

# Local Regression based Colorization Coding

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**Abstract:** A new image coding technique for color image based on colorization method is proposed. In colorization based image coding, the encoder selects the colorization coefficients according to the basis made from the luminance channel. Then, in the decoder, the chrominance channels are reconstructed by utilizing the luminance channel and the colorization coefficients sent from the encoder. The main issue in colorization based coding is to extract colorization coefficients well such that the compression rate and the quality of the reconstructed color becomes good enough. In this paper, we use a local regression method to extract the correlated feature between the luminance channel and the chrominance channels. The local regions are obtained by performing an image segmentation on the luminance channel both in the encoder and the decoder. Then, in the decoder, the chrominance values in each local region are reconstructed via a local regression method. The use of the correlated features helps to colorize the image with more details. The experimental results show that the proposed algorithm performs better than JPEG and JPEG2000 in terms of the compression rate and the PSNR value.

## 1 INTRODUCTION

Colorization refers to the method of colorizing the monochrome image using only a few number of color components. In this method, the chrominance components can be reconstructed with a few number of colorization coefficients combined together with the luminance channel. In contrast, colorization based coding makes use of the fact that the numbers required to colorize the luminance channel is small, and tries to select the colorization coefficients in the encoder which give the best colorization result in the decoder.

(Cheng and Vishwanathan, 2007) utilizes a segmentation method and selects the pixels that represent the colors of the segmented regions iteratively. But there is no way to reduce the redundancy of the initially selected representative pixels. (Ono et al., 2010) reduces the redundancy of the representative pixel set and extracts further required pixels for colorization. However, the final result relies on the randomness of the initially chosen representative pixels. Furthermore, the Levin's colorization process has to be carried out for each iteration step.

(Lee, et al., 2013) constructs a colorization matrix which consists of colorization basis by using a multi-scale meanshift segmentation method. Then, by solving the  $L_0$  minimization problem, the optimum colorization coefficients for colorization are selected. The quality of the output color image is dependent on how well the colorization matrix is designed.

In this paper, we propose a novel approach that designs the colorization matrix based on the use of linear regression. The proposed method is based on the fact that the distribution of the chrominance components in a local region are somewhat correlated with that of the luminance components. Therefore, the chrominance components can be estimated from the luminance components using a linear mean square error (LMSE) estimator. The parameters minimizing the LMSE are obtained from the  $L_0$  minimization problem with respect to the proposed regression based colorization matrix. As can be seen from the experimental results, the compression rate and the quality of the reconstructed color image becomes good and outperforms those from the JPEG and the JPEG2000 standard schemes.

## 2 RELATED WORKS

### 2.1 Levin's Colorization

(Levin et al., 2004) proposes an algorithm that reconstructs the color image from the monochrome luminance image with a few representative pixels containing chrominance information. It is based on the assumption that neighbouring pixels in the chrominance channel have similar values if they have similar values in the luminance channel. Let  $\mathbf{y}$  denote the luminance vector,  $\mathbf{u}$  denote the chrominance vector to be reconstructed, and  $\mathbf{x}$  is a sparse vector which has chrominance value only at the positions of the representative pixels. Then, the cost function of (Levin et al., 2004) is defined as

$$J(\mathbf{u}) = \|\mathbf{x} - \mathbf{A}\mathbf{u}\|^2, \quad (1)$$

where  $\mathbf{A} = \mathbf{I} - \mathbf{W}$ ,  $\mathbf{I}$  is a  $n$  by  $n$  identity matrix,  $n$  is the size of  $\mathbf{u}$ , and  $\mathbf{W}$  is an  $n$  by  $n$  matrix composed of weighting components  $\omega'_{rs}$  defined as

$$\omega'_{rs} = \begin{cases} 0 & \text{if } r \in \Omega \\ \omega_{rs} & \text{otherwise} \end{cases}, \quad (2)$$

$$\omega_{rs} \propto e^{(\mathbf{y}(r) - \mathbf{y}(s))^2 / 2\sigma_r^2}. \quad (3)$$

$\Omega$  is the set of the representative pixels,  $\omega_{rs}$  is the weight between  $\mathbf{y}(r)$  and  $\mathbf{y}(s)$  which are the luminance pixels at  $r$  and  $s$  position, and  $\sigma_r$  is a variance of luminance in the 8-neighbourhood pixels of position  $r$ . To minimize the cost function defined in Eq. (1),  $\mathbf{u}$  can be solved directly as

$$\mathbf{u} = \mathbf{A}^{-1}\mathbf{x} \quad (4)$$

which is due to the fact that  $\mathbf{A}$  is invertible, a fact proved in previous works.

