motion estimation. The mask of segmentation, given by the segmentation of the object in the previous frame, is the main parameter to compute a set of markers to segment the same objects in the current frame. This paper introduces a new version of the watershed from propagated markers. In this proposal, the set of markers and its associated model graph are constructed in function of the mask of segmentation. The input graph is constructed given by the hierarchical segmentation of the next frame. The graph matching between the model graph and the input graph provides a pre-segmentation mask that will be used to compute the initial markers to the next frame. Experiments were done to illustrate the performance of the new version and its comparison to methods found in the literature and to previous versions of the watershed from propagated markers.

1 INTRODUCTION

Object segmentation in image sequences is the frame-to-frame segmentation of objects whose semantics remains unchanged, and it is a very important step in video processing frameworks (Ngan, 2011). Basically, it consists in a segmentation-and-tracking framework: the object of interest is segmented in the current frame in order to be tracked to the next frame, where it will be segmented again, and so on. Techniques that do object segmentation in image sequences may be classified into the following segmentation categories: automatic (or non-supervised), assisted (or supervised or also interactive) and semi-automatic. In the automatic segmentation, the objects are detected automatically in the initial frame and they are tracked through the following frames, without user intervention.

Assisted methods offer the option to alter the segmentation results: user may choose the object to be segmented and fix the segmentation/tracking results. Usually four properties are desirable for assisted methods: interactivity, generality, rapid response and progressive manual edition (Flores and Lotufo, 2010a).

Some methods may be classified into a semi-automatic category: the user, for instance, selects an

object in a small set of frames, and, then, he may not intervene in the segmentation process anymore.

The watershed from propagated markers (Flores and Lotufo, 2010a) is an assisted method for object segmentation in image sequences, given by the combination of the classical watershed from markers (Beucher and Meyer, 1992) technique with motion estimation (Beauchemin and Barron, 1995). Initially, the objects are segmented interactively, in order to semantically define the objects of interest. Then, the objects are segmented in a frame-to-frame basis, where the segmentation of such objects in the previous frame provides markers that will be used to track and segment the same objects to the current frame, if there are errors in the segmentation, the user can interactively fix the results by inserting or removing markers.

Watershed from propagated markers is not a hard concept. It allows the research of alternative ways to segment and track the objects by designing new types of markers. This work proposes the application of graph matching in the segmentation framework. Graph matching consists in to find a correspondence among sets of vertices in graphs, where each set may contain attributes about appearance, local features, and / or their relationships with other sets. This information is used to calculate the matching. Graph

matching methods have several applications such as 2D and 3D image analysis, biometrics, biomedical and biological frameworks, etc (Conte et al., 2004).

1.1 Literature Review

There are in the literature previous versions of the watershed from propagated markers include the one based on binding of markers (Flores and Lotufo, 2010a), spatial temporal gradient (Flores and Lotufo, 2010b), and border tracking by graph matching (Ortoncelli and Flores, 2013).

In (Flores and Lotufo, 2010a) binding of markers created around the border of the segmentation mask were propagated by the Lucas and Kanade optical flow computation method (Beauchemin and Barron, 1995) in order to segment the next frame, the same method is used to do the propagation in (Flores and Lotufo, 2010b), but this watershed from propagated markers variation also uses a spatio-temporal gradient to segment the images. This gradient is computed with a 3-D structuring element.

Another version of the watershed from propagated markers is based on graph matching (Ortoncelli and Flores, 2013). In this version, graph matching is applied to propagate markers from the previous frame to the current one. A model graph represents markers in the previous frames, and a frame graph represents the hierarchical segmentation of current frame. An energy function is used to match each edge of model graph with one edge of the frame graph, the result of this matching it is used as markers to segment the object of interest in the current frame.

In the literature, graph matching is also used to segment images without the watershed from propagated markers. In (Noma et al., 2012), the user inputs markers that will be used to create a model graph. Then, a graph matching method is used to compute the correspondence between this graph and another one based on the image regions, the result of this matching is the segmented image. In (Noma et al., 2012) also is presented a semi-automatic variation of the proposed method, in which a image sequence is segmented with the same set of markers computing the matching for each image of the sequence.

