Modeling and Algorithm for Dynamic Multi-objective Weighted Constraint Satisfaction Problem

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Abstract: A Constraint Satisfaction Problem (CSP) is a fundamental problem that can formalize various applications related to Artificial Intelligence problems. A Weighted Constraint Satisfaction Problem (WCSP) is a CSP where constraints can be violated, and the aim of this problem is to find an assignment that minimizes the sum of weights of the violated constraints. Most researches have focused on developing algorithms for solving static mono-objective problems. However, many real world satisfaction/optimization problems involve multiple criteria that should be considered separately and satisfied/optimized simultaneously. Additionally, they are often dynamic, i.e., the problem changes at runtime. In this paper, we introduce a Multi-Objective WCSP (MO-WCSP) and develop a novel MO-WCSP algorithm called Multi-Objective Branch and Bound (MO-BnB), which is based on a new solution criterion called (l,s)-Pareto solution. Furthermore, we first formalize a Dynamic MO-WCSP (DMO-WCSP). As an initial step towards developing an algorithm for solving a DMO-WCSP, we focus on the change of weights of constraints and develop the first algorithm called Dynamic Multi-Objective Branch and Bound (DMO-BnB) for solving a DMO-WCSPs, which is based on MO-BnB. Finally, we provide the complexity of our algorithms and evaluate DMO-BnB with different problem settings.

1 INTRODUCTION

A *Constraint Satisfaction Problem* (CSP) (Mackworth, 1992) is a problem that finds a consistent assignment of values to variables. A surprisingly wide variety of Artificial Intelligence problems can be formalized as CSPs, e.g., resource allocation problem (Cabon et al., 1999), scheduling (Verfaillie et al., 1996), combinatorial auctions (Sandholm, 1999), and bioinformatics (Backofen and Gilbert, 2001). A *Weighted Constraint Satisfaction Problem* (WCSP) (Schiex et al., 1995; Larrosa and Schiex, 2004) is a CSP where constraints can be violated. The aim of this problem is to find an assignment that minimizes the sum of weights of the violated constraints.

Most researches on CSPs and WCSPs have focused on developing algorithms for solving static mono-objective problems. However, many real world satisfaction/optimization problems involve multiple criteria that should be considered separately and satisfied/optimized simultaneously. Additionally, they are often dynamic, i.e., the problem changes at runtime.

In this paper, we formalize a *Multi-Objective Weighted Constraint Satisfaction Problem* (MO-WCSP) which is a WCSP that involves multiple criteria. We also develop a novel MO-WCSP algorithm called Multi-Objective Branch and Bound (MO-BnB), which is based on a new solution criterion called (l,s)-Pareto solution. This algorithm utilizes branch and bound technique and depth-first search strategy, and finds a subset of Pareto front using adjustable parameters l and s. In MO-WCSPs, generally, since trade-offs exist among criteria, there does not exist an ideal assignment, which minimizes all criteria simultaneously. Thus, we characterize the solution of MO-WCSPs using the concept of *Pareto optimality*. Solving a MO-WCSP is to find the Pareto front. The Pareto Front is a set of (weighted) cost vectors obtained by Pareto optimal solutions. An assignment is a Pareto optimal solution, so there does not exist another assignment that improves all of the criteria. A MO-WCSP can be represented using a graph called constraint graph, in which a node represents a variable and an edge represents a constraint. In MO-

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WCSPs, even if a constraint graph has the simplest tree structure, the size of the Pareto front, i.e., the number of Pareto optimal solutions, is often exponential in the number of cost vectors. In such problems, finding all Pareto optimal solutions is not realistic.

Furthermore, we first introduce a *Dynamic Multi-Objective Weighted Constraint Satisfaction Problem* (DMO-WCSP) which is defined by a sequence of static MO-WCSPs. As an initial step towards developing an algorithm for DMO-WCSPs, we focus on the change of weights of constraints, i.e., we assume that only the weight of each constraint changes at runtime. However, the change is unpredictable, i.e., it is not known in advance how the weights will change in the next problem in a sequence. This assumption requires a *reactive approach*, i.e., we need to solve each MO-WCSP in a sequence one by one. We also develop the first algorithm called Dynamic Multi-Objective Branch and Bound (DMO-BnB) for DMO-WCSPs, which is based on MO-BnB. Finally, we provide the complexities of these two algorithms, respectively. In section evaluation, we evaluate the performance of DMO-BnB with different problem settings.

A *Multi-Objective Constraint Optimization Problem* (MO-COP) (Rollon and Larrosa, 2006; Perny and Spanjaard, 2008; Marinescu, 2010) is the extension of a mono-objective Constraint Optimization Problem (COP) (Dechter, 2003; Schiex et al., 1995). A COP is a problem where the goal is to find an assignment of values to variables so that the sum of the resulting costs is optimized. This problem is quite similar to a WCSP. An MO-COP is a COP that involves multiple criteria. For MO-COPs, various complete algorithms have been developed (Rollon and Larrosa, 2006; Rollon and Larrosa, 2007; Marinescu, 2009). In an MO-COP, the size of the Pareto front is exponential in the number of variables, i.e, all assignments are Pareto optimal solutions in the worst case. Since finding all Pareto optimal solutions becomes easily intractable, it is important to consider fast but incomplete algorithms for large-scale applications. Also, various incomplete algorithms have been developed (Rollon and Larrosa, 2006; Perny and Spanjaard, 2008; Marinescu, 2010). Compared to these MO-COP algorithms, our proposed algorithm DMO-BnB is for solving a Dynamic MO-WCSP.