The matrix  $\mathbf{A}$  is constructed from the luminance  $\mathbf{y}$  and the representative pixels, and varies for different sets of representative pixels.

### 2.2 Colorization Coding using Optimization

As mentioned in the previous section the selection of the representative pixels affects the reconstruction of the color image. Colorization coding tries to choose a set of representative pixels in the encoder, which size is small and which gives a good colorization

result in the decoder.

In (Lee, et al., 2013), the colorization process is generalized using the colorization matrix  $\mathbf{C}$ , as following:

$$\mathbf{u} = \mathbf{C}\mathbf{x}. \quad (5)$$

The Levin's colorization algorithm can be expressed by letting  $\mathbf{C}$  equal to  $\mathbf{A}^{-1}$ . Furthermore, in (Lee, et al., 2013), the matrix  $\mathbf{C}$  is not necessarily square. The extraction of the colorization coefficients is formulated as an  $L_0$  minimization problem

$$\arg \min_{\mathbf{x}} \|\mathbf{x}\|_0, \text{ s.t. } \mathbf{u} = \mathbf{C}\mathbf{x}. \quad (6)$$

To satisfy the restricted isometry property (RIP) condition (Candés et al, 2005) which guarantees that the solution from the  $L_1$  minimization is equivalent to that from the  $L_0$  minimization, a Gaussian random matrix  $\mathbf{R}_G$  is multiplied to both sides of equation in Eq. (6) as following:

$$\arg \min_{\mathbf{x}} \|\mathbf{x}\|_0, \text{ s.t. } \mathbf{R}_G\mathbf{u} = \mathbf{R}_G\mathbf{C}\mathbf{x}. \quad (7)$$

Then, Eq. (7) can be changed into an  $L_1$  minimization problem:

$$\arg \min_{\mathbf{x}} \|\mathbf{x}\|_1, \text{ s.t. } \mathbf{R}_G\mathbf{u} = \mathbf{R}_G\mathbf{C}\mathbf{x}. \quad (8)$$

The problem in Eq. (8) can be solved by Basis Pursuit (Chen et al., 1998) or Orthogonal Matching Pursuit (Tropp and Gilbert, 2007). Actually, (Lee, et al., 2013) solves the minimization problem

$$\arg \min_{\mathbf{x}} \|\mathbf{u} - \mathbf{C}\mathbf{x}\|^2, \text{ s.t. } \|\mathbf{x}\|_0 < L \quad (9)$$

to set a limit on the number of colorization coefficients, where  $L$  is a positive number controlling the desired compression rate.

The colorization matrix is constructed directly using the multi-scale meanshift segmentation (Comaniciu and Meer, 2002) on the luminance channel. The luminance channel is segmented with kernels of different bandwidths and photometric and geometric distance weights.

Thus, the colorization matrix can be expressed as

$$\mathbf{C} = [\mathbf{C}_1, \mathbf{C}_2, \dots, \mathbf{C}_k], \quad (10)$$

where  $\mathbf{C}_l$  is a sub matrix constructed from a

meanshift segmentation of scale  $l$ . Here, the entries of the colorization matrix are either 1 or 0 depending on whether the corresponding pixel belongs to the segmented region or not.

### 3 PROPOSED METHOD

In this paper, we change the entries of the colorization matrix such that the correlation in the distribution of the values in the luminance channel is taken into account. We also show that this is equal to perform a total regression on the local sub regions.

Similar to the method in (Lee, et al., 2013), the meanshift segmentation algorithm is performed on the luminance channel in the encoder to produce regions from which the colorization vectors are generated. The meanshift segmentation is performed using kernels of different bandwidths, where the different bandwidths are generated using different photometric and geometric weights.