The seeded region growing (SRG) algorithm (Adams and Bischof, 1994) is similar to graph matching methods in the context of image segmentation. The SRG algorithm consists in a sequential labelling technique, in which each iteration labels only one pixel that neighbours the already labelled pixels and the aggregation criterion is given by a dissimilarity measure (in graph matching methods the all the regions are labelled in only one iteration). In the liter-

ature there are automatic (Fan et al., 2001) and semi-automatic (Zhi and Jie, 2004) methods that extends SRG algorithm from the pixel level for the regional one in order to segment image sequences. In (Zhi and Jie, 2004) it is also proposed a method variation in which is possible correct interactively the segmentation errors.

1.2 Contributions

Despite the good segmentation results provided by the support of graph matching techniques, The interactive correction of errors and resegmentation due the mispropagation of the markers (or in the segmentation) may be costly, because graph matching techniques are sometimes heavy tasks, the interactive approach would contradict one of the desirable properties of the interactive segmentation methods: rapid response (Flores and Lotufo, 2010a).

To solve this problem the combination between watershed from propagated markers and graph matching is a good alternative, that is improved in (Ortoncelli and Flores, 2013). In this paper it was proposed a new way to combine these methods. Given the mask of segmentation from the previous frame, the model graph is computed from this mask, and this model is matched (with a graph matching method (Noma et al., 2012)) to the previously computed graph (that represents the hierarchical segmentation of the current frame) in order to find a pre-segmentation mask of the objects of interest in the current frame in order to create the markers that will be applied to the segmentation of the current frame with watershed from markers.

This new approach is better than use only a graph matching method, because the matching is done just once in the beginning of the processing of the current frame, in order to create markers. Given the computed markers, watershed from markers is applied to compute the segmentation of the objects of interest. If there are errors in the segmentation, the user can correct then interactively, adding or removing markers, but how this time the segmentation is done with watershed from markers, the response for the user is very fast. Figure 1 illustrates the watershed from propagated markers variation proposed in this paper.

This approach differs from the watershed from propagated markers variation proposed in (Ortoncelli and Flores, 2013), because in this approach it was computed a pre-segmentation mask of the frame by a graph matching method in order to compute de markers, and in (Ortoncelli and Flores, 2013) a graph matching method is used to directly propagate the markers. This new approach showed more efficient



Figure 1: Proposed approach: (a) Marker imposed by the user in the image j ; (b) Image segmented with the method watershed from markers; (c) The scribbles generated from the segmentation mask of the image; (d) Input Graph; (e) Model Graph based on the scribbles from the image c ; (f) Graph matching result. (g) Marker propagated; (h) Marker with user intervention; (i) Final segmentation of the image $j+1$ (after refinement).

and robust results.

The advantage of the proposed approach is that it leads to the creation of more accurate markers. More, since watershed from propagated markers meets all desirable features of interactive segmentation methods (Flores and Lotufo, 2010a), the proposed method is efficient and robust, what can be seen in the experimental results (Section 4), which shows the comparison among the new proposal with others methods: the standard watershed from markers (Beucher and Meyer, 1992) (in which the user input interactively makes for each frame, without propagation, in order to create the ground truth), tree variations of the watershed from propagated markers (Flores and Lotufo, 2010a; Flores and Lotufo, 2010b; Ortoncelli and Flores, 2013), and two semi-automatic methods, also based on the morphological hierarchical segmentation (Noma et al., 2012; Zhi and Jie, 2004).

1.3 Paper Organization

This paper is organized as follows: Section 2 presents some preliminary concepts, needed in the proposal of this paper. Section 3 presents the proposed method. The experimental results are in the Section 4. Finally, the conclusion and the proposal of future works are in Section 5.