Furthermore, there exists several works on Dynamic CSPs (DynCSP) (Dechter and Dechter, 1988; Faltings and Macho-Gonzalez, 2002). Compared to these previous works, there exists no work on considering multiple criteria in a dynamic environment, as far as the authors are aware. Also, compared to evolutionary algorithms (Deb et al., 2002; Bringmann et al., 2011) for Multi-Objective Optimization Problems (MOOPs), the advantage of our algorithm is that it can guarantee to find all Pareto optimal solutions and DMO-BnB is an algorithm for DMO-WCSPs.

About application domains of DMO-WCSPs, we believe that sensor networks would be a promising area. This problem is a kind of resource allocation problems which can be formalized as a COP and a WCSP. For example, consider a sensor network in a territory, where each sensor can sense a certain area in this territory. When we consider this problem with multiple criteria, e.g., data management, quality and quantity of observation data, and electrical consumption, it can be formalized as a MO-WCSP. Also, when we consider this problem in a dynamic environment, e.g., when we need to sense some objectives that invade this territory and move to different areas at run time, we can apply DMO-WCSP technique.

2 PRELIMINARIES

A Weighted Constraint Satisfaction Problem (WCSP) (Larrosa and Schiex, 2004) is defined by a tuple $< V, D, C_W >$, where V is a set of variables, D is a set of domains, C_W is a set of weighted constraints. A variable x_i takes its value from a finite, discrete domain D_i. A constraint relation (i, j) means there exists a constraint relation between x_i and x_j. Each constraint relation (i, j) has a weight w_{ij}, where $\sum_{(i,j)\in C_W} w_{ij} = 1$. For x_i and x_j, which have a constraint relation, the cost for an assignment $\{(x_i, d_i), (x_j, d_j)\}$ is defined by a weighted cost function

$$f_{w_{ij}}(d_i, d_j) = \begin{cases} 0 & (i,j)\ is\ satisfied. \\ w_{ij} & (i,j)\ is\ unsatisfied. \end{cases} \quad (1)$$

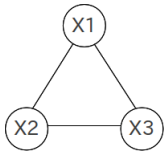
For a value assignment to all variables A, let us denote

$$R(A) = \sum_{(i,j)\in C_W, \{(x_i,d_i),(x_j,d_j)\}\subseteq A} f_{w_{ij}}(d_i, d_j), \quad (2)$$

where $d_i \in D_i$ and $d_j \in D_j$. Solving a WCSP is to find an assignment that minimizes the sum of the value of all weighted cost functions. When all (weighted) constraints are satisfied, the resulting weighted cost is zero, and in case all constraints are unsatisfied, the total cost is one. A WCSP can be represented using a graph called constraint graph, in which a node represents a variable and an edge represents a constraint.

In thi paper, we assume that all constraints are binary for simplicity. However, relaxing this assumption to general cases is relatively straight forward.

Example 1 (WCSP)**.** Figure 1 shows a graph coloring problem with three variables x_1, x_2 and x_3. Each variable has to choose a different color with



w_{ij}	weight
w_{12}	0.1
w_{13}	0.3
w_{23}	0.6

Figure 1: Example of WCSP.

its neighbors and takes its value from a discrete domain $\{black, white\}$. The right table represents the weights of all constraints, e.g., when all variables choose *black*, all constraints are not satisfied, and the total weighted cost is $w_{12} + w_{13} + w_{23} = 1$. The solutions of this problem, i.e, an assignments which minimizes the sum of the weighted costs, are $\{(x_1, black), (x_2, black), (x_3, white)\}$ and $\{(x_1, white), (x_2, white), (x_3, black)\}$, and the resulting weighted cost is 0.1.

3 MULTI-OBJECTIVE WCSP

In this section, we formalize a Multi-Objective Weighted Constraint Satisfaction Problem (MO-WCSP). Furthermore, we introduce a new solution criterion called (l,s)-Pareto solution and develop an algorithm for obtaining all (l,s)-Pareto solutions.

3.1 Model

A *Multi-Objective Weighted Constraint Satisfaction Problem* (MO-WCSP) is defined by a tuple $< V, D, \mathbb{C}_W >$, where V is a set of variables, D is a set of domains, $\mathbb{C}_W = \{C^1_{W^1}, C^2_{W^2}, ..., C^m_{W^m}\}$ is a set of multi-objective weighted constraints, where m is the number of objectives. For an objective h $(1 \leq h \leq m)$ and a value assignment to all variables A, let us denote

$$R^h(A) = \sum_{(i,j) \in C^h_{W^h}, \{(x_i,d_i),(x_j,d_j)\} \subseteq A} f^h_{w^h_{ij}}(d_i, d_j), \quad (3)$$

where $d_i \in D_i$ and $d_j \in D_j$. Then, the sum of the values of all weighted cost functions for m objectives is defined by a cost vector, denoted $R(A) = (R^1(A), \ldots, R^m(A))$. When all constraints are satisfied, the resulting cost vector is $(0,0,...,0)$, and in case all constraints are unsatisfied, the resulting cost vector is $(1,1,...,1)$. Finding an assignment that minimizes all objective simultaneously is ideal. However, in general, since trade-offs exist among objectives, we cannot obtain such an ideal assignment. Thus, the (optimal) solution of a MO-WCSP is characterized by using the concept of *Pareto optimality*. A MO-WCSP can be also represented using a constraint graph.