The colorization matrix  $\mathbf{C}$  is composed of  $k$  sub-matrices, where each sub-matrix corresponds to a certain scale. Each sub-matrix  $\mathbf{C}_l$  is made up by using the segmentation result of the scale- $l$  meanshift segmentation. Denoting the number of segmented regions corresponding to the scale  $l$  by  $n_l$ , the sub matrix  $\mathbf{C}_l$  can be expressed as

$$\mathbf{C}_l = [\mathbf{c}_{l,1}, \mathbf{c}_{l,2}, \dots, \mathbf{c}_{l,2n_l}]. \quad (11)$$

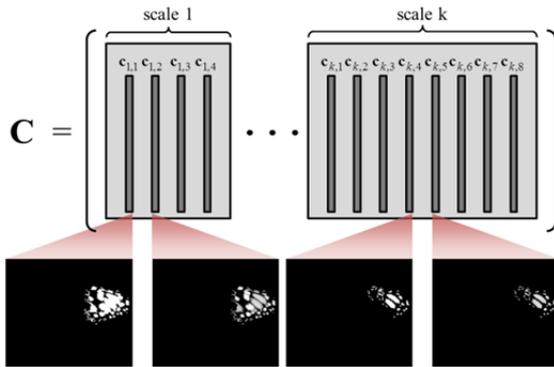


Figure 1: Construction of colorization matrix.

The main difference between the proposed method and the method in (Lee, et al., 2013) is that we construct two types of colorization vectors for each segmented region. The first type consists of the value 1 and 0, where the value 1 is assigned to the segmented regions and 0 for the others. The second type replaces the value 1 with the luminance value at the same position in the luminance channel and 0 for

the others. The type 1 vectors are placed at the odd positions in the sub-matrix  $\mathbf{C}_l$  and type 2 vectors are placed at the even places:

$$\mathbf{c}_{l,2i-1}(j) = \begin{cases} 1 & \text{if } j \in \Omega_l^i \\ 0 & \text{otherwise} \end{cases}, \quad (12)$$

and

$$\mathbf{c}_{l,2i}(j) = \begin{cases} \mathbf{y}(j) & \text{if } j \in \Omega_l^i \\ 0 & \text{otherwise} \end{cases}. \quad (13)$$

The rationale for adding colorization vectors of type 2, i.e., the vectors expressed in Eq. (13), is based on the assumption that the chrominance components have a similar distribution as the luminance components in the same local region.



Figure 2: single scale meanshift segmentation.

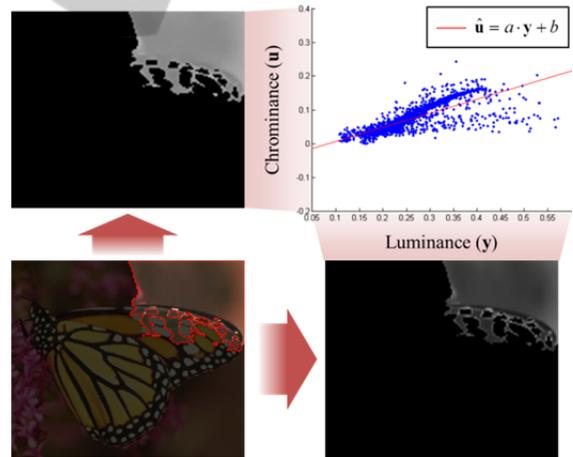


Figure 3: Linear regression of the chrominance function versus the luminance function in a local region segmented from Figure 2.

We demonstrate this fact using an exemplary result. Figure 2 shows the segmentation result using meanshift segmentation with a certain scale. Figure 3 illustrates the distribution of the chrominance values versus the distribution of the luminance values for the right-top segmented region in Figure 3.

It can be observed that the chrominance values are correlated with the luminance values.

Furthermore, it can be observed that the correlation is nearly linear.

