Light-trail based Hierarchy
The Optimal Multicast Route in WDM Networks Without Splitters and Converters

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Abstract: Multicasting in WDM core networks is known as an efficient way of communications in high-speed multimedia applications. However, costly and complicated fabrication prevents multicast capable switches (splitters) from deploying in the proposed architectures. Besides, in practical routing cases, the state of the network is given by a directed graph. Accordingly, this paper investigates the multicast routing without splitters in directed asymmetric topologies. The objective is to minimize the number of wavelengths used and then find the best cost solution among those requiring the same number of wavelengths. In the case of no splitters, a set of light-paths starting from the multicast source covering all the destinations is known as the traditional solution. In this paper, we introduce two new concepts namely *light-trail based hierarchy* and *light-path based hierarchy*, and develop two ILP formulations for them. Theoretical analysis and simulation results show that the optimal solution is a set of light-trail based hierarchies. Particularly, our light-trail based solution achieves fewer wavelengths required (up to 21.95% saved) while keeping slightly lower cost (up to 3.79% saved) compared to light-path based solution.

1 INTRODUCTION

Being capable of supporting heavy load communications, all-optical networks are promising to be serious candidates for high-speed backbone networks. In pure optical routing, the messages are transmitted using Wavelength Division Multiplexing (WDM) technology without electronic processing in the condition that the computed routes should satisfy optical constraints (Zhang et al., 2000).

1.1 Multicast Routing Problems under Optical Constraints

Multicast is known as the efficient way of communications to perform data transmission from a source to several destinations. In traditional IP electronic networks, the solutions are known as spanning trees computed in the topology graph. In optical networks, however, the multicast routes do not necessarily correspond to trees but some structures complying several (optical) constraints. Among the constraints, the availability of light splitters in the switches are often the most hard ones. Light splitters (or multicast capable switches) are special nodes capable of splitting an incoming signal from a predecessor to several successors. Nevertheless, splitters are expensive and complicated in fabrication. Besides, splitting induces considerable power loss (inversely proportional with the number of outgoing ports (Ali and Deogun, 2000b)). Therefore, we assume to study the multicast routing on the WDM networks without splitters. In addition, since wavelength converters (the devices that can shift a passing signal from one wavelength to the other (Mukherjee, 2006)) are costly and immature enough, we also exclude them from this study.

Regarding the objectives of multicast problems, the requirements of economizing the networks resources (e.g., the wavelengths) are first thing to concern. Besides, among the possible routes, the least-cost one is preferred. The total cost of the routes is defined as the summation of the costs of the individual links of the routes and the cost of each link could be any types of metrics including distance, monetary cost, etc., depending on the network entity that we are trying to minimize. However, it is usually hard to simultaneously minimize both metrics. Therefore the trade-off solution is more interesting.

Another important aspect to consider is the kind of examined networks. Most of the studies in optical

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multicast routing problems are carried out on *symmetric networks* under the assumption that there are two opposite directed fibers between every pair of connected switches. In this case the networks are well modeled by *undirected* (Zhang et al., 2000), (Din, 2009) or *bidirected* graphs (Ali and Deogun, 2000a), (Zhou et al., 2010). However, it is more practical and general to investigate the routing problems on *asymmetric networks* that can be modeled by an arbitrary *directed* graphs (or *digraphs*) where each arc represents a directed fiber between a pair of nodes. It is more practical because even if the network is designed to be bidirected, when some demands hold some of the network resources, the resultant topology graph is then no longer bidirected but (arbitrarily) directed, therefore the routing for subsequent demands will be calculated on a digraph. It is also more general because bidirected graphs are special cases of digraphs where every arc has its reverse one.

1.2 Related Works

Due to its interest, WDM multicast routing has been investigated intensively in the literature and several propositions exist to adapt multicast routing algorithms to the optical constraints (cf. (Zhang et al., 2000) for some basic algorithms and (Zhou and Poo, 2005) for a survey).

The problem of minimizing the number of used wavelengths was investigated at first in (Li et al., 2000). The considered network is assumed to be equipped with splitters and wavelength converters, and it is considered as a set of wavelength graphs where the arcs representing the wavelengths available in the corresponding fibers. The objective is to construct a light-tree satisfying optical constraints such that the number of required wavelengths is minimized. The NP-hardness of the problem is proved, and an approximation algorithm has been proposed.