Definition 1 (Dominance)**.** For a MO-WCSP and two cost vectors $R(A)$ and $R(A')$ obtained by assignments

w^1_{ij}	weight
w_{12}	0.1
w_{13}	0.3
w_{23}	0.6

w^2_{ij}	weight
w_{12}	0.6
w_{13}	0.3
w_{23}	0.1

(x_1,x_2,x_3)	cost vector
(b,b,b)	(1,0)
(b,b,w)	(0.1,0,4)
(b,w,b)	(0.3,0.7)
(w,b,b)	(0.6,0.9)
(w,w,b)	(0.1,0.4)
(w,b,w)	(0.3,0.7)
(b,w,w)	(0.6,0.9)
(w,w,w)	(1,0)

Figure 2: Example of bi-objective-WCSP.

A and A', we say that $R(A)$ *dominates* $R(A')$, denoted by $R(A) \prec R(A')$, iff $R(A)$ is partially less than $R(A')$, i.e., (i) it holds $R^h(A) \leq R^h(A')$ for all objectives h, and (ii) there exists at least one objective h', such that $R^{h'}(A) < R^{h'}(A')$.

Definition 2 (Pareto Optimal Solution)**.** For a MO-WCSP and an assignment A, we say A is the *Pareto optimal solution*, iff there does not exist another assignment A', such that $R(A') \prec R(A)$.

Definition 3 (Pareto Front)**.** For a MO-WCSP, a set of cost vectors obtained by Pareto optimal solutions is called a *Pareto front*. Solving a MO-WCSP is to find the Pareto front.

Example 2 (MO-WCSP)**.** We use the same problem as in Example 1. Figure 2 shows the tables for weights and cost vectors for a bi-objective WCSP, where w^1_{ij} represents a weight for objective one and w^2_{ij} for objective two. For objective one, constraints are same as in Example 1, i.e., each variable must choose a different color with its neighbors. For the constraints of objective two, we require that each variable has to choose the same color with its neighbors. The right table shows the weighted cost table obtained by all combination of assignments. When all variables choose *black*, the total weighted cost for objective one is 1, i.e., all constraints are unsatisfied, and 0 for objective two, i.e., all constraints are satisfied. The Pareto optimal solutions of this problem are $\{\{(x_1,b), (x_2,b), (x_3,b)\}, \{(x_1,b), (x_2,b), (x_3,w)\}, \{(x_1,w), (x_2,w), (x_3,b)\}, \{(x_1,w), (x_2,w), (x_3,w)\}\}$ and the obtained Pareto front is $\{(1,0),(0.1,0.4)\}$.

For a MO-WCSP with m objectives, a vector $R(A) = (r^1,...,r^m)$ obtained by an assignment A, a constant vector $l = (l^1,...l^m)$, and a non-negative integer s, where r^h and l^h are costs for objective h $(1 \leq h \leq m)$, we define the following novel solution criteria, l-weak, s-sum, and (l,s)-Pareto solutions.

Definition 4 (l-weak Pareto Solution)**.** For a MO-WCSP, we call that A is an *l-weak Pareto solution*, iff A is a Pareto optimal solution and it holds $r^h \leq l^h$ for all objectives h $(1 \leq h \leq m)$.

Definition 5 (s-sum Pareto Solution)**.** For a MO-WCSP, we call that A is an *s-sum Pareto solution*, iff A is a Pareto solution and it holds $r^1 + ... + r^m \leq s$.

Definition 6 ((l,s)-Pareto Solution)**.** For a MO-WCSP, we call that A is a *(l,s)-Pareto solution*, iff A is an l-weak and an s-sum Pareto solution.

Intuitively, we try to avoid violated constraints with hight weighted costs adjusting a constant vector l and want to restrict the sum of weighted costs of all objectives with a parameter s. In our Example 2, when we set l to $(0.6, 0.6)$, and s to 0.7, we can ignore the Pareto optimal solutions $\{\{(x_1,b), (x_2,b), (x_3,b)\}, \{(x_1,w), (x_2,w), (x_3,w)\}\}$. The (l,s)-Pareto solutions of this problem are $\{\{(x_1,b), (x_2,b), (x_3,w)\}, \{(x_1,w), (x_2,w), (x_3,b)\}\}$. For the cost vector $(0.1, 0.4)$ obtained by these two assignments, they are l-Pareto solutions, since the weighted costs, i.e., 0.1 for objective one and 0.4 for objective two, are less than 0.6 ($= l^1 = l^2$). They are also s-sum Pareto solutions, since the total weighted costs of two objectives, i.e., $0.1 + 0.4 = 0.5$, are less than 0.7 ($= s$). Thus, they are the (l,s)-Pareto solutions. When we set $l = (1,1)$ and $s = 2$, i.e., no restriction, the set of (l,s)-Pareto solutions is the same as the set of all Pareto optimal solutions.