Therefore, the correlation can be expressed by a line estimated by the linear mean square error (LMSE) estimator. Then, the chrominance components can be estimated from the luminance components by the following linear equation:

$$\hat{\mathbf{u}} = a \cdot \mathbf{y} + b \cdot \mathbf{1}, \quad (14)$$

where

$$a = \frac{\text{cov}(\mathbf{u}, \mathbf{y})}{\text{var}(\mathbf{y})}, \quad b = \mu_{\mathbf{u}} - a \cdot \mu_{\mathbf{y}}, \quad (15)$$

and  $\mathbf{1}$  denotes the vector having the same size as  $\mathbf{y}$  and with all the entries having the value 1.

Here,  $\mu_{\mathbf{u}}$  is the mean of the chrominance components, and  $\mu_{\mathbf{y}}$  is the mean of the luminance components. The object function to be minimized for each segmented region is

$$e_{l,m} = \|\mathbf{u}_{l,m} - \hat{\mathbf{u}}_{l,m}\|^2, \quad (16)$$

where the subscript  $l, m$  denotes the region corresponding to the  $m$ th basis vector of the scale -1 sub matrix. The local approximated chrominance function based on the linear model in Eq. (14) can be expressed as

$$\hat{\mathbf{u}}_{l,m} = a_{l,m} \cdot \mathbf{y}_{l,m} + b_{l,m} \cdot \mathbf{1}_{l,m}. \quad (17)$$

Now we can observe the fact that the vector  $\mathbf{y}_{l,m}$  corresponds to the vector  $\mathbf{c}_{l,2i}$  in Eq. (13) and the vector  $\mathbf{1}_{l,m}$  to the vector  $\mathbf{c}_{l,2i-1}$  in Eq. (12). Therefore, by solving the problem in Eq. (9) with the matrix containing the colorization vectors in Eq. (12) and Eq. (13), we are actually computing the linear coefficients  $a_{l,m}$ ,  $b_{l,m}$  of the local linear regression model. However, since the minimization problem of Eq. (9) produces the coefficients that minimize the total error in the chrominance differences between the original and the reconstructed color image, the linear coefficients are computed to minimize the following total error in the linear regression model:

$$E = \sum_l \sum_m e_{l,m}. \quad (18)$$

We also use an initial color reconstruction method to further reduce the compressed file size. For the sub-matrix corresponding to scale 1, all the coefficients corresponding to all the basis vectors in

the sub-matrix are extracted in the encoder and sent to the decoder. Then, in the decoder, an initial reconstruction of the color components is done using these coefficients. This initial reconstruction of the color components covers the whole domain of the reconstructed color image, and thus prevents that some regions become uncoloured. Furthermore, since all the coefficients are sent for the sub-matrix of scale-1, the coefficients can be sent in a pre-defined order, and therefore, there is no need to send the position information of the coefficients. This reduces the size of data to be sent and thus gives a higher compression rate.

Then, for the residual image, i.e., the image obtained from the subtraction of the original color image and the initially reconstructed color image, the coefficients are extracted by the  $L_0$  minimization.

The whole system flow of the proposed algorithm is described in Figure 5. In the encoder, the original color image is divided into the luminance channel and the chrominance channels. The chrominance channels are sub-sampled according to the 4:2:0 format, since the resolution of chrominance channels are lower than that of luminance channel. The luminance channel is encoded with conventional methods such as JPEG or JPEG2000. The encoded bit stream is sent to the decoder, and the decompressed luminance channel is also sub-sampled to the size of the chrominance channel. The decompressed and sub-sampled luminance channel is segmented with the multi-meanshift algorithm. Then, the type 1 and type 2 colorization basis vectors are constructed for each segmented region. After that, the initial reconstruction of the chrominance channels is performed using the sub matrix of scale-1. Then, further colorization coefficients are extracted from the residual image.

The bit stream of the luminance channel and the colorization coefficients are sent to the decoder. The bit stream of the luminance channel is decoded and sub-sampled. Using the decompressed luminance channel, the colorization matrix is constructed in the same manner as in the encoder. The colorization process is performed by multiplying the colorization matrix and the colorization coefficients sent from the encoder. Then, the colorized chrominance channels are up-sampled to the size of the luminance channel. An inverse YUV conversion of the decompressed luminance channel and the reconstructed chrominance channels reconstructs the color image.

