Development of 3D Simulation System for Multi-Axis Turn-Mill Machining

Hong-Tzong Yau, Jhih-Long Chen, Bing-Rau Yu and Tsan-Jui Yang
Advanced Institute of Manufacturing with High-Tech Innovations, National Chung Cheng University, Chia-Yi, Taiwan

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Abstract: This paper proposes a 3D simulation system for multi-axis turn-mill machining. With the diversification of technology products, traditional three-axis machining has been unable to meet the needs of existing industries. Multi-axis machining system with high processing efficiency and accuracy has gradually become the mainstream, and is widely used to produce high-value-added products, such as aerospace components and medical devices. Since multi-axis machining hardware is very expensive, how to ensure the generation of correct NC paths and avoid machine collisions from happening become important challenges. In this paper, the triangular mesh model is used to represent the cutter, machine parts and materials. The actual movement of the machine can be simulated after G codes are parsed. For the material removal, taking processing speed into consideration, this paper proposes a hybrid simulation mechanism including 2D intersection, Z-map and Boolean operations. This proposed method can obtain better result in terms of processing efficiency and machining precision. At the same time, collision detection is used to find interference between tools and machines. Using experiments, this paper demonstrates a variety of turn-mill machining examples to verify the feasibility of the proposed method.

1 INTRODUCTION

Recently, computer numerical control (CNC) multi-axis milling machine has become a main approach for maching complex objects. The advantages of multi-axis milling include efficiency, accuracy and cost-effectiveness. The multi-axis milling machine is more suitable for milling complex free-form surfaces than the traditional 3-axis milling machine. Moreover, it can provide better surface quality and less surface roughness. However, after milling, most mechanical components still require turning, grinding, drilling and tapping. Therefore, part moving, mounting and tool calibration need to be repeated in the subsequent procedures.

In order to overcome this bottleneck, a CNC turn-mill machine provides a more complete solution. The advantages of the CNC turn-mill machine include shortened machining time, increased production, and decreased cost.

With the improvement of CAD/CAM technology, intellegent or smart machine is growing vigorously. The intelligent monitoring and interactive 3D machining simulation can increase the added values of NC machines. However, a voxel-based rendering is used in most simulation software. This kind of rendering is inefficient because it needs much memory space and computing power to provide accurate visualization. In this paper, a polygon-based representation model with Z-map and Boolean operations is proposed to simulate the practical machining process in the turn-mill machine.

The aim of this paper is to propose a 3D simulation system for multi-axis turn-mill machining. After loading the machining parameters and NC paths, the simulated workpiece, tooling turret and cutter can be generated automatically. With 3D collision detection, the simulation can be carried out more intuitively in different kinds of machining situations.

2 LITERATURE SURVEY

2.1 NC Simulation

Real-time simulation for virtual machining process is an important key, and pre-processing and good hardware are necessary to enhance the performance

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efficiency (Kerning, 2010). Generally, NC simulation can mainly be categorized into three approaches. The first approach uses direct Boolean intersection of solid models to calculate the volume of material removal during machining (Huang, 1995). This approach is theoretically capable of providing accurate NC simulation, but this method has the drawback of expensive computation. The second approach uses spatial partitioning representation to represent a cutter and work-piece (Kim, 2006). In this approach, a solid object is decomposed into a collection of basic geometric elements, which includes Z-map, voxel and ray representation. Voxel and Octree (Jang, 2000) is another popular 3D decomposition approach which is used to develop NC simulation system. However, the disadvantages of Voxel and Octree are mainly the high memory requirement and poor boundary accuracy. The third approach uses discrete vector intersection (Chang, 1991). Discrete vector method is one of the most efficient methods in NC simulation. Z-map simulation can be classified as a special case of discrete vector method. However, this method does not directly generate a solid model for the machined part and is not suitable for cases with dramatic normal changes during simulation.