2 GRAPH MATCHING AND IMAGE SEGMENTATION

An Attributed Relational Graph (ARG) is a graph which vertices and/or edges are associated to feature vectors (Tsai and Fu, 1979). An ARG may be formally denoted by $G = (V, E, \mu, \sigma)$, in which V is a set of vertices, E is a set of directed edges, μ represents the vertex attributes, and σ represents the edge

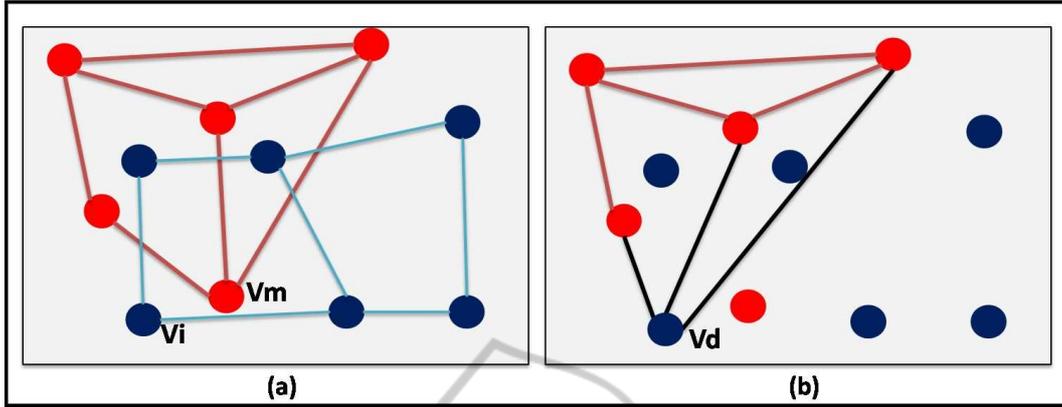


Figure 2: Creation of the deformed graph: (a) graphs G_m (red) and G_i (blue); e (b) $G_d(v_m, v_i)$.

attributes. Let $|V|$ and $|E|$ be, respectively, the cardinalities of the vertices set and of the edges set.

Let $v \in V$ be a vertex of V and $e(v_1, v_2) \in E$ an edge of E . Let $v_1, v_2 \in V$, such that v_1 is adjacent to v_2 . Let the set $\mu(v) = (id, color, ctd)$ be composed by three attributes: i) *id* is the label of the vertex. ii) *color* is the triple given by the mean of each color band inside the region represented by the vertex. iii) *ctd* is the centroid of the region.

Let $e = (v_1, v_2) \in E$, $v_1, v_2 \in V$. Let $\sigma(e) = (ori)$ be an unique attribute given by the vector which angle and norm are defined by the centroids of v_1 and v_2 .

The graph matching method proposed in (Noma et al., 2012) consists in to find a good correspondence between two ARG'S: G_i and G_m . The first one, the *input graph*, $G_i = (V_i, E_i, \mu_i, \sigma_i)$ represents all image regions. Its computation is given by the hierarchical segmentation of the image (Meyer, 2006; Vachier, 1995): regions define the set of vertices and the edges are defined by the neighbourhood among these regions. The *model graph*, $G_m = (V_m, E_m, \mu_m, \sigma_m)$, is given by a subgraph of G_i , where vertices are selected by the intersection with a set of markers (that represents the foreground and background). If a region of the hierarchical segmentation is intercepted by a marker, its corresponding vertex in the input graph is included as a vertex in the model graph. Markers imposition and vertices selection may be done, for instance, manually (Noma et al., 2012). The choice of vertices in the proposal introduced in this paper is described below.

The computation of the correspondence between each vertex of G_m to each vertex of G_i , is a very expensive task, due combinatorial reasons. An alternative to overcome this problem is through the deforming graph strategy (Noma et al., 2012).

The *deformed graph* represents a deformation of the model graph by replacing a vertex of G_m by a

vertex of G_i . Consider a pair (v_i, v_m) of vertices, such that $v_i \in V_i$ and $v_m \in V_m$. The deformed graph is denoted by $G_d(v_i, v_m)$, and it is computed as follows (the Figure 2 illustrates the creation of a deformed graph):

- For each vertex $v_m \in V_m$, replace it by each vertex $v_i \in V_i$ in order to obtain $|V_i|$ deformed graphs.
 - The replacement of v_m for a vertex $v_i \in V_i$ is done in a way that the adjacency relations of v_m remains unchanged. Let the replacing vertex be denoted by v_d . The replacement will define an deformed graph.
 - Given this deformed graph, the matching between v_m and v_i is assessed by a cost function that measures the dissimilarities between the attributes of, respectively, v_d and v_m . This cost function is presented below.
- The best matching between v_m and a vertex $v_i \in V_i$ is the one whose deformed graph provides the minimum value from the cost function applicaton. The region represented by v_i receives the label of v_m .