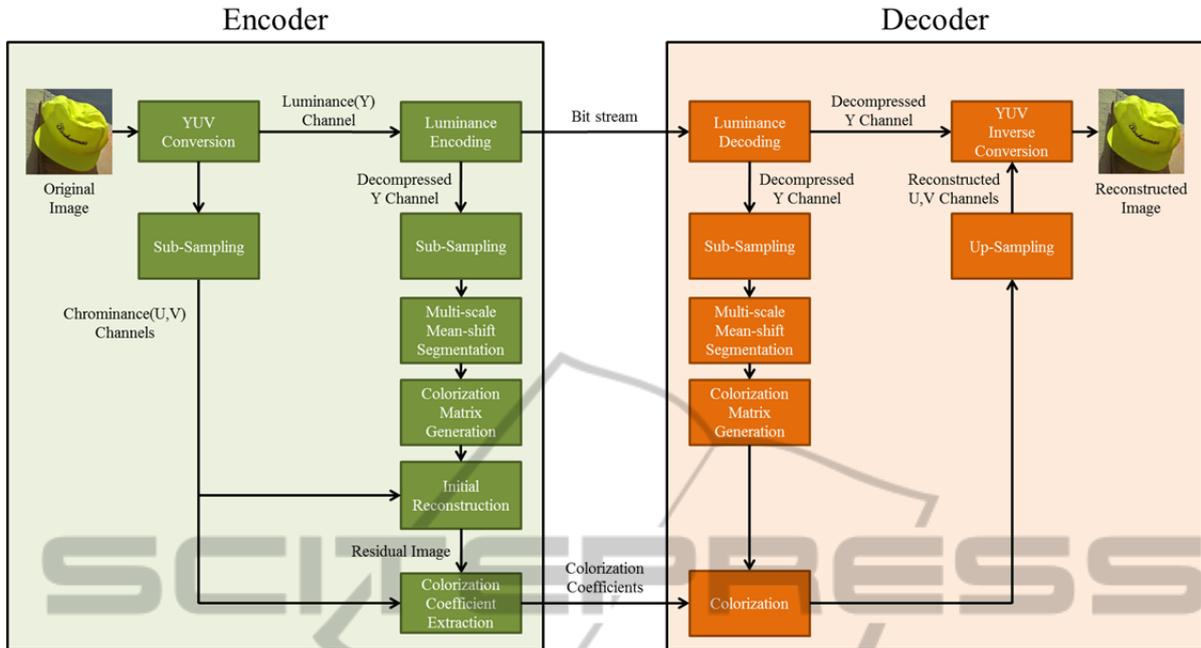


Figure 4: Overall system of proposed method.

## 4 EXPERIMENTAL RESULTS

We compared the results of the proposed algorithm with those of the JPEG and JPEG 2000 standards. The relative spatial distance weights to the photometric distance are set to 1, 2, 4, and 8. For better performance, extra wavelet basis vectors are cascaded to the colorization matrix to compensate for the artifact due to the uncorrelated chrominance components and the compression error in the luminance channel.

Three different sized color images are used in the experiment. The first one is called *House* (256 by 256 pixel size), the second one is called *Cap* (256 by 256 pixel size), and the last one is called *Butterfly* (294 by 250 pixel size).

We used a total of 200 colorization coefficients in the encoder. A number of 8, 20, and 14 colorization coefficients were extracted in advance for the initial color reconstruction for the *House*, *Cap*, and *Butterfly* image respectively. For pre-extracted coefficients from initial reconstruction, 16 bits are used for each pixel, i.e., 8 bits per chrominance value. For the other coefficients, additional 6 bits are used to index the corresponding colorization vectors. Thus, 0.570, 0.527, and 0.522, [kB] are required for encoding chrominance channels of *House*, *Cap*, and *Butterfly* respectively. The luminance channels are encoded by JPEG2000

in the experiment. The file sizes of luminance channel of *House*, *Cap*, and *Butterfly* are 6.05, 3.68, and 4.08 [kB] respectively.

Table 1: Comparison of file sizes and PSNR values.

Image	Method	Size(KB)	PSNR(dB)
House	JPEG	6.86	25.2405
	JPEG2000	6.64	26.7628
	Proposed	6.62	26.8898
Cap	JPEG	4.27	30.0350
	JPEG2000	4.20	31.1059
	Proposed	4.20	31.5197
Butterfly	JPEG	4.80	23.8253
	JPEG2000	4.63	25.7853
	Proposed	4.60	25.9530

For evaluating the performance of each method, the PSNR measure is used. Table 1 demonstrates that the objective quality of the reconstructed color image using the proposed method is superior to that using the JPEG or JPEG 2000 standards. Figure 5 to Fig. 10 show that the visual quality is better than those using JPEG or JPEG2000 standards.

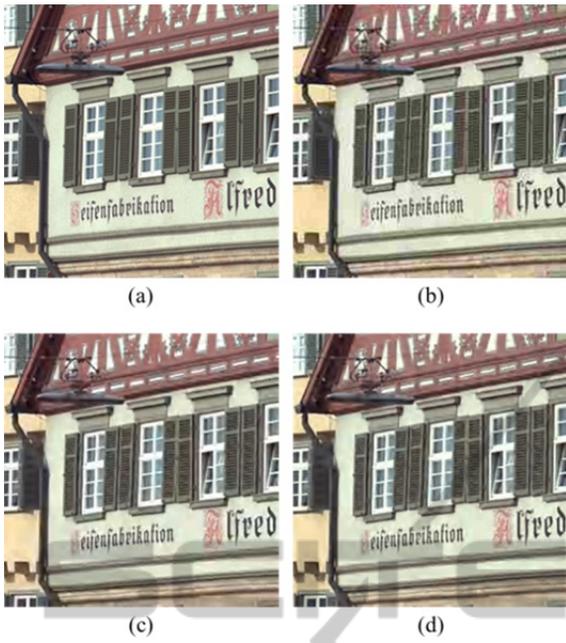


Figure 5: Experimental results with 256×256 “House”. (a) Original. (b) JPEG. (c) JPEG2000. (d) Proposed.

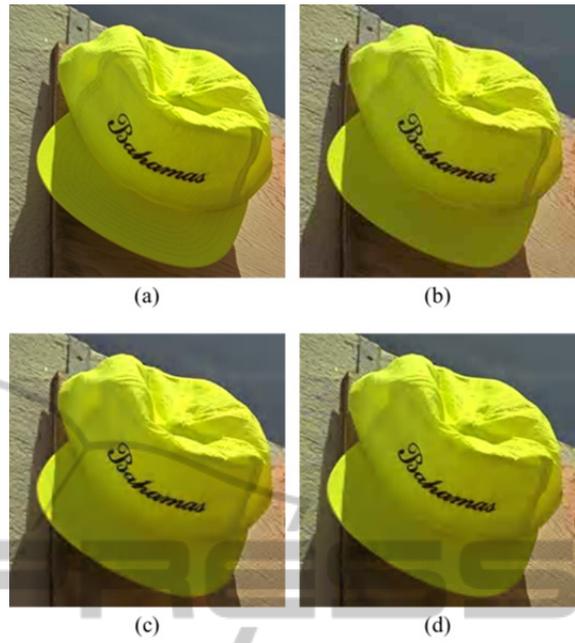


Figure 7: Experimental results with 256×256 “Cap”. (a) Original. (b) JPEG. (c) JPEG2000. (d) Proposed.



Figure 6: Zooming results of “House”. (a) Original. (b) JPEG. (c) JPEG2000. (d) Proposed.