3.2 Algorithm

We introduce a novel algorithm called Multi-Objective Branch and Bound (MO-BnB) which can find all (l,s)-Pareto solutions for MO-WCSPs. This algorithm utilizes a branch and bound technique and depth-first search strategy that are well-known techniques for solving constraint optimization problems.

Algorithm 1 and 2 show the pseudo-code of MO-BnB. We assume that a variable ordering is given. The input is a MO-WCSP, a constant vector l, and a non-negative integer s, and the output is the entire set of (l,s)-pareto solutions (line 1 and 2 in Algorithm 1). MO-BnB starts with an empty pareto front and a null cost vector and solves the first variable according to the variable ordering (line 7-10 in Algorithm 1). It chooses a value for the variable and updates the cost vector according to the cost tables (step 1 in Algorithm 2). At this moment the obtained cost vector c has to ensure three properties:

- c is not dominated by the constant vector l
- the sum of the elements of c is not larger than s
- c is not dominated by the current Pareto front PS

If one of the three properties is violated, MO-BnB branches on the next value of the variable. When its domain is completely explored, the search branches to the previous variable and continues the solving (step 2 in Algorithm 2). When a complete assignment is formed, i.e. no variable left to be assigned, a new solution is added to the Pareto front PS. All previous dominated solutions are removed from the Pareto front and the search continues with the next values of the variable (step 3.1 in Algorithm 2). If c fulfills the three properties, it continues the solving with the next variable according to the ordering (step 3.2 in Algorithm 2). The search stops when the whole search space has been covered by the Branch and Bound search. MO-BnB finally outputs the entire set of (l,s)-Pareto solutions of the MO-WCSP.

Algorithm 1: MO-BnB.

```
1: INPUT : a MO-WCSP P, a constant vector l and a non-negative integer
   s.
2: OUTPUT : the set of all (l,s)-pareto solutions PS
3: Root: the root of P
4: AS: an assignment of variables
5: Cost: the cost vector of AS
6: PS: a set of pairs <cost vector, set of assignments>
7: AS ← ∅
8: PS ← ∅
9: Cost ← null vector
   // Launch solving from the root
10: PS ← first.solve(AS,Cost,PS,l,s)
11: return PS
```

Algorithm 2: Solve(AS,Cost,PS).

```
1: INPUT : <AS,Cost,PS,l,s>
2: OUTPUT : PS the set of all (l,s)-pareto solutions
3: for each value v1 of the variable domain do
4:    AS ← v1
5:    local_cost ← null vector
      // step 1: Compute local cost of the choice
6:    for each constraint with an ancestor a do
7:       v2 ← value of a in AS
8:       local_cost ← local_cost + cost(v1,v2)
         // cost(v1,v2) is a vector given by the constraint
9:    end for
10:   new_cost ← Cost + local_cost
      // step 2: Bound checking
11:   if r^i > l^i, r^i ∈ new_cost, l^i ∈ l then
12:      AS ← (AS \ v1)
13:      continue
14:   end if
15:   if Σ_{r^i ∈ new_cost} r^i > s then
16:      AS ← (AS \ v1)
17:      continue
18:   end if
19:   if new_cost is dominated by PS then
20:      AS ← (AS \ v1)
21:      continue
22:   end if
      // step 3.1: New pareto solution
23:   if AS is complete then
24:      E ← all elements of PS dominated by new_cost
25:      PS ← PS \ E
26:      PS ← PS ∪ {(new_cost,AS)}
27:      continue
28:   end if
      // step 3.2: Continue solving
29:   PS ← next.solve(AS,Cost,PS)
30:   AS ← (AS \ v1)
31: end for
32: return PS
```

Theorem 1. For a MO-WCSP, the memory required by MO-BnB is $O(md^n)$ and the required computa-

tion time is $O(nmd^n + md^{2n})$. (which belongs to $O(md^{2n})$), where m is the number of objectives, d is the maximal domain size of variables, and n is the number of variables.

Proof. Let P be a MO-WCSP and PF be the pareto front of P. In the worst case, all possible combinations of assignments of variables are Pareto optimal solutions, i.e., there exists d^n Pareto optimal solutions. Thus, with each assignment being associated with a weighted cost vector of size m, the number of Pareto optimal solutions is bounded by $O(d^n)$. Since MO-BnB only requires to store the Pareto front during solving, its memory use is bound by $O(md^n)$.

Let ST be the number of partial assignment of P and k be a non-negative integer $(1 \leq k \leq n)$. Consider that only one variable can change its value at a given time. When no pruning occurs, and since there are d^k possible assignments for the first k variables in a variable ordering, the number of partial assignments is given by $\sum_{k=1}^{n} |D|^k = \frac{|D|^{n+1}-1}{|D|-1} - 1$. Each time a new value is assigned to a variable, MO-BnB :

- updates the current cost vector c, this operation is linear in the size of the vector, i.e., $(n-1) \times m$,
- checks if c is dominated by a constant vector l, this operation is linear in the size of the vector: m,
- checks if the sum of the element of c is superior to the parameter s, this operation is linear in the size of the vector: m.
- checks if c is dominated by the current pareto front, this operation is linear in the size of the pareto front: $|PF|$.