2.2 Z-map Simulation

For Z-map machining simulation, the larger grid size will lead to faster simulation speed but poor accuracy. On the contrary, smaller grid size can improve the simulation result but resulting in low efficiency. (Lee, 2002) proposed enhanced Z-map method which can satisfy the requirements of efficiency and accuracy. Some algorithms (Maeng, 2003~2004) calculate the intersection of cutter swept volume and work-piece, and adjust its z value to achieve the simulation results. (Cai, 2010) presented a general simulation methodology based on a Z-map model of work-piece for predicting surface topographic features and roughness formed in the finish milling process. (Tsai, 2013) calculated mesh position and z coordinates of tool scanning surface according to tool path and initial tool scanning boundary in reducing the long computation time. This method can simulate not only straight line and arc but also nonlinear path, like helical and spline curves.

2.3 Boolean Operation

The difference set of Boolean calculation is widely used in CAD/CAM and reverse engineering and is considered to be able to achieve the purpose of accurate material removal. About the type of computation, (Tayebi, 2011) assort them into four different categories: exact arithmetic methods, approximate arithmetic methods, interval computation methods and volumetric methods. (Mei, 2013) described a simple and robust algorithm for triangular mesh Boolean operators; their method can deal with both manifold and non-manifold cases. (Wang, 2011) presented a new approach to compute the approximate Boolean operations of two freeform polygonal mesh solids efficiently with the help of Layered Depth Images. (Reqiuicha, 1985) explored the boundary condition and merge of solid model, including dealing with coplanar issues.

2.4 Collision Detection

The main concept of collision detection is to test the intersection of bounding volumes of objects. If there is no collision between bounding volumes of objects, it means that there is no collision for measured models and does not need further interference tests. (Chan, 2003) proposed an algorithm which can specifically define the axis-aligned bounding box of an object. (Hutter, 2007) added triangular mesh information into bounding volume hierarchy to increase the effectiveness of collision detection. (Curtis, 2008) extended the above mentioned method and simplify the detection rules to accelerate calculation efficiency.

3 RESEARCH METHOD

This paper presents a 3D simulation system for multi-axis turn-mill machining. In the machining simulation system, the main consideration is the simulation efficiency. In order to obtain the balance between efficiency and accuracy, this paper uses the mainstream triangular mesh model as our basis format to represent the work-piece. And in our machining simulation algorithm of material removal, we propose a hybrid mechanism to choose among the algorithms of Deformation of Intersection Point (DIP), Z-map and Boolean. The system will automatically determine the appropriate algorithm according to the different machining processes. For example, DIP is for turning process, Z-map is for milling process and Boolean takes all situations that Z-map can't handle, like drill or chamfer. The blue arrow in Figure 1 shows it can support the deformation of DIP and Z-map. Besides, the DIP and Z-map are also of mutual assistance. That's why our system can handle the mixed case of turning and milling process. Additionally, to increase the simulation value and ensure the machining accuracy of the tool-path, the system will detect the collisions between tools and the machine in the whole simulation process.

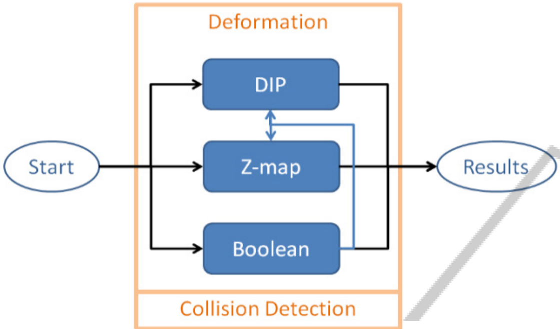


Figure 1: Flow chart of our simulation process.

4 ENVIRONMENT CONSTRUCTION

4.1 Parameter Setting

First, the program will load the related files and parameters before the machining simulation starts. The detailed descriptions of each of these files are as follows:

1. Initial parameters of program: material types, jaw types and G30 coordinates of origin.
2. NC file: data of NC path.
3. Material file: User-defined 2D point group of raw material block.
4. Jaw file: User-defined 2D point group of jaw.
5. File of process and parameter: name of processes, parameter of tools and of tool colours.

4.2 Work-piece Construction

Based on the 2D plane concept of a turning machine, the system will draw the material profile on a 2D plane. Then we use the polar coordinates to rotate material profile 360 degrees on the Z axis and build the triangular mesh model. Users can adjust the mesh density to their needs. The material shape is drawn by users. However, the system will automatically determine the mesh size of the material model by considering the follow-up deformation and the situation of the NC code. The following figure shows a cylindrical material was made by this method.

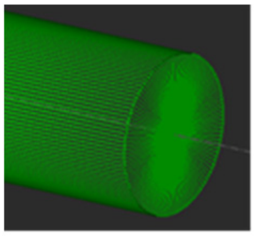


Figure 2: Work piece shown in triangular grid.

5 SIMULATION OF MATERIAL REMOVAL

5.1 Deformation of Intersection Point (DIP)

In turning simulation process, we can get several intersections by each step using vertical line, horizontal line or every connected line of two NC points. As long as with correct orders of the NC path track, we can get the 2D shape's point group after deformation. Furthermore by revolving the original point group 360 degrees, we can create the 3D space points after the material removal. The line-line intersection equation of this paper is as follows:

There are two line equations L1 and L2 in the 2D space. Two points on the line of segment L1 are (x1,y1), (x2,y2). The other points on the line of segment L2 are (x3,y3), (x4,y4). The intersection points of two segment lines are as follows:

$$P_x = \frac{\begin{vmatrix} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} & \begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix} \\ \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} & \begin{vmatrix} x_3 & 1 \\ x_4 & 1 \end{vmatrix} \end{vmatrix}}{\begin{vmatrix} \begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix} & \begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix} \\ \begin{vmatrix} x_3 & 1 \\ x_4 & 1 \end{vmatrix} & \begin{vmatrix} y_3 & 1 \\ y_4 & 1 \end{vmatrix} \end{vmatrix}} \quad P_y = \frac{\begin{vmatrix} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} & \begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix} \\ \begin{vmatrix} x_3 & y_3 \\ x_4 & y_4 \end{vmatrix} & \begin{vmatrix} y_3 & 1 \\ y_4 & 1 \end{vmatrix} \end{vmatrix}}{\begin{vmatrix} \begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix} & \begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix} \\ \begin{vmatrix} x_3 & 1 \\ x_4 & 1 \end{vmatrix} & \begin{vmatrix} y_3 & 1 \\ y_4 & 1 \end{vmatrix} \end{vmatrix}} \quad (1)$$

$$(P_x, P_y) = \left(\frac{(x_1y_2 - y_1x_2)(x_3 - x_4) - (x_1 - x_2)(x_3y_4 - y_3x_4)}{(x_1 - x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 - x_4)}, \frac{(x_1y_2 - y_1x_2)(y_3 - y_4) - (y_1 - y_2)(x_3y_4 - y_3x_4)}{(x_1 - x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 - x_4)} \right) \quad (2)$$

If the two lines are parallel, then

$$(x_1 - x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 - x_4) = 0 \quad (3)$$

In Figure 3, the points in order 1,2,3,4 are points of the tool shape. The program will determine the point is taken or not, based on if it is inside or outside. If it is inside then it is taken, if not find the intersection point with the other points of the tool shape.

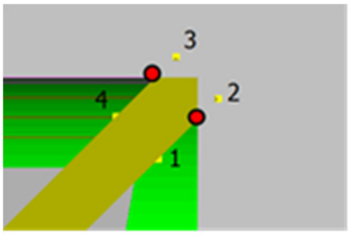
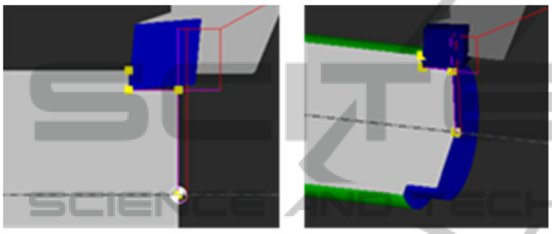


Figure 3: Illustration of deformation algorithm.

Figure 4 is the result of turning simulation.
In Figure 4(a), it is a simplified schematic drawing showing the intersection points and the deformation result of turning. In Figure 4(b), we show the result of simulation with a tilted angle.



(a) (b)

Figure 4: The result of turning simulation.

5.2 Z-map

Z-map is the second method of material removal in simulation in this paper. The Z-map method has one advantage: it does not break up the mesh model. It has been widely used in 3-axis milling simulation. The main principle is changing the depth of point group of the mesh to get simulation results of material removal by the path of the sweeping tool. In implementation, our research uses Tree data structure to accelerate the search efficiency. Along the cutter movement, the G01 NC segment will be divided into three areas: Circle A (red), Circle B (yellow), and Rectangular C (blue). In real situation, path is continuous. When the mesh grids are marked under these three areas and ready for deformation, the system only deals with the parts of Circle B and Rectangular C. Figure 5 is a mesh model and we use it's triangular grid endpoints to establish the Z-map. Figure 6 shows the tool is moved linearly during the simulation. Figure 7 shows the tool is moved in an arc during the simulation. Marked sections of grid are Circle A(red), Circle B(yellow) and fan-shaped C(pink), and we only need to deal with B and C.

In general, if we only use the Z-map to push grid points, it is difficult to keep the accuracy in simulation. Just like Figure 8, the only thing one can

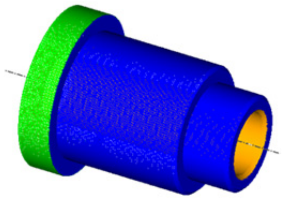


Figure 5: Mesh model for Z-map.

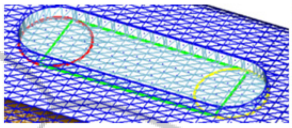


Figure 6: Straight path.

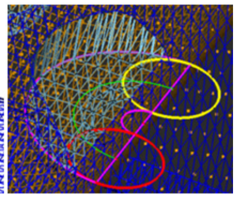


Figure 7: Circular path.

do is to increase the grid density to approximate the real machining results. Hence, we propose the concept of "grid folding" as shown in Figure 9. It is using the idea of taking apart one grid into five grids. In the middle there will be two segments of sharp boundary generated intersection points. When the Z-map pushes the grids with a flat end-mill, the boundary grids need to be "extended" by the additionally generated intersection points. At this moment, using the intersection points as folding points, we can obtain the sharp edges in the boundary. Figure 9 shows that the required accuracy can be reached using this method.

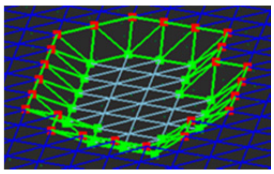


Figure 8: Inaccuracy situation.

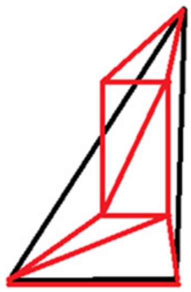
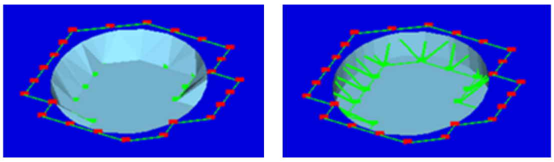


Figure 9: Grid folding.

The actual test results are as follows, Figure 10(a) is the result of the original grid, and Figure 10(b) is the result after grid folding. Using such method, we can get the good simulation result without the increase of grid density by subdivision.

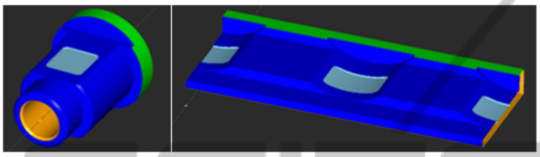
In the simulation of the mixed case of turning and milling, the Z-map method can be further extended by combining the use of polar coordinates transferred to the Y axis and the C axes. The polar coordinates can be "flattened out" as in Figure 11(b),



(a) (b)

Figure 10: The results before and after.

the simulation is performed by the Z-map, and the result is then “folded back” as in Figure 11(a). Figure 12 shows the simulation result using this extended Z-map approach.



(a) (b)

Figure 11: The results of “folded back” and “flattened out”.

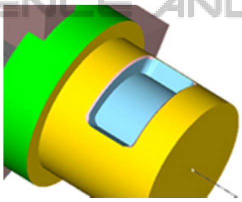


Figure 12: Simulation results.

5.3 Boolean Operation

Boolean operation plays an important role in geometric processing, which aims to obtain union, subtraction and intersection of geometric models. Theoretically, an accurate material removal model may be obtained through the Boolean operation. But the drawback is expensive computational loading. For machining simulation, the required efficiency may not be achieved. The purpose of this paper is to achieve the material removal simulation process, and therefore only consider the difference operator. Here, we implement a simple and robust Boolean algorithm proposed by (Mei, 2013); the operational flowchart of difference operators is as follows:

Step 1: Roughly search the candidate intersection triangle-pairs by bounding box detection and removal of coplanar triangle-pairs with plane projection algorithm.
Step 2: Compute the intersection line for each pair of triangles. Merge all the segments into a loop as expansion boundary.
Step 3: Split cutter and work-piece according to the intersection loop.
Step 4: Decide seeds on both cutter and work-piece. Execute region growing method to define removal part based on mesh topology.
Step 5: Merge all the sub-surfaces which are defined as “reserve” to form a new model.

Since the using Boolean operators must pay a higher cost, if we simply use Boolean operators as a way to remove the material, it will result in inefficient simulation with a higher cost. Therefore, in general, Z-map will be considered as the first method for deformation. The timing of using Boolean is when the Z-map fails to generate correct deformation. In this system Boolean is mostly used for drilling and chamfering editing and other related actions, for example, reaming of thread or chamfer. Figure 13 and 14 indicate drilling and chamfering examples where Boolean operations are used.

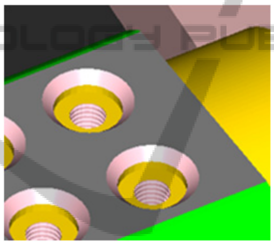


Figure 13: Drilling.

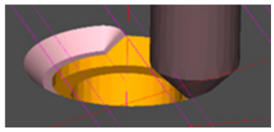


Figure 14: Chamfering.

In addition, in drilling there could be a special case of coplanar problem and we have a special process to deal with it. For example, Figure 15 is the drilling case we used. Using NC path points to exclude the grids and the lines on the co-plane, there will be no incorrect intersection lines during the process of Boolean. This can help avoid mistakes and prevent Boolean failure from happening.

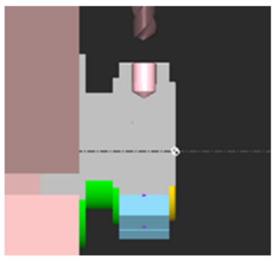


Figure 15: The drilling case has co-planar situation.

6 SIMULATION OF TURN-MILL PROCESS

The main purpose of this work is to construct 3D simulation of turn-mill process. In addition to adopting triangular mesh model to represent every work-piece, we also create realistic tools and turrets for the simulation of the turn-mill process. This includes different characteristics of turning and milling machines so it can be widely used in various practical planning situations. In this section we will show actual industry examples and demonstrate the feasibility of the proposed system.

6.1 Emulated Turret Construction

In the beginning, this paper mentioned that we hope to create an emulated turn-mill machine. Therefore, our goal is to construct turrets and tools based on real machine information as much as possible. The database of tool and turrent (STL files) was provided by a CNC factory. Figure 16 shows that if the user opens turret viewing function, he can examine the turret at the front view (or any views). This will help the user easily check the tools one by one by rotating the tools counter-clockwise.

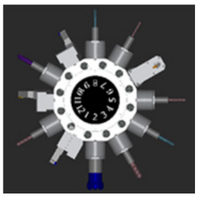


Figure 16: The realistic turret.

6.2 Turning

The first part of this software development is turning. This system not only includes outside turning and inside turning, but also circular and screw cutting paths. Unlike other general lathe simulation software which provides only 2D turning simulation, this system provides true 3D simulation using triangular solid meshes. This not only provides realistic material removing process, but also is very useful for 3D collision detection between work-piece and machine structures including turrets. Figure 17 and 18 show two such examples.

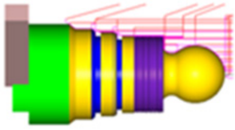


Figure 17: Result 1.

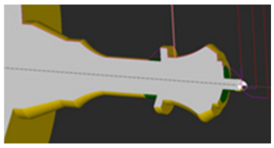


Figure 18: Result 2.

6.3 Milling

The second important part of this software development is milling. At the end of each turning process, this system will convert the object into a triangular mesh for Z-map or Boolean simulation. Most milling simulations are performed using the Z-map method. Boolean operations are employed for chamfering or other complex functions such as drilling or internal threading. Figure 19 and 20 are other two difficult simulation cases.

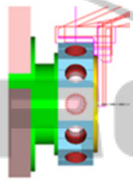


Figure 19: Results.

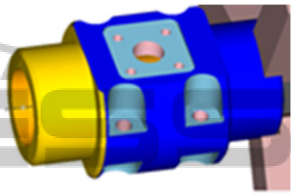


Figure 20: Results.

6.4 Sub-spindle

Most turn-mill machines often come with a sub-spindle. Therefore, our system also adds a sub-spindle module, making this simulation more versatile. Figure 21 shows machining the work-piece under the spindle and the subsequent sub-spindle clamping of the work-piece. Simulations carried out in the sub-spindle need to consider the reversed cutting directions.

Figure 21: Work piece is moved from main spindle to sub-spindle.

6.5 Cylindrical Interpolation/Polar Coordinate Interpolation

This paper also deals with special G code commands (G07.1, G12.1). This method is discussed in section 5.2. It can also be applied to the sub-spindle module. Figure 22 and 23 are simulation results of polar cases.

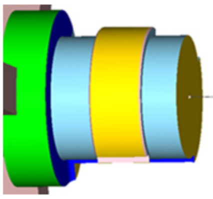


Figure 22: Polar case 1.

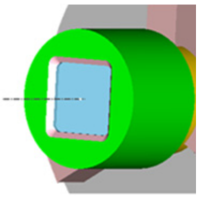
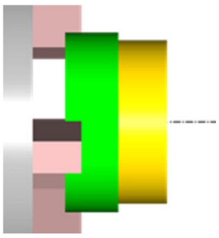


Figure 23: Polar case 2.

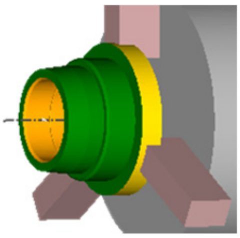
6.6 Hybrid Turning/Milling/Drilling

In this work, combination of different machining processes need to be considered, for example, from turning to milling and then to turning, or from milling to turning and then to milling. These changes tend to disrupt the original data structure of the system. We overcome this problem by recording the accumulated profile of the turning process, which can be converted to a solid triangular mesh anytime. In the subsequent process of milling or drilling, Boolean operation can be used to obtain the final geometry. A turning/drilling/turning case is shown in Figure 24.

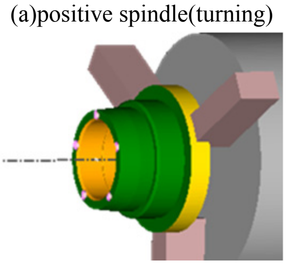
(a) the work-piece is turned in positive spindle
(b) move to the sub-spindle, and turn again
(c) the work-piece is drilled
(d) the work-piece is turned again to obtain the final result



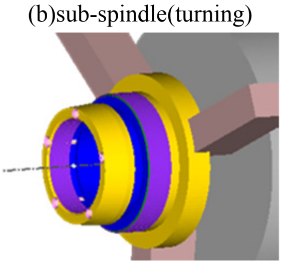
(a)positive spindle(turning)



(b)sub-spindle(turning)



(c) sub-spindle(drilling)

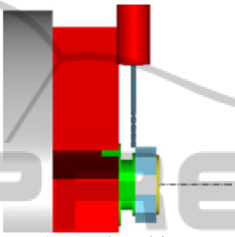


(d) final results

Figure 24: Different steps of simulation.

6.7 Collision Detection

Collision detection is a very important function in turn-mill simulation. We can use the collision detection function to verify whether the NC path is correct or not, so we can avoid significant damage because of the collision between the tool and machine before machining. In this paper, we use three steps to do collision detection. First, we record the bounding box of tool when we create the tool database, then we use an AABB tree to filter collision components in the simulation. Finally, we use mesh intersection algorithm to detect collision components. We display the collision detection by changing the colour of the collision components to red to warn the user. Figure 24 (a) shows the collision result between tool and jaws. Figure 24(b) shows the collision result between tool and work-piece.



(a)Tool and jaw interference

(b) Tool and work-piece interference

Figure 25: Two different situations of interference.

7 DISCUSSION

Complete turn-mill machining simulation has been presented in the previous sections. This paper demonstrates that turn-mill machining simulation is possible through the integration of 2D Intersection, Z-map and Boolean algorithms with balanced efficiency and accuracy. In this section, will discuss the problems encountered during the simulation process and their solutions. First, for the turning simulation, this work achieves the desired result by 2D intersection calculation. The surface of the work-piece will be deformed according to the intersection profile. Then, the turned profile is rotated 360 degrees to construct a 3D solid mesh. The result shows that this method is feasible, robust and highly efficient. For the milling simulation, this paper removes materials by the Z-map algorithm. The Z-map method can provide an excellent simulation result similar to the Boolean operation but with better efficiency. Meanwhile, this paper also extends the Z-map method into polar coordinates system to simulate turn-mill machining. Furthermore, the work-piece must be subdivided and then deformed in order to achieve the simulation of material removal. It can predict that the system performance will decrease due to huge polygon data after long time milling simulation. Thus, polygon merge is necessary during the simulation process. Further, the Z-map can't perform the drill function to remove materials due to its algorithm limitation. The Boolean operation is the only solution to simulate drilling operation.

The difference set of Boolean can be used to remove material from the original object. Theoretically, Boolean operation can produce precise machining results. However, its drawback is that Boolean calculation needs higher computation cost and will decrease performance during simulation. In addition, coplanar contacts between the cutter and work-piece are quite common during the simulation. For the surface-based model representation, coplanar issue is still a challenge.

8 CONCLUSIONS

The main contribution of this paper is the simulation of turn-mill machining in 3D with accuracy and efficiency. Triangular mesh model is used to represent the cutter and work-piece, including tooling turret and clamp. The advantage of turn-mill machining is that it can shorten processing time, increase productivity, reduce storage space, and maintain accuracy. The turn-mill process is usually complicated and involves multiple cutters and spindles. The danger and cost of collision is high, therefore accurate and efficient simulation of the process becomes critical. To achieve this purpose and balance between efficiency and precision, this paper proposes a hybrid simulation method by combining DIP, Z-map and Boolean operations. The software will determine the appropriate material removal method according to the NC instructions. In the simulation processing, in order to check if the path is correct or not, mesh collision detection algorithm is used to find out interferences. For the future work, first, this research will focus on the improvement of Boolean operation. The efficiency of Boolean operation is still a bottleneck of simulation. In addition, the Boolean operators still need to exclude some special cases such as coplanar in advance to ensure the success of the simulation. In the future, this work will also be extended to the on-line turn-mill simulation/monitor by integrating the simulation software with a PC-based controller.

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