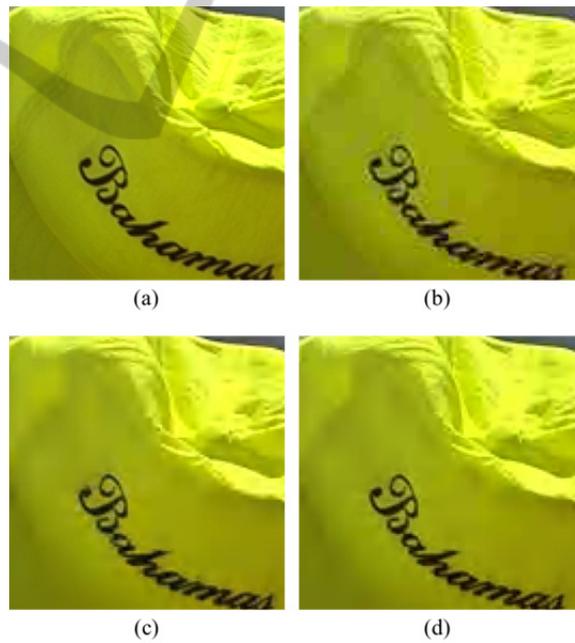


Figure 8: Zooming results of “Cap”. (a) Original. (b) JPEG. (c) JPEG2000. (d) Proposed.

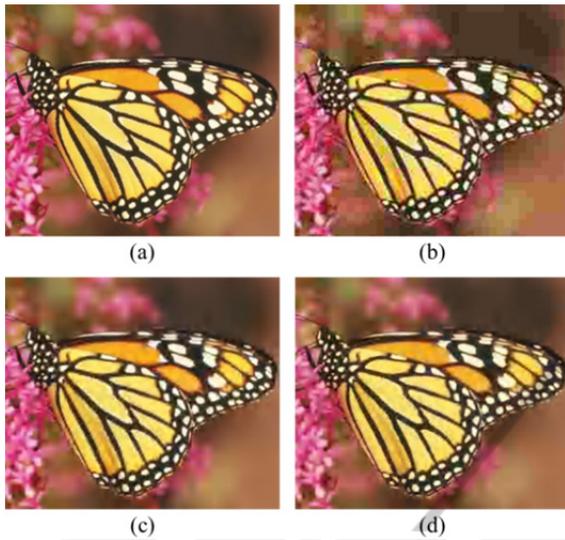


Figure 9: Experimental results with 250×294 “Butterfly”. (a) Original. (b) JPEG. (c) JPEG2000. (d) Proposed.

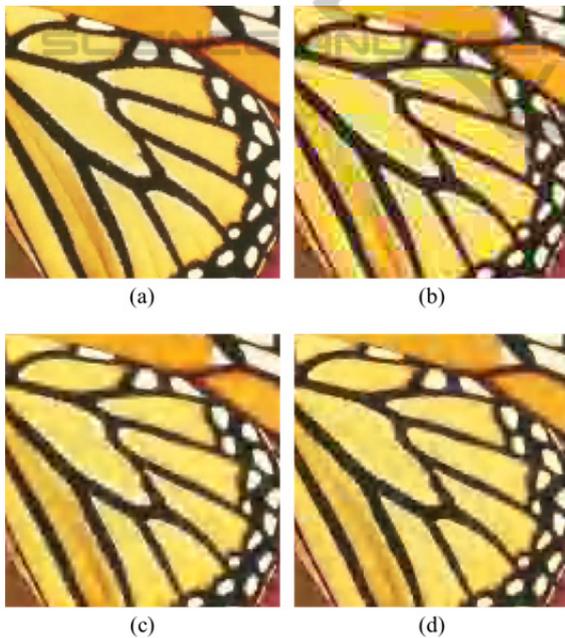


Figure 10: Zooming results of “Butterfly”. (a) Original. (b) JPEG. (c) JPEG2000. (d) Proposed.

## 5 CONCLUSIONS

In this paper, we proposed a colorization coding method based on the analysis of the local correlation of the luminance and chrominance components as a linear regression model. We designed two types of colorization vectors for each segmented region, and composed the colorization matrix using these

vectors. The proposed algorithm outperforms the JPEG and JPEG2000 standards in terms of the compression rate and the PSNR value. If a lossless coding such as the run-length coding is applied to the proposed algorithm, the compression rate can be enhanced.

For future work, extra colorization vectors which can depict the uncorrelated region between the luminance and the chrominance components should be investigated.

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