As stated above, the cost function is given by the measurement of some dissimilarities between attributes. Before the formalization of the cost function, the dissimilarity measures need to be reviewed.

The relative position between two vertex is given by the following equation. It compares two given vectors, \vec{v}_1 and \vec{v}_2 , by considering the angle between them and their norms:

$$Cvec(\vec{v}_1, \vec{v}_2) = \lambda_2 \frac{\cos\theta - 1}{2} + (1 - \lambda_2) \frac{||\vec{v}_1| - |\vec{v}_2||}{C_s}$$

where $\cos\theta$ denotes the cosine of the angle between \vec{v}_1 and \vec{v}_2 . $|\vec{v}|$ denotes vector norm. C_s is a normalization term and $0 \leq \lambda_2 \leq 1$ is a prioritization factor (in this work, it was used $\lambda_2 = 0.5$).

The evaluation of structural dissimilarities between v_i and v_m is given by,

$$dS(Gd(v_i, v_m), Gm) = \frac{1}{|E(v_d)|} \sum_{e_d \in E(v_d)} Cvec(\sigma(e_d), \sigma(e_m))$$

where $E(v_d)$ is the set of deformed edges, connected to v_d , and e_m is the model edge related to v_d .

The evaluation of appearance dissimilarities between v_d and v_m is given by,

$$dA(v_d, v_m) = \frac{Eucl(\mu(v_d).color, \mu(v_m).color)}{C_A}$$

in which $Eucl$ computes the Euclidean distance between two vectors, and the constant C_A is a normalization value for the color difference between v_d and v_m , that is the maximum distance among the color triples.

Let Gd be the deformed graph, and v_d its respective deforming vertex. The cost function $E(v_i, v_m)$ is given by

$$E(v_i, v_m) = \lambda_1 dA + (1 - \lambda_1) \sum_{\forall e \in E(v_d)} dS,$$

in which $0 \leq \lambda_1 \leq 1$ is a prioritization factor (in this work, it was used $\lambda_1 = 0.5$).

3 THE PROPOSED METHOD

Let $Z = (z_1, z_2, \dots, z_n)$ be an input sequence of images, where z_1 is the first frame and z_n is the last one. Let G_i^j and G_m^j are respectively the input ARG and the model ARG of the frame j .

The watershed from propagated markers based on hierarchical segmentation and on the graph matching method described in the previous Section is given as follows:

1. $j \leftarrow 1$.
2. The user segments z_j by the interactive watershed from markers (Fig. 1 (a) and 1 (b)). How better is the segmentation quality, best will be the markers propagated.
3. The markers (scribbles) that will be used for generate G_m^{j+1} are given by the segmentation mask from z_j (Fig. 1 (b)) : the contours of the dilation of the segmentation mask give the external markers, and the contours of the erosion of the same mask provide the internal ones (Fig. 1 (c)).
4. G_i^{j+1} is created. Its vertex represents the regions given by the hierarchical segmentation of z_{j+1} , according an area parameter α (Meyer, 2006) (Fig. 1 (d)).
5. G_m^{j+1} is given by all vertex of G_i^{j+1} which regions are intercepted by the markers computed in Step 3 (Fig. 1 (e)).

6. The graph matching between G_i^{j+1} and G_m^{j+1} is done, generating a pre-segmentation mask of z_{j+1} (Fig. 1 (f)).
7. The intersection between the pre-segmentation mask of the frame z_{j+1} and the dilation of the segmentation mask of z_j , to correct eventual small segmentation errors.
8. Markers are generated around the pre-segmentation mask (Fig. 1 (g)). Such markers are generated in a similar way to approach of binding of markers (Flores and Lotufo, 2010a): the contour of erosion of the pre-segmentation mask and the contour of the erosion of the negation of the pre-segmentation mask are obtained. These contours are broken in short segments forming the set internal and external markers for each object.
9. If there are segmentation errors, the user interactively fixes the segmentation by inserting or removing markers (Fig. 1 (h)), in order to finish the segmentation of the object of interest (Fig. 1 (i)).
10. $j \leftarrow j + 1$
11. if $j < n$, go to Step 3.

Figure 3 illustrates the proposed method with a block diagram.

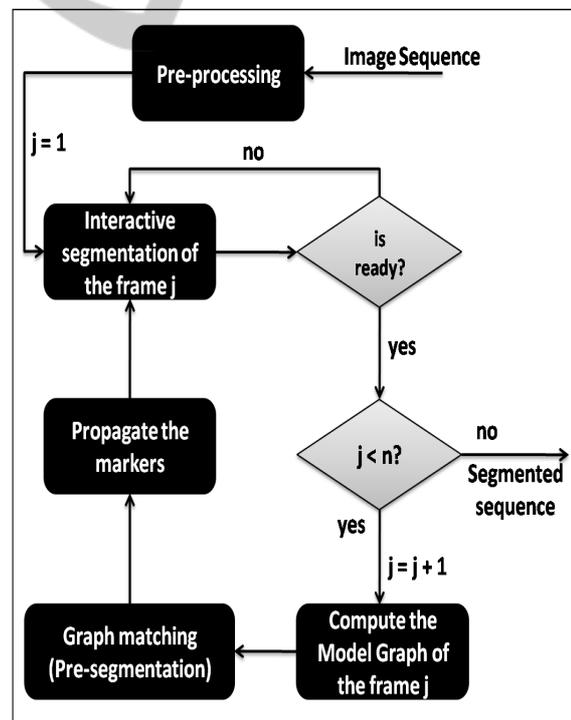


Figure 3: Block diagram.

The graph matching computation is a costly task, since each vertex $v_i \in V_i$ is matched with all vertices

$v_m \in V_m$, in order to find the best correspondence. Considering $n = |V_i|$ and $m = |V_m|$, the graph matching computation has complexity $\Theta(n \cdot m)$.

To overcome this problem, the graph G_i is computed in an unique pre-processing step where a matching cost matrix (MC) is also computed. This matrix contains the matching cost between vertices $v_i, v_j \in V_i$. MC is similar to an adjacency matrix (Cormen et al., 2011), but each element of the matrix contains the correspondence cost between two vertices.

Note that graph G_m not is considered in the pre-processing computation, since there is no way to mount this graph in this step, because it depends of the object segmented from the previous frame. But, since G_m is a subgraph of G_i , the matching between some vertices $v_m \in V_m$ and some vertices $v_i \in V_i$ is contained in MC.

The complexity of MC computation is $\Theta(n^2)$, but it is still advantageous, since it improves the segmentation time, then helps to preserve the property of fast response, from the interactive segmentation methods (Flores and Lotufo, 2010a).

4 EXPERIMENTAL RESULTS

Experiments were done in order to comprove the efficiency and robustness of the proposed approach. The experiments were assessed by the application of a quantitative benchmark (Flores and Lotufo, 2010a) for evaluation of assisted segmentation of objects in image sequences, that measures: the number of user interferences, the time spent, the segmentation error (in relation of the ground truth sequence obtained by the interactively manual segmentation) and the object movement (computed only for the ground truth sequence, not is used to determine the efficiency and robustness of a method, but it helps to understand the results of the benchmark).

Two big sets of experiments were done in order to evaluate the proposed method and to compare it to other methods: the standard watershed from markers (Beucher and Meyer, 1992) (used to create the ground truth sequence), two semi-automatic methods (Noma et al., 2012; Zhi and Jie, 2004), and tree recent variations of the watershed from propagated markers (WFPM), the binding of markers (Flores and Lotufo, 2010a), the spatio-temporal gradient support (Flores and Lotufo, 2010b), and the border tracking by graph matching (Ortoncelli and Flores, 2013).

All methods compared in this experiment and a graphical user interface was designed and implemented in Python. The experiments were done on a computer with the following configuration: Processor

Intel i5 (2.53 Ghz), with 6 GB of memory and Windows 7 operating system.

4.1 Evaluated Methods

The ground truth was obtained by the use of the standard watershed from markers (SWFM). The user imposed interactively markers to the objects of interest in each frame from the sequence, that was segmented by watershed from markers method (Beucher and Meyer, 1992).

Experiments were done with others three watershed from propagated markers (WFPM) variations, for (Flores and Lotufo, 2010a) the markers are propagated by the Lucas and Kanade optical flow computation method (Beauchemin and Barron, 1995), for (Flores and Lotufo, 2010b) the same propagation method is used but with spatio-temporal gradients created with two 3-D structuring elements presented in (Flores and Lotufo, 2010b), B5 and B6. For (Ortoncelli and Flores, 2013) that is based on the hierarchical image segmentation, it was used two extinction area parameters to create the frame graphs (25 e 50), let these graphs computed in a pre-processing step.

The semi-automatic methods (Noma et al., 2012; Zhi and Jie, 2004) evaluated also are based on the hierarchical image segmentation, for (Noma et al., 2012) it was used two area parameters (25 and 50), and for (Zhi and Jie, 2004) it was used only the parameter 50, but for the interactive and semi-automatic versions of the method. In both of the semi-automatic methods evaluated the graphs are computed in a pre-processing step.

In the experiments, for all the evaluated methods it was used a color gradient, computed by the union of the gradients from each band of the input color image under the RGB color space.

4.2 Quantitative Results

Two experiments were done to illustrate the efficiency and robustness of the proposed method. In the first experiment, it was segmented the man in the 150 first frames of Foreman sequence. The second experiment is a repetition of the first one using the Carphone sequence (<http://trace.eas.asu.edu/yuv/>). The Tables 1 and 2 show the segmentation results, the columns of this tables represents from left to right the respectively the following information: the segmentation method, the mean of user interactions by frame, the mean time spent to segment a frame (user interaction and processing time) and finally the percentage of segmentation error.

Table 1: Experimental Results: Foreman Sequence.

Method	Interactions	Time (seconds)		Error (%)
		User	Proc.	
Manual (SWFM)	22.453	50.575	0	0
Proposed Approach ($\alpha = 25$)	1.626	3.834	3.668	0.811
Proposed Approach ($\alpha = 50$)	1.62	3.98	3.489	0.845
WFPM - (Flores and Lotufo, 2010a)	1.733	4.41	4.918	1.306
WFPM - (Flores and Lotufo, 2010b) B6	1.706	4.224	4.916	1.455
WFPM - (Flores and Lotufo, 2010b) B26	1.713	4.231	4.88	1.465
WFPM - (Ortoncelli and Flores, 2013) - 25	4.486	9.139	10.28	0.93
WFPM - (Ortoncelli and Flores, 2013) - 50	3.68	8.161	9.241	0.95
(Noma et al., 2012) - 25	0.053	0.118	2.077	3.572
(Noma et al., 2012) - 50	0.053	0.118	1.747	3.491
(Zhi and Jie, 2004) - semi-automatic	0.06	0.133	24.291	51.912
(Zhi and Jie, 2004) - interactive	72.053	55.167	24.291	1.276

Table 2: Experimental Results: Carphone Sequence.

Method	Interactions	Time (seconds)		Error (%)
		User	Proc.	
Manual (SWFM)	15.56	39.608	0	0
Proposed Approach ($\alpha = 25$)	1.933	4.571	3.609	0.436
Proposed Approach ($\alpha = 50$)	2	4.969	3.302	0.445
WFPM - (Flores and Lotufo, 2010a)	2.106	4.999	4.883	0.457
WFPM - (Flores and Lotufo, 2010b) B6	1.806	4.491	4.946	0.832
WFPM - (Flores and Lotufo, 2010b) B26	1.713	4.123	4.897	0.851
WFPM - (Ortoncelli and Flores, 2013) - 25	3.346	7.161	11.148	0.57
WFPM - (Ortoncelli and Flores, 2013) - 50	3.746	7.908	9.338	0.471
(Noma et al., 2012) - 25	0.046	0.11	2.86	3.107
(Noma et al., 2012) - 50	0.046	0.11	1.854	3.309
(Zhi and Jie, 2004) - semi-automatic	0.046	0.114	24.691	26.637
(Zhi and Jie, 2004) - interactive	55.553	42.204	24.691	0.889

Details about the experimental results in a frame-by-frame way, are illustrated in the Figures 4 and 6 by graphics, this graphics represents the informations about interaction, time and segmentation error, for some of the methods analyzed in the experiments: (i) the proposed methods with $\alpha = 25$; (ii) the SWFP used to create the ground truth; (iii) the WFPM variation proposed in (Flores and Lotufo, 2010a), that gets the best results between the interactive methods of the experiment (not considering the new approach); and (iv) (Noma et al., 2012) that gets the best results between the semi-automatic methods of the experiment. The motion information about the segmented object, are illustrated in the Figures 6 and 7 by graphics.

More details about the experimental results are available at <http://www.din.uem.br/~fcflores/work/visapp2014.html>. This website shows all the experimental results in a frame-by-frame way, and has several graphics showing the performance variation through the image sequence for each assessed method. Besides tables and graphs, the website also

contains videos with the segmentation results for each experiment.

5 CONCLUSIONS

This paper introduces a new version to the watershed from propagated markers that combines graph matching to hierarchical segmentation. This improvement combines the efficiency and robustness of the watershed from propagated markers with the segmentation quality of the graph matching.

Experiments were done in order to assess the impact of the graph matching method to the watershed from propagated markers framework. The proposed approach showed better results than others variations of the WFPM method, except for (Flores and Lotufo, 2010b) that had better results in relation to the number of interactions and time of user interference (only for the Foreman sequence), but the approach proposed

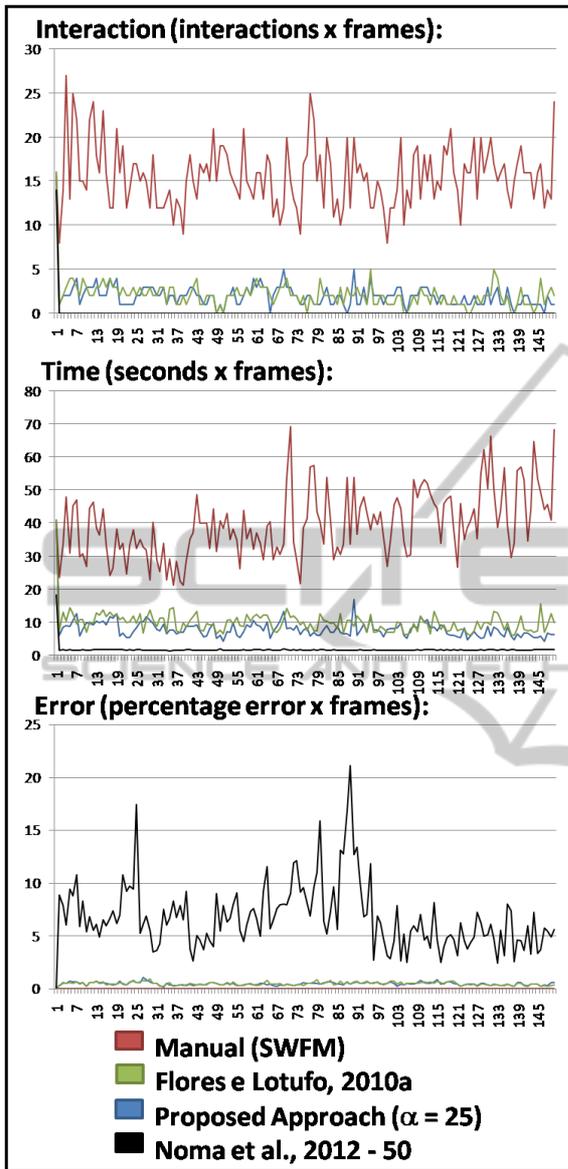


Figure 4: Carphone Sequence - frame-by-frame information.

obtained very close results in these parameters and provided a better segmentation results.

In relation of the semi-automatic methods (Noma et al., 2012; Zhi and Jie, 2004), they also had better results in relation to the number of interactions and time of user interference, as is expected of such methods, in which the user imposes markers in only a limited set of frames. But the proposed approach, as has already been said, had better segmentation results.

It was also concluded that the use of the new improvement in this paper is advantageous even considering the need for a pre-processing step, because the obtained results are better to the ones provided by the

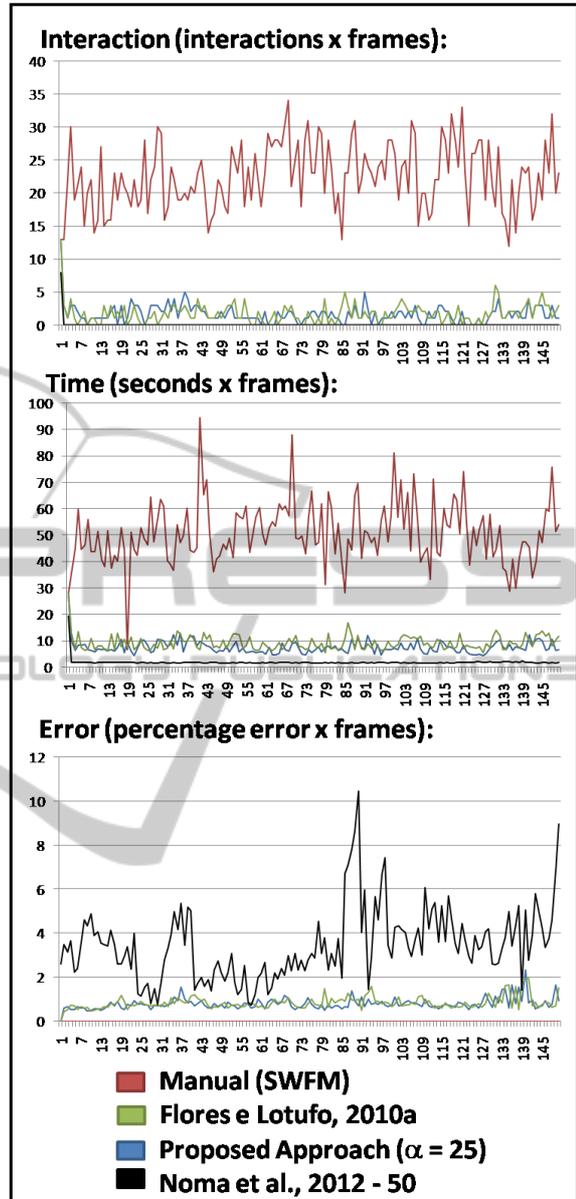


Figure 5: Foreman Sequence - frame-by-frame information.

other variations of the WFPM, and in relation of the semi-automatic methods, the segmentation error was very lower in the proposed approach.

Future works include the exploitation of alternative graph algorithms and their implementation in a parallel paradigm, in order to reduce the processing time, and perhaps eliminate the need of the pre-processing step.

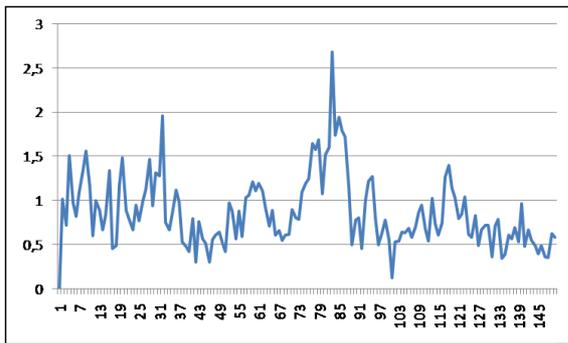


Figure 6: Carphone Sequence - frame-by-frame information.

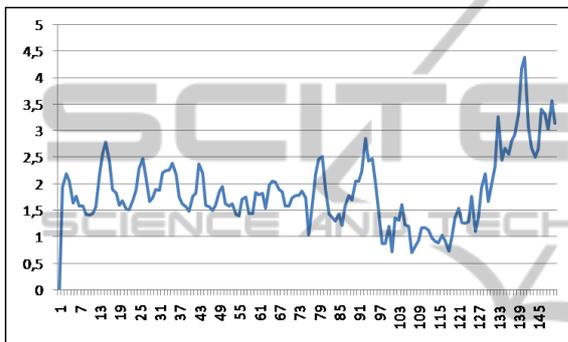


Figure 7: Carphone Sequence - frame-by-frame information.

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