The case of switching without splitters in *symmetric networks* has been discussed in (Ali and Deogun, 2000a). The problem is to find a Multiple-Destination Minimum Cost Trail that starts from a source and spans all the destinations with minimizing the total cost of the links traversed. To ensure a feasible solution, a low-cost cross-connect architecture called Tap-and-Continue (TaC) has been proposed to replace splitters. TaC cross-connects can tap a signal with small power at the local station and forward it to one of its output ports. The problem is proved to be NP-hard and then a heuristic (namely MDT) is proposed. The advantage of MDT heuristic is that only one wavelength (and one transmitter) is sufficient for each multicast request (i.e, the wavelength is minimized). However, due to multitude of round-trip traversing, a large number of links is required in both directions, and the total cost of the light-trail is always very high.

The multicast routing problems without splitters in *asymmetric networks* has been studied in (Le et al., 2013). The problem is proved to be NP-hard, and two heuristics namely Farthest First and Nearest First are proposed. These heuristics based on light-trails. Thanks to the interesting properties of light-trails, the number of wavelengths can be considerably saved. In comparison to the heuristics proposed in (Din, 2009), they provide better solutions with fewer wavelengths required and lower total cost. However there are no exact solutions given to calculate their approximation ratios.

In this paper, we study the multicast routing in asymmetric WDM networks without splitters and converters. Our objective is to minimize the number of used wavelengths and then try to minimize the total cost. To solve the problem we introduce a new concept called *light-trail based hierarchies* and develop two ILP formulations to search for the exact solutions. We theoretically and experimentally show that the optimal solution is a set of light-trail based hierarchies. The structure of the paper is the following. Section 2 presents the problem modeling and performance metrics. The concept of light-trail based hierarchy and its benefits are given in Section 3. Then the ILP formulation for it is presented in Section 4, followed by the experimental results on their performances in Section 5. We conclude our paper in Section 6.

2 PROBLEM MODELING AND METRICS

The considered network is modeled by the topology graph $G = (V,A)$, a simple digraph in which each arc represents the availability of a directed fiber between a pair of nodes (we suppose that there are at most two opposite directed fibers between any pair of nodes). As mentioned in Section 1, we deal the multicast problem in the networks which are not equipped with any splitters but TaC cross-connects. We suppose that each fiber has the same set W of available wavelengths and each arc $a \in A$ is associated with a positive value $cost(a)$. Given the multicast request $r = (s,D)$, in which $s \in V$ is the source node and $D \subseteq V \setminus \{s\}$ is the set of destinations, the routing problem is to compute the light-structures (e.g., light-trees) from s covering all the destinations simultaneously. These light-structures must comply the following constraints (Zhou and Poo, 2005):

- *Wavelength Continuity Constraint:* In the absence of wavelength converters, the same wavelength should be used continuously on all the links along a light-structure.
- *Distinct Wavelength Constraint:* Two light-structures should be assigned with different wavelengths unless there are edges (or arcs) disjoint.
- *Degree Constraint:* In the absence of light-splitters, all the nodes (except the source) in every light-structure should have the degree that do not exceed two.

Without loss of generality, let LS be the set of light-structures LS_i, $i = 1,\dots,k$ computed for the given request r. Since each light-structure consumes a distinct wavelength, the number of wavelengths needed to perform the multicast request r is equal to k: $No.\ Wavelengths(r) = k$. The total cost of the light-structures is the summation of cost all the arcs in all light-structures LS_i:

$$TotalCost(r) = \sum_{i=1}^{k} cost(LS_i) = \sum_{i=1}^{k} \sum_{a \in LS_i} cost(a).$$

In our study, we first minimize the number of used wavelengths, then try to minimize the total cost among the solutions with the same minimal wavelengths.

Traditionally, the solutions correspond to light-trees in general cases or light-paths in the case of no splitters and no converters (as it is considering in this paper). However, the nodes can be traversed several times with the same wavelength as long as there are different incoming and outgoing ports for each passing (Zhou et al., 2010). Consequently, the solutions are not necessarily sets of light-paths but sets of light-trails. In Section 3, we introduce a new light-structure based on light-trails call *light-path based hierarchies*. We will prove that the problem with light-path based solutions is NP-hard. We then compare it with the light-path based solution to find a better solution for the considering problem. Its ILP formulation is given in Section 4.