If no pruning occurs, ST represents the maximal occurrence of the four above operations. Solving a MO-WCSP by MO-BnB is then bound by $O(|ST| \times ((n-1)m+m+m+|PF|)) = O((\frac{d^{n+1}-1}{d-1} - 1) + ((n+1)m + md^n))$, which belongs to $O(nmd^n + md^{2n})$. Thus, the runtime complexity belongs to $O(md^{2n})$. □

4 DYNAMIC MO-WCSP

In this section, we formalize a Dynamic Multi-Objective Weighted Constraint Satisfaction Problem (DMO-WCSP) which is the extension of a MO-WCSP. Furthermore, we develop the first complete DMO-WCSP algorithm called Dynamic Multi-Objective Branch and Bound (DMO-BnB) which is based on MO-BnB. We also provide its complexity.

4.1 Model

A *Dynamic Multi-Objective Weighted Constraint Satisfaction Problem*, we denote DMO-WCSP, is defined by a sequence of static MO-WCSPs

$$< MO\text{-}WCSP_0,\ MO\text{-}WCSP_1, ...,\ MO\text{-}WCSP_k > . \quad (4)$$

Solving a DMO-WCSP is to find a sequence of Pareto fronts

$$< PF_0,\ PF_1, ..., PF_k >, \quad (5)$$

where PF_i $(0 \leq i \leq k)$ is the Pareto front of MO-WCSP$_i$. In our model, it allows to change the number of variables, constraints, objectives, domain size, and weights of constraints. In this paper, as an initial step towards developing an algorithm for solving a DMO-WCSP, we focus on the change of the weights of constraints. Since the change is unpredictable, i.e., it is not known in advance how the weights will change in the next problem in a sequence, it requires a *reactive* approach. For dynamic problems, there exist two approaches, i.e., proactive and reactive. For a proactive approach, all MO-WCSPs in a sequence are known in advance. Since we know the changes among MO-WCSPs in a sequence, one possible goal of this approach is to find a common solution in a sequence. On the other hand, for a reactive approach, since the next problem MO-WCSP$_{i+1}$ becomes known only after current problem MO-WCSP$_i$ was solved, it requires to solve each MO-WCSP in a sequence one by one. The goal is to find a sequence of Pareto fronts.

4.2 Algorithm

We develop the first DMO-WCSP algorithm called Dynamic Multi-Objective Branch and Bound (DMO-BnB) which is based on MO-BnB. In DMO-BnB, we reuse the (l,s)-Pareto solutions of the previous problem, and compute the weighted cost vector which is utilized as an upper bound for the current problem. More specifically, we modified our algorithm MO-BnB as follows. The input contains PS_{i-1} which is the Pareto front of the previous problem. DMO-BnB initializes the Pareto front of the new problem with PS_{i-1} by replacing line 8 in Algorithm 1 by: $PS \leftarrow compute(P, PS_{i-1})$. For each assignment of PS_{i-1}, it computes the new cost vector. The rest of its pseudo-code is almost the same as that of MO-BnB.

Theorem 2. Let k be the size of the sequence of a DMO-WCSP. The memory required by DMO-BnB is given by $O(md^n)$ and the required computation time is given by $O(kmd^{2n})$, where m is the number of objectives, d is the maximal domain size of variables, n is the number of variables.

Proof. Solving a DMO-WCSP has the same memory complexity as solving one MO-WCSP with MO-BnB. To solve the initial MO-WCSP of a sequence, DMO-BnB needs to store its Pareto front in the memory, where the complexity belongs to $O(md^n)$. For the next MO-WCSP, DMO-BnB reuses the Pareto front of the previous MO-WCSP. It updates the previous Pareto front by computing the new cost vectors of each (l,s)-Pareto solution and checks the dominance. The memory use of this operation is bounded by the size of the Pareto front, i.e., $O(md^n)$. Since DMO-BnB solves the problems one by one, the memory use of this algorithm is bounded by $O(md^n)$.

For the computation time, it solves the initial MO-WCSP of a sequence from scratch. Its complexity belongs to $O(md^{2n})$. To solve the next MO-WCSP, DMO-BnB starts by revising the previous Pareto front. For each assignment, it computes a new cost vector. For each variable, it checks at most $n-1$ constraints. Since there are n variables in an assignment, the complexity of this operation belongs to $O(mn^2)$. Since there is d^n possible assignments, this operation belongs to $O(d^n \times mn^2)$. We also check the dominance among l, s, and previous $|PF|$, i.e., $m+m+|PF|$. The complexity of the whole preprocessing is $O(d^n \times (mn^2+m+m+|PF|)) = O(d^n \times (mn^2+2m+md^n)) = O(mn^2d^n+2md^n+md^{2n})$ which belongs to $O(md^{2n})$ This preprocessing is done before solving each MO-WCSP from a sequence, except the initial problem. The complexity of solving a DMO-WCSP with k MO-WCSPs belongs to $O((k-1)(md^{2n})+k(md^{2n}) = O(2kmd^{2n}-md^{2n})$ that belongs to $O(kmd^{2n})$. □

5 EVALUATION

In this section, we evaluate the performance of DMO-BnB. In our experiments, we use the following problem instances. We generate DMO-WCSPs, where the sequence contains ten MO-WCSPs. All the tests are made with density one, i.e., complete graph which is the most difficult problem. For each objective of a MO-WCSP in a sequence, we generate the same complete graph and choose the weight of a constraint uniformly at random so that the sum of the weights of all constraints is one. Each data point in a graph represents the average of 50 problem instances. We implemented DMO-BnB in C++ and carried out all experiments on one core running at 2.6GHz with 12GB of RAM. For a constant vector l, we mostly use setting $l = (l^1,...,l^m)$ with $l^i = l^j$, $1 \leq i,j \leq m$ and set ten hours time limit for the experimentation.

Figure 3 represents the run time of DMO-BnB

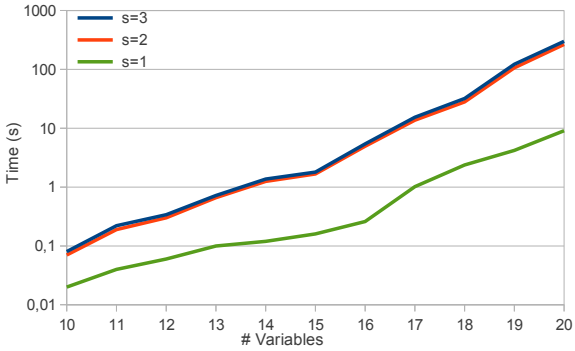


Figure 3: Results of run time for $s = 1$, 2, and 3.

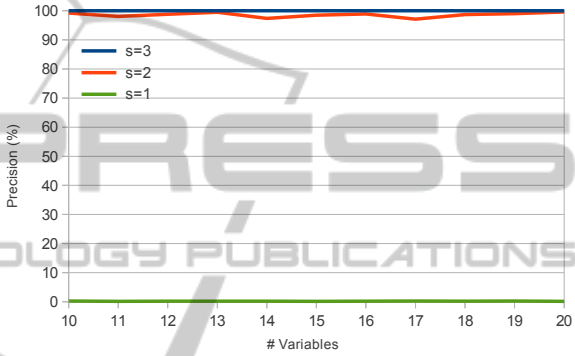


Figure 4: Results of ratio for $s = 1$, 2, and 3.

with different s, varying the number of variables, domain size is two, the number of objectives is three, and each value of l is one, i.e., no restriction with l. The x axis represents the number of variables and y axis shows the run time. Note that three is the maximal value for s, since the number of objectives is three. We can see that the results for $s = 2$ and $s = 3$ are almost same, but the run time improves when we set s is equal to one. This is because when we set small s, the solution space becomes smaller, i.e., the number of (l,s)-Pareto solutions that DMO-BnB finds, in general, becomes smaller [1]. Figure 4 shows the ratio of the number of (l,s)-Pareto solutions obtained by DMO-BnB in all Pareto optimal solutions. When we set that s is three, i.e., no restriction, DMO-BnB finds all Pareto optimal solutions which means 100%. In case s is two, the (l,s)-Pareto solutions are more than 95% of all Pareto optimal solutions. However, when we decrease s to 1, i.e., strong restriction, the obtained (l,s)-Pareto solutions are less than 1% of all. This is because, in our experiments, most Pareto

[1]There is no guarantee that the run time always becomes smaller, when we set small s. It depends on the Pareto surface which we don't know in advance. In a special case, it may happen that Pareto solutions exist only in a specific solution area so that the run times for all s are almost same.

Table 1: Results of run time and ratio of obtained (l,s)-Pareto solutions. Naive represents the results where MO-BnB is iteratively applied without any re-used of previous results.

# Var	Run time/Precision (and Run time improvement)				
	Naive	s=4 (l=1.0)	s=3 (l=1.0)	s=2 (l=1.0)	s=1 (l=1.0)
10	2.28s	2.11s/100% (7.45%)	2.1s/100% (0.47%)	2.03s/99.21% (3.79%)	0.13s/0.09% (93.83%)
11	7.15s	6.54s/100% (8.53%)	6.51s/100% (0.45%)	6.31s/98.03% (3.51%)	0.46s/0.12% (92.96%)
12	14.97s	14.01s/100% (6.41%)	13.95s/100% (0.42%)	13.45s/98.78% (3.99%)	1.05s/0.07% (92.5%)
13	60.72s	56.18s/100% (7.47%)	55.95s/100% (0.46%)	54.04s/99.46% (3.81%)	2.94s/0.06% (94.76%)
14	207.14s	192.68s/100% (6.54%)	192.51s/100% (0.55%)	186.67s/97.36% (3.57%)	5.88s/0.08% (96.96%)
15	826.72s	762.39s/100% (7.78%)	758.35s/100% (0.52%)	737.54s/98.488% (3.26%)	22.36s/0.03% (97.06%)

optimal solutions exist in the solution space [2], where $o^1 \leq 1$, $o^2 \leq 1$, $o^3 \leq 1$, and $o^1 + o^2 + o^3 > 1$. We can see that there is a trade-off between the run time and the ratio of obtained (l,s)-Pareto solutions, i.e., when we set a strong restriction like $s = 1$, we can find (l,s)-Pareto solutions quickly, but the ratio of obtained (l,s)-Pareto solutions becomes low.

Figure 5 represents the run time of DMO-BnB with different l, varying the number of variables, domain size is two, the number of objectives is three, and the parameter s is three, i.e., no restriction with s. Note that one is the maximal value for each element of a constant vector l. We can see that all results are almost same [3], i.e., adjusting l does not help to decrease the run time. However, the ratio of obtained (l,s)-Pareto solutions increases significantly compared to that of in Figure 4. Figure 6 shows the ratio of obtained (l,s)-Pareto solutions in all Pareto optimal solutions. We can see that the ratio of obtained (l,s)-Pareto solutions is more than 75% for all tests. Specifically, when the number of variables is 20, DMO-BnB can find more than 90% of all Pareto optimal solutions. We consider that adjusting l is suitable for obtaining more (l,s)-Pareto solutions, but not for the run time. We will examine in greater detail why the ratio becomes higher, when the number of variables increases.

Table 1 represents the results of run time and ratio of (l,s)-Pareto solutions obtained by DMO-BnB, where the domain size increases from two to three, and from three to four for the number of objectives. Because of the page limitation, we only show the results for different values of s. We obtained the similar results as in Figure 3 and 4. The run time of DMO-BnB for $s = 2$, 3, and 4 are almost same, but it can find (l,s)-Pareto solutions very quickly in case $s = 1$. When the number of variables is 15, the run times for $s = 2,3$, and 4 are more than 730s, but in case $s = 1$, it

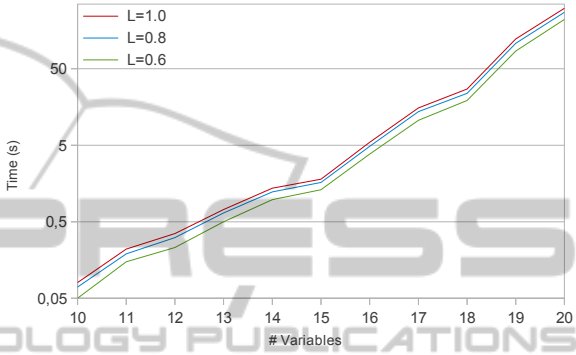


Figure 5: Results of run time for $l = 0.6$, 0.8, and 1.0.

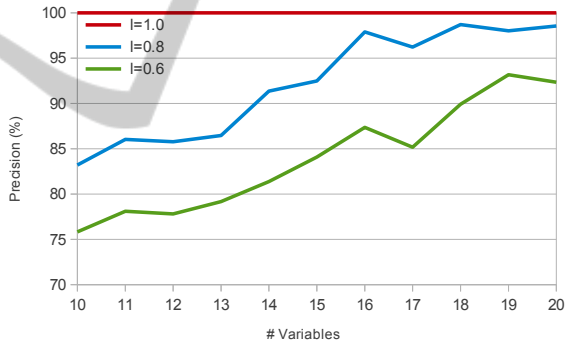


Figure 6: Results of ratio for $l = 0.6$, 0.8, and 1.0.

is 22.3s, i.e., 97% improvement. The ratio of obtained (l,s)-Pareto solutions is more than 97% for $s = 2,3$, and 4, while it is less than 0.03% for $s = 1$. For a constant vector l, we also obtained the similar results as in Figure 5 and 6.

In summary, these experimental results reveal that adjusting parameter s is suitable when we want to find (l,s)-Pareto solutions quickly. On the other hand, in case we want to have a number of (l,s)-Pareto solutions, adjusting a constant vector l is a good strategy.

As we shown in Figure 4, we obtained the extreme results for the ratio of (l,s)-Pareto solutions, i.e., almost everything for $s = 2$ and 3, or almost nothing for $s = 1$. One might imagine that is it possible to obtain the results between them. We additionally examine with a real number of s using the same problem instances of Figure 3 and 4. We show the run time and

[2] Even when we chose the weight of each constraint uniformly at random, we observed these results. We will examine it for different graph structures, e.g., scale-free, small world, and tree instead of complete graph.

[3] In case l is less than 0.5, we could not obtain any (l,s)-Pareto solutions in these problem instances.

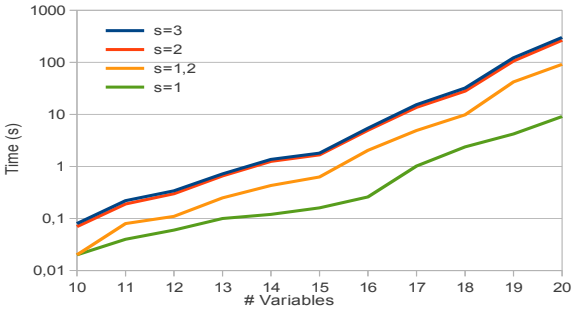


Figure 7: Results of run time for $s = 1$, 1.2, 2, and 3.

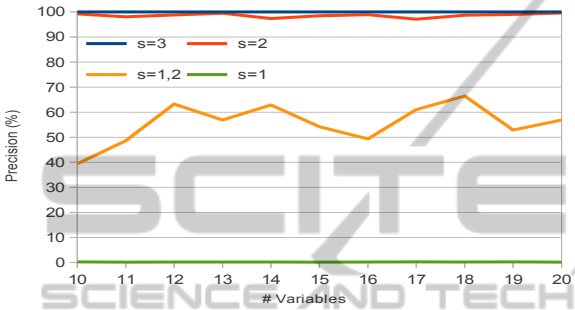


Figure 8: Results of ratio for $s = 1$, 1.2, 2, and 3.

the ratio of (l,s)-Pareto solutions for $s = 1.2$ in Figure 7 and 8. We can see that it is possible to obtain the results between extreme results for $s = 1$ and 2, e.g., we can obtain the ratio from 40% to 65% for $s = 1.2$.

6 CONCLUSIONS

In this paper, we introduced a MO-WCSP and developed a novel algorithm MO-BnB, which is based on a new solution criterion called (l,s)-Pareto solution. Furthermore, we first introduced a DMO-WCSP which is defined by a sequence of static MO-WCSPs. As an initial step towards developing an algorithm for solving a DMO-WCSP, we focused on the change of weights of constraints and developed the first algorithm DMO-BnB which is based on MO-BnB. We also provided the complexity of our algorithms. In the experiments, we evaluated our algorithm DMO-BnB with different problem settings. Our experimental results showed that adjusting parameter s is suitable when we want to find (l,s)-Pareto solutions quickly. In case we want to have a number of (l,s)-Pareto solutions, adjusting a constant vector l is good strategy.

For future work, we want to abandon the assumption of this paper that considers only changes of weights of constraints, and develop a novel algorithm for DMO-WCSPs. We also consider to develop an incomplete algorithm for DMO-WCSPs. Since finding all Pareto optimal solutions becomes easily intractable, it is important to consider to develop a fast but incomplete algorithm for large-scale applications. Finally, we will develop algorithms based on scalarization methods (Ehrgott, 2005; Marler and Arora, 2004; Miettinen, 1999). Also, we intend to apply our algorithm on challenging real world problems, e.g., sensor network and scheduling problems.

REFERENCES

Backofen, R. and Gilbert, D. (2001). Bioinformatics and constraints. *Constraints*, 6(2/3):141–156.

Bringmann, K., Friedrich, T., Neumann, F., and Wagner, M. (2011). Approximation-guided evolutionary multi-objective optimization. In *IJCAI*, pages 1198–1203.

Cabon, B., de Givry, S., Lobjois, L., Schiex, T., and Warners, J. P. (1999). Radio link frequency assignment. *Constraints*, 4(1):79–89.

Deb, K., Agrawal, S., Pratap, A., and Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Trans. Evolutionary Computation*, 6(2):182–197.

Dechter, R. (2003). *Constraint Processing*. Morgan Kaufmann Publishers.

Dechter, R. and Dechter, A. (1988). Belief maintenance in dynamic constraint networks. In *AAAI*, pages 37–42.

Ehrgott, M. (2005). *Multicriteria Optimization*. Springer.

Faltings, B. and Macho-Gonzalez, S. (2002). Open constraint satisfaction. In *CP*, pages 356–370.

Larrosa, J. and Schiex, T. (2004). Solving weighted csp by maintaining arc consistency. *Artificial Intelligence*, 159(1-2):1–26.

Mackworth, A. (1992). Constraint Satisfaction. In *Encyclopedia of Artificial Intelligence*, pages 285–293.

Marinescu, R. (2009). Exploiting problem decomposition in multi-objective constraint optimization. In *CP*, pages 592–607.

Marinescu, R. (2010). Best-first vs. depth-first and/or search for multi-objective constraint optimization. In *ICTAI*, pages 439–446.

Marler, R. and Arora, J. (2004). Survey of multi-objective optimization methods for engineering. *Structural and Multidisciplinary Optimization*, 26(6):369–395.

Miettinen, K. (1999). *Nonlinear Multiobjective Optimization*. Kluwer Academic Publishers, Boston.

Perny, P. and Spanjaard, O. (2008). Near admissible algorithms for multiobjective search. In *ECAI*, pages 490–494.

Rollon, E. and Larrosa, J. (2006). Bucket elimination for multiobjective optimization problems. *Journal of Heuristics*, 12(4-5):307–328.

Rollon, E. and Larrosa, J. (2007). Multi-objective Russian doll search. In *AAAI*, pages 249–254.

Sandholm, T. (1999). An algorithm for optimal winner determination in combinatorial auctions. In *IJCAI*, pages 542–547.

Schiex, T., Fargier, H., and Verfaillie, G. (1995). Valued constraint satisfaction problems: Hard and easy problems. In *IJCAI*, pages 631–639.

Verfaillie, G., Lemaître, M., and Schiex, T. (1996). Russian doll search for solving constraint optimization problems. In *AAAI/IAAI, Vol. 1*, pages 181–187.