New Learning Rules for Three-layered Feed-forward Neural Networks based on a General Learning Schema

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Abstract. We propose a general schema for learning rules in neural networks, the General Enforcing Rule Schema (GERS), from which we infer new simplified learning rules for in particular supervised learning of three-layered feed-forward networks, namely Enforcing Rule Supervised ERS and ERS2. These rules are comparatively simpler than the established rule of Backpropagation and their performance is at least equivalent. The new rules are compared with Backpropagation in different experiments; an additional comparison is performed with real data for the prediction of parcel delivering.

1 Introduction

Most of the established learning rules for neural networks are, as is well known, “Hebbian” ones, i.e. they are based on the principle of Hebb that learning is the changing of connections between the (artificial) neurons. Although Hebb speaks of the changing of “cells” rather than of connections [1] it seems safe to transfer his principle to the changing of connections. In this sense Hebb may be called the (unintentional) father of learning neural networks.

The Hebbian learning rules and their numerous variants have been applied for a long time and they have frequently demonstrated their usefulness. Yet despite their obvious success they have a slight methodical and theoretical blemish:

From a *methodical* point of view it seems a bit strange that the activation values or output values of the neurons respectively are used on the one hand as cause variables and on the other as performance variables. Consider, e.g., a standard version of the Backpropagation rule, which we shall use also for comparison reasons:

$$\delta_j = \begin{cases} f'_j\left(net_j\right)\left(t_j - o_j\right) & \text{, if j is an output neuron} \\ f'_j\left(net_j\right)\sum\limits_k\left(\delta_k w_{ik}\right) & \text{, if j is a neuron from the hidden layer} \end{cases} \tag{1}$$

Here the activation or output value respectively of the neuron j represents as a part of the learning rule a cause variable or an independent variable respectively. On the other hand the performance of the network as a result of the application of the learning rule is measured by the output values and that means the activation values of the different output neurons. Hence the activation values are also used as performance

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variables or dependent variables.

To be sure, this double usage of the activation values is no logical circle. Yet it seems rather redundant to use the activation values twofold. Hence it would be desirable to construct learning rules where the activation or output values are used only as performance variables. Mathematically the rules would become simpler, which would be an advantage for teaching, programming, and application purposes.

In addition, from a *theoretical* point of view it is rather unsatisfactory that the specific learning rules for different types of learning, particularly supervised learning and self-organized learning, have not much in common, besides the general aspects of Hebb's principle. The Backpropagation rule as a paradigm for supervised learning for example has on a first and second sight not much to do with the well-known "Winner-takes all" rule that is used in Kohonen Feature Maps for self-organized learning. Hence it would be desirable to construct a general learning schema that would be the basis for learning rules, applied to different types of learning, and that is also based on Hebb's proven principle - in neuroinformatics and biology (cf. e.g. [6]).

In a *formal* sense its essence certainly is the increasing or decreasing respectively of the weight values between sending neurons and a receiving artificial neuron, as many learning rules take into account. By leaving out the output values of the respective neurons we obtain a general learning schema in its simplest form as

$$\Delta w_{ij} = \pm c \text{ and } 0 \leq c \leq 1. \qquad (2)$$

The constant c has the same function as the learning rate η in different standard learning rules. If one uses for example the linear activation function

$$A_j = \sum w_{ij} A_i , \qquad (3)$$

then in large networks the activation values frequently become too large. In order to avoid such an increasing equation (2) can be extended by introducing a "dampening factor". Then the schema becomes

$$\Delta w_{ij} = \pm c * | (1 - w_{ij}(t) | \qquad (4)$$

if $w_{ij}(t)$ is the according weight value at time t, i.e. before changing. The dampening factor is used in order to keep the weight values in the interval (-1, 1).

Equations (2) and (4) are just schemas, namely the General Enforcing Rule Schemas (GERS). The application to different types of learning needs in the two types investigated by us, namely supervised learning and self-organized learning, additional components. We chose these types of learning because they are by far the most important ones in the usage of neural networks. By the way, when dealing with reinforcement learning equation (4) can be used directly as the according learning rule.

For example, an according learning rule for self-organized learning, which we developed for a self-organized learning network (Self Enforcing Network, SEN), is

$$w(t+1) = w(t) + \Delta w \text{ and} \qquad (5)$$
$$\Delta w = c_* v_{sm},$$

c is again a learning rate and v_{sm} is the according value in a so-called semantical matrix, i.e. the data base for the learning process (cf. [2]; [3]). Obviously this is a direct application of GERS to this type of learning without taking into regard the output values. The main emphasis in this article, however, is put on the application of GERS to supervised learning, in particular learning of three-layered feed-forward networks.