3 LIGHT-TRAIL BASED HIERARCHIES

Before defining the new concept *light-trail based hierarchy*, let us first introduce the concept *light-hierarchy* proposed in (Molnar, 2011).

Based on the fact that the multicast routes are not necessarily sub-graphs but any types of structures that retain the connectivity and spanning properties, a *hierarchy* is proposed to replace the traditional solutions (e.g., paths, trees, etc.). It is a graph related structure obtained by a homomorphism of a tree in a graph. Recall that in graphs, a homomorphism can be defined as follows. Let $Q = (W,F)$ and $G = (V,E)$ be two (both *undirected* or *directed*) graphs. Q is called the *base graph*, and G is the *target graph*. An application $h: W \to V$ maps a vertex in W to each vertex in V is a homomorphism if the mapping preserves the adjacency: $(u,v) \in F \Rightarrow (h(u),h(v)) \in E$. If Q is a connected graph without cycle (a tree) then the triple (Q,h,G) defines a *hierarchy* in G. If both graphs Q and F are directed, the triple (Q,h,G) defines a *directed hierarchy*[1] in G (Molnar, 2011). In term of optical routing, *light-hierarchy* is defined as a hierarchy using a single wavelength. Equivalently, a light-hierarchy is a hierarchy that has no duplicated arc but is free of repetition of nodes (Zhou et al., 2010).

According to the definition of light-hierarchy, when the base graph Q is a rooted tree without branching vertices (except the root corresponding to the multicast source[2]), i.e, Q is a *star*, the triple (Q,h,G) defines a special light-hierarchy. It corresponds to a set of rooted arc-disjoint trails in the target graph G, so a single wavelength is needed to serve it. For this reason, we call it *light-trail based hierarchy (LTH)*.

Especially, if the mapping h is injective (i.e., each vertex in W associates with only one vertex in V), then the hierarchy has no duplicated vertices (and so no duplicated arcs well as), and it corresponds to a set of rooted *elementary* trails (trails without repetition of vertices) or paths, in G. This has been considered as the traditional solution for the problem we are examining (Din, 2009). We will call it a *light-path based hierarchy* (LPH) in order to distinguish with a general light-trail based hierarchy.

Figure 1 shows an example of a light-trail based hierarchy. Each vertex of the star Q is associated with an unique vertex of the graph G. In the reverse direction, some vertices of G are mapped from several vertices in Q (nodes a and f). A vertex in Q can be labelled by the vertex in G which it is associated. To distinguish the occurrences related to the same vertex v in G, we will use the labels $v^1, v^2, \dots, v^k$ in Q (and in the hierarchy H as well). Notice that the degree of a vertex occurrence v^i in the hierarchy H is defined as the degree of the corresponding vertex occurrence v^i in the base graph Q (Molnar, 2011). It is important to verify the *degree constraint* stated in Section 2.

With LTH solutions, the considering problem is

[1] In this paper we just consider directed hierarchies, but sometimes the word *directed* is omitted for the sake of simplicity.

[2] Because the source can be equipped with multiple transmitters, so it can inject the same wavelength to different successors.

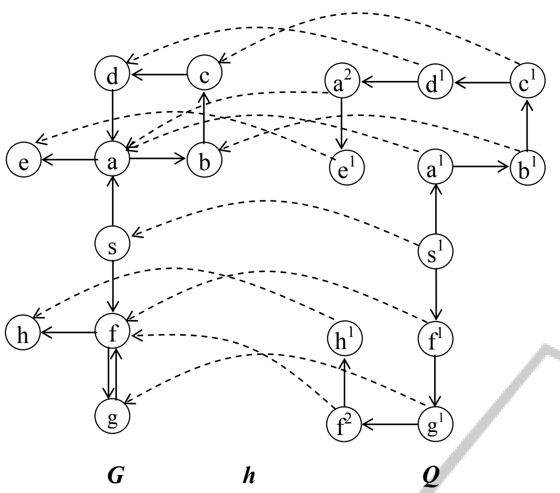

Figure 1: Mapping of vertices from a star for a light-trail based hierarchy.

NP-hard. It can be deduced from the following theorem.

Theorem 3.1. *Let k be any fixed positive integer. If $P \neq NP$, then there is no polynomial time algorithm to check whether $W^* \leq k$, where W^* is the minimum number of wavelengths needed for the given multicast request.*

Proof. We reduce the well-known directed Hamiltonian Path problem to our problem. It is known that to decide whether a given graph is Hamiltonian is NP-complete (Garey and Johnson, 1979).
Let $G = (V, A)$ be a given directed graph. We build a graph G' by replacing each vertex $v \in V$ by two new vertices v^1 and v^2 and linking v^1 to v^2 by the arc (v^1, v^2). Each predecessor of v becomes a predecessor of v^1 and each successor of v becomes a successor of v^2.
We build a graph H by making k copies of G', $G'_1 = (V'_1, A'_1)$, $G'_2 = (V'_2, A'_2)$, ..., $G'_k = (V'_k, A'_k)$, and adding two new vertices s, z connected by the arc (s, z) (s is considered as the source). Then we make z adjacent (predecessor) to all v^1-vertices of each copy of G' (Figure 2). We suppose that $D = V'_1 \cup V'_2 \cup \ldots \cup V'_k$. It is easy to check that H admits a solution $W^* \leq k$ for the light-trail based problem if and only if G admits a directed Hamiltonian path. □

Lemma 3.1. *For the problem of minimizing the number of wavelengths, the path-based solution is not optimal.*

Proof. Let us consider the topology given in Figure 1. We suppose that the source node is s and $D = V - \{s\}$

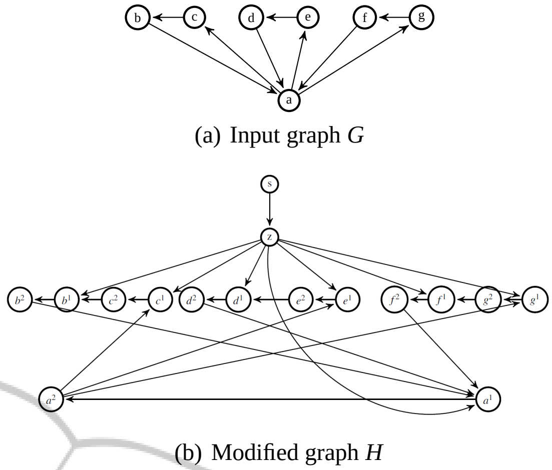

Figure 2: Illustration of the proof (polynomial transformation of G). For simplicity we suppose that $k = 1$.

is the set of destinations. The two solutions: light-trail based hierarchy (LTH) and the light-path based hierarchy (LPH) are shown in Figure 3. As shown in Figure 3a), only one LTH is sufficient to span all the destinations satisfying the aforementioned *degree constraint*. On the other side, as shown in Figure 3b), two LPHs (each using one wavelength) are required, i.e., two different wavelengths needed to span all the destinations. Hence, in this case, the light-path based solution can not be optimal. □

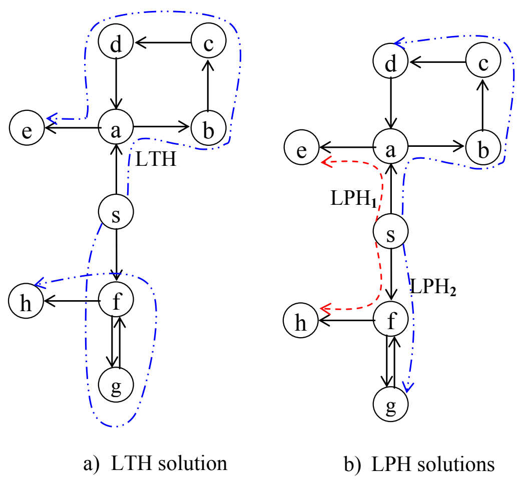

Figure 3: LTH and LPH solutions for the same multicast request.

Theorem 3.2. *The optimal solution for the problem of minimizing the number of wavelengths in non-splitter WDM networks is a set of light-trail based hierarchies. The number of required wavelengths is at least* *one (in the best case) and limited by that needed for the optimal light-path based hierarchies.*

Proof. Assume that the problem always has feasible solutions, i.e., there is at least one directed path from the multicast source to each destination and there are enough wavelengths to route.

Let us first recall the definition of light-trail based hierarchy. The base graph Q is a star, so it satisfies the aforementioned *degree constraint*. According to the definition of vertex degree in the corresponding hierarchy, the hierarchy H also satisfies the degree constraint.

Besides, to guarantee the *distinct wavelength constraint*, the mapping h associates a vertex of Q to a vertex of G such that no two arcs in Q are associated with the same arc in G, i.e., no duplicated arcs in H. The worst case happen when the mapping function (h) is injective, i.e., there are no duplicated vertices (hence no duplicated arcs) in the resultant hierarchies. These light-trail based hierarchies correspond to sets of light-paths (as we call light-path based hierarchies). Thus, even if the duplication of nodes is not possible to diminish the number of wavelengths, this number of wavelengths needed for light-trail based solution is equal to the number of light-path based hierarchies in the worst case.

However, in general cases, the mapping can generate several duplicated vertices in the resultant hierarchies. As shown in the above example, these vertices can help to visit more destinations in one trail. As the result, these duplicated vertices reduce the number of wavelengths required to cover all the destinations. In the best case, a set of trails which can be colored by just one wavelength is sufficient (Figure 3a)). □

Lemma 3.2. *The optimal solution for the problem of minimizing the number of wavelengths does not necessarily minimize the total cost of the solution in nonsplitter WDM networks.*

Proof. Consider an example in Figure 4 where there is a trail that spans all destinations nodes $(s,\ 0,\ 1,\ 0,\ 2,\ 0,\ 3,\ 0,\dots,\ k-2,\ 0,\ k-1,\ 0,\ k)$. Just only one wavelength is sufficient for this trail. Therefore, this light-trail based hierarchy is the optimal solution in term of number of wavelengths. In contrast, the light-path based solution can be found corresponding to the set of paths $\{(s,\ 0,\ 1),\ (s,\ 0,\ 2),\ (s,\ 0,\ 3),\dots,\ (s,\ 0,\ k-1),(s,\ 0,\ k)\}$. All these paths share the arc $(s,\ 0)$. So the number of wavelengths needed to perform the multicast is equal to k.

Now we suppose that the cost of arc $(s,\ 0)$ is equal to 1, all the others have costs of 10. Accordingly, the light-trail based solution consumes $cost(LTH) = 1+2*(k-1)*10+10) = 20*(k-1)+11$. Whereas, the light-path based solution consumes $cost(LPH) = k*1+k*10 = 11*k$. Obviously, $cost(LTH) > cost(LPH), \forall k > 1$. Hence, the lemma follows. □

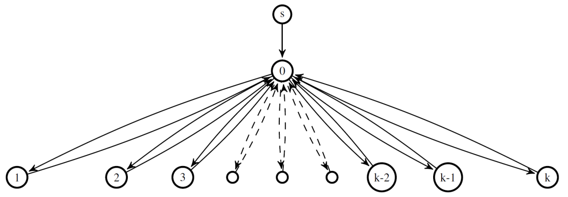

Figure 4: Graph $G=(V,A)$, a source node s and a set $D = \{1\ ,2\ ,3\ ,\dots\ ,k-1\ ,k\ \}$.

4 ILP FORMULATION FOR LIGHT-TRAIL BASED HIERARCHIES

In this section, we formulate the considering problem with the solution corresponding to a set of *light-trail based hierarchies*. Let us recall that each LTH can be composed by a set of rooted arc-disjoint trails (and thus, each requires a distinct wavelength). The fact that one wavelength can may not sufficient to cover all the destinations, several LTHs (i.e, several wavelengths) may be needed.

Notations and Network Parameters:

- $G=(V,A)$: The directed graph with a set V of nodes and a set A of arcs
- W: The set of wavelengths available on each arc
- λ: A wavelength $\lambda \in W$
- Δ: An big enough integer such that $\Delta > \sum_{a \in A} cost(a)$
- $In(m)$: The set of nodes which have incoming arcs to node m in G
- $Out(m)$: The set of nodes which have outgoing arcs from node $m \in V$
- (s,D): A multicast request
- $Indeg(m)$: The in degree of node m
- $Outdeg(m)$: The out degree of node m
- $a(m,n)$: The arc from node m to node n
- $c_{m,n}$: The cost of the arc $a(m,n)$

ILP Variables:

- $L^{\lambda}_{m,n}$: Binary variable. Equal to 1 if wavelength λ is used on arc $a(m,n)$ on wavelength λ; equal to 0 otherwise.

- $F^{\lambda}_{m,n}$: Commodity flow, integer variable. Denotes the number of destinations served by the arc $a(m,n)$ on wavelength λ.
- $w(\lambda)$: Binary variable. Equal to 1 if wavelength λ is used by the light-trails, equal to 0 otherwise.

ILP Formulation:
The primary objective is to minimize the number of wavelengths required. Secondly, among the wavelength optimal solutions, the one with the lowest cost will be chosen. To achieve this, a big enough integer Δ is introduced which is superior to the summation of costs of all the arcs in the graph, i.e., $\Delta > \sum_{a \in A} c_{m,n}$. Accordingly, the general objective function can be expressed as follows.

$$Minimize: \Delta \cdot \sum_{\lambda \in W} w(\lambda) + \sum_{\lambda \in W} \sum_{n \in V} \sum_{m \in Out(n)} c_{m,n} \cdot L^{\lambda}_{m,n} \quad (1)$$

This objective function is subject to a set of constraints which are listed below.

LTH Structure Constraints:
Source Constraint:

$$\sum_{\lambda \in W} \sum_{m \in In(s)} L^{\lambda}_{m,s} = 0 \quad (2)$$

$$1 \leq \sum_{\lambda \in W} \sum_{n \in Out(s)} L^{\lambda}_{s,n} \leq |D| \quad (3)$$

Constraints (2) and (3) ensure that the source s must not have any incoming arcs in a LTH, but must have at least one outgoing arc on some wavelength and the total number of outgoing arcs from s should not exceed the number of destinations, i.e., $|D|$.

Destination Constraint:

$$1 \leq \sum_{\lambda \in W} \sum_{m \in In(d)} L^{\lambda}_{m,d} \leq |D| - 1, \forall d \in D \quad (4)$$

Constraint (4) guarantees that each destination should be spanned in at least one LTH but at most $|D|-1$ LTHs.

Non-source Node Constraint:

$$\sum_{n \in Out(m)} L^{\lambda}_{m,n} \leq \sum_{n \in In(m)} L^{\lambda}_{n,m}, \forall \lambda \in W, \forall m \in V \setminus \{s\} \quad (5)$$

Since all the nodes are MI nodes that are equipped with TaC option, they can be transited several times. However, constraint (5) ensures that the number of outgoing arcs should not exceed the number of incoming ones for every LTH.

Non-member Nodes Constraint:

$$\sum_{n \in Out(m)} L^{\lambda}_{m,n} = \sum_{n \in In(m)} L^{\lambda}_{n,m}, \forall \lambda \in W, \forall m \in V \setminus (s \cup D) \quad (6)$$

Constraint (6) makes sure that non-member nodes can be either not used or served only as the intermediate nodes. In this case, the number of outgoing arcs is equal to the number of incoming ones in every LTH. Constraints (5) and (6) also imply that only destinations can be leaf nodes.

Relationship between $L^{\lambda}_{m,n}$ and $w(\lambda)$:

$$w(\lambda) \geq L^{\lambda}_{m,n}, \forall m,n \in V, \forall \lambda \in W \quad (7)$$

$$w(\lambda) \leq \sum_{m \in V} \sum_{n \in V} L^{\lambda}_{m,n}, \forall \lambda \in W \quad (8)$$

Constraints (7) and (8) indicate that wavelength λ is used in a LTH if and only if at least one arc uses it.

However, the above set of constraints is not enough to guarantee the connectivity of the LTHs as shown in (Zhou et al., 2010). To solve this problem, we use the community method that is proposed in (Yu and Cao, 2005). We introduce an other variable, commodity flow $F^{\lambda}_{m,n}$, as the support of the variable $L^{\lambda}_{m,n}$ in order to make sure the continuity and connectivity of the resultant LTHs.

Connectivity Constraints:
Source Constraint:

$$\sum_{\lambda \in W} \sum_{n \in Out(s)} F^{\lambda}_{s,n} = |D| \quad (9)$$

Constraint (9) indicates that the sum of flows emitted by the source is equal to the number of destinations in the given multicast session.

Destinations Constraints:

$$\sum_{\lambda \in W} \sum_{n \in In(d)} F^{\lambda}_{n,d} = \sum_{\lambda \in W} \sum_{n \in Out(d)} F^{\lambda}_{d,n} + 1, \forall d \in D \quad (10)$$

$$\sum_{n \in In(d)} F^{\lambda}_{n,d} - 1 \leq \sum_{n \in Out(d)} F^{\lambda}_{d,n} \leq \sum_{n \in In(d)} F^{\lambda}_{n,d}, \forall \lambda \in W, \forall d \in D \quad (11)$$

Constraints (10) and (11) ensure that each destination must be consumed totally one and only one flow in all the LTHs. These constraints also guarantee that each destination is reachable from the source.

Non-member Nodes:

$$\sum_{n \in In(m)} F^{\lambda}_{n,m} = \sum_{n \in Out(m)} F^{\lambda}_{m,n}, \forall \lambda \in W, \forall m \in V \setminus (s \cup D) \quad (12)$$

Equation (12) ensures that non-member nodes are only served as intermediate nodes without consuming any flows.

Relationship between $L^{\lambda}_{m,n}$ and $F^{\lambda}_{m,n}$:

$$F^{\lambda}_{m,n} \geq L^{\lambda}_{m,n}, \forall m,n \in V, \forall \lambda \in W \quad (13)$$

$$F^{\lambda}_{m,n} \leq |D| \cdot L^{\lambda}_{m,n}, \forall m,n \in V, \forall \lambda \in W \quad (14)$$

Equations (13) and (14) indicate that an arc should carry a positive number of flows if it is used in a LTH, and this number should not exceed the total flows emitted by the source.

It is worth noting that with the supplementary connectivity constraints, the constraints (3) and (4) are now relaxed.

5 EXPERIMENTAL RESULTS

In this section we present the experimental results of the LTH solution for the concerned problem in comparison to the traditional LPH solution. In order to make the comparison, we also develop an ILP formulation for the LPH solution.

5.1 LPH Structure Constraint

Like the difference between light-path and light-trail structures, LTH allows cycles whereas LPH does not. In other words, for LPH structures, there is at most one incoming arc to every node (except the source) for every given wavelength. Thus, to make ILP formulation for LPH we just add one more constraint to the constraints of the ILP formulation for LTH presented in Section 4. This constraint can be formulated as follows:

$$\sum_{n \in In(m)} L^{\lambda}_{n,m} \leq 1, \forall \lambda \in W, \forall m \in V \setminus \{s\} \quad (15)$$

5.2 Simulation Settings

The two ILP formulations are implemented in C++ using GLPK v4.45 package (Makhorin, 2010). We have carried out series of simulations with random graphs generated using LEDA v.6.3 library (Mehlhorn and Naeher, 2010). All the considered graphs are directed. Due to the fact that ILP programs do not scale well, we just test with relative small graphs in which the number of nodes $N = \{20, 30, 40, 50\}$. The density value (the ratio between the number of arcs and the number of nodes) is fixed to 2. Graphs with this density are considered as sparse graphs. We suppose that sparse graphs well reflect the common core optical networks.

The costs of arcs are randomly selected from the set of integer $\{1, 2, .., 20\}$, and the set of destinations D are also randomly selected with different size $|D| = \{10\%, 20\%, .., 50\%\}$ of the number of nodes. To ensure that there is a feasible solution for all instances, the selected graph must be connected and have at least one directed path from the source to each destination. Moreover, in order to guarantee an acceptable confidence interval, for a certain graph and for each size $|D|$, we run 100 simulations with different sources and destination sets. For each simulation, the number of wavelengths used and the total cost of the routes (hierarchies) are computed as the resultant performance metrics to evaluate the two ILP formulations.

Besides, to accelerate the ILP computation speed, we first employ the Farthest First heuristic proposed in (Le et al., 2013) to the light-trail based hierarchy ILP and the Farthest Greedy heuristic proposed in (Din, 2009) to the light-path based hierarchy ILP to get the upper bound for the number of wavelengths used. These heuristics are known to be good ones for the same concerned problem applying the two considering approaches respectively. With these heuristics, we benefited much of the time saved to accomplish the simulations.

5.3 Simulation Results

The overall simulation results are presented in Table 1. As it is expected, light-trail based hierarchy solution (marked as LTH in the table) outperforms light-path based hierarchy counterpart (marked as LPH) in both number of wavelengths used and the total cost. For the number of wavelengths used, the LTH solution always consumes fewer wavelengths than LPH one. In particular, the ratio of saved wavelengths of LTH solution is up to 19.84% (N = 20), 21.47% (N = 30), 16.09% (N = 40) and 21.95% (N = 50). On average, this ratio is 11.43%, 10.75%, 9.82% and 12.71% respectively. This is obvious, because in general, allowing the repetition of nodes in a LTH, more destinations can be covered by a LTH than by a LPH. Therefore, in term of minimizing the number of wavelengths, LTHs requires fewer or at most equal to the number of wavelengths needed by LPH solution. This is compatible with the Theorem 3.2.

For the total cost, there are few instances in which the total costs are better with LPH solution, shown as some negative reduced ratios in the table. This is compatible with the Lemma 3.2. However, in general the LTH solution results in lower costs with the saved ratio up to 3.79% (N=50), and on average this ratio is 1.31% (N=20), 0.46% (N=30), 0.90% (N=40), and 2.49% (N=50). In short, even though the total cost is the second optimized objective, it is also better with LTH based solution.

Table 1: Performance comparison between light-trail based hierarchy (LTH) and light-path based hierarchy (LPH) solutions.

Size	Wavelengths			Total cost		
N=20						
|*D*|	LPH	LTH	↘	LPH	LTH	↘
2	100	100	0%	5758	5630	2.22%
4	138	113	18.12%	9762	9543	2.24%
6	123	121	1.63%	10591	10565	0.25%
8	126	101	19.84%	8997	8884	1.26%
10	199	164	17.59%	14004	13922	0.59%
AVG			11.43%			1.31%
N=30						
|*D*|	LPH	LTH	↘	LPH	LTH	↘
3	102	100	1.96%	6416	6281	2.10%
6	114	109	4.39%	11474	11253	1.93%
9	163	128	21.47%	14701	14377	2.20%
12	161	151	6.21%	20598	20277	1.56%
15	203	163	19.70%	22069	23281	-5.49%
AVG			10.75%			0.46%
N=40						
|*D*|	LPH	LTH	↘	LPH	LTH	↘
4	120	109	9.17%	9198	9132	0.72%
8	170	154	9.41%	18436	18349	0.47%
12	174	146	16.09%	22640	22158	2.13%
16	196	171	12.76%	26584	26255	1.24%
20	302	297	1.66%	33441	33467	-0.08%
AVG			9.82%			0.90%
N=50						
|*D*|	LPH	LTH	↘	LPH	LTH	↘
5	114	105	7.89%	14417	13981	3.02%
10	151	126	16.56%	19859	19165	3.49%
15	246	192	21.95%	28187	28310	-0.44%
20	204	179	12.25%	33614	32750	2.57%
25	266	253	4.89%	54694	52620	3.79%
AVG			12.71%			2.49%

6 CONCLUSIONS

In this paper we address the multicasting problem in all-optical networks without splitters and converters. The problem is to find the multicast routes which minimize the number of required wavelengths with a low cost. The problem is proved to be NP-hard, and two exact solutions are presented in the forms of ILP formulations (one for light-path based hierarchy, and the other for light-trail based hierarchy). The theoretical analysis pointed out that the optimal solution for the problem in term of wavelength minimization corresponding to a set of light-trail based hierarchies. The simulations are carried out to verify it. Once again, the experimental results showed that not a set of light-path based hierarchies but a set of light-trail based hierarchies are the optimal solution. Moreover, although it does not necessarily deduce the cost optimal solution, the light-trail based solution also appears to be a good solution when consuming a lower cost compared to the traditional light-path based solution in general.

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