2 ERS – Enforcing Rule Supervised

Three-layered networks need as learning rules strictly speaking two components, namely the variation of the weights between the hidden and the output layer and the variation of the weights between the input layer and the hidden one. The first component of our new rule is computed by

$$\Delta w_{ij} = c * \left|\left(1-\left|w_{ij}(t)\right|\right)\right| * \delta_j * \mathrm{sgn}(o_i) \qquad (6)$$

δ_j is of course the distance between the output neuron j and the respective component in the target vector, i.e. $\delta_j = t_j - o_j$. Note that the factor sgn(o_i) is used only as the sign of the whole product; the absolute numerical value of o_i does not matter. For brevity's sake we call this rule ERS – Enforcing Rule Supervised. The second component of ERS is:

$$\delta_j = \begin{cases} t_j - o_j & \text{, if j is output neuron} \\ \sum_k w_{jk} * \delta_k & \text{, else} \end{cases} \qquad (7)$$

For experimental and methodical purposes we also constructed a variant of ERS by adding the factor o_i:

$$\Delta w_{ij} = c * \left|\left(1-\left|w_{ij}(t)\right|\right)\right| * \delta_j * o_i \qquad (8)$$

This variant is more similar to the established rules because of the factor o_i; for the sake of brevity we call this variant ERS2. The computation of the weight variation between the hidden and the output layer is the same as in equation (6).

Because we wished to compare ERS with standard learning rules we performed our experiments also with a standard version of the Backpropagation Rule (BP), already shown in equation (1).[1] There are numerous other versions of BP but we used the standard version usually introduced in textbooks on neural networks (e.g. [5]).

Equation (7) shows that ERS and ERS2 are also "back propagating" rules as they propagate the error to the weight values between input and hidden layer. Recently a learning rule for multilayered feed forward networks has been proposed, namely a so-called "No-Prop" algorithm, which just uses the Delta rule for the variation of the weight values between output and hidden layer and leaves the other weight values fixed [8]. As far as we know this algorithm has been analyzed just for a few examples.

3 Experimental Design and Results

We analyzed the performance of ERS and ERS2 in comparison to BP with networks

[1] Attempts to improve standard BP can be found, e.g., in Orr and Müller[4].

of different size and different topologies.[2] The input and target values are generated at random from the interval between 0.00 and 1.00. The initial weight values of the networks are also generated at random from the interval (-0.5, 0.5) or (-1, 1). A learning process is considered as successful if $0 \leq \delta \leq 0.01$. If such a value is not reached after mainly 2.000 learning steps the learning run stops and is marked as unsuccessful. In each single experiment ERS, ERS2, and BP got the same initial matrix and the same input and target patterns. The learning rate c for ERS and ERS 2 was in the first series c = 0.2 and 0.4 respectively; when enlarging the networks we decreased the learning rate to c = 0.04. In all experiments in the case of BP was η = 0.9. For error computation we used the Euclidean distance.

The activation function is tangent hyperbolicus tanh(x) for ERS and ERS2; for BP we used the logistic function. To simplify the experiments we chose the same size of the patterns as the number of patterns to be learned, i.e. if the networks had for example to learn 20 patterns the patterns consisted of 20 components. When we speak of "pattern" we mean strictly speaking a pair of patterns, namely an input pattern and a target pattern.

The first experimental series was performed with 10 different patterns for each network, generated at random. In all 1000 different tests were performed and the maximal iteration number, i.e. number of learning steps, was limited to 2.000. Table 1 shows the results for the most suited topologies respectively for all learning rules; the topologies are obtained from according experiments. The values given for the mean refer to all experiments and *only* to successfully learned patterns; SD (standard deviation) was computed for all experiments, i.e. also the unsuccessful ones. LR is the learning rate and AF the respective activation function. Min / Max refers to minimal or maximal learning steps respectively.

Table 1. Performance for "ideal" topologies; the numbers in the first line represent the numbers of successful learning processes.

Learning Rule	**BP**	**ERS**	**ERS2**
Topology	10—25—10	10—65—10	10—40—10
Weight intervals	(-1.0 , 1.0)	(-1.0 , +1.0)	(-0.5 , 0.5)
Performance	**998**	**1000**	**1000**
LR /AF	0.9 / logistic	0.1 / TanH	0.4 / TanH
Min / Max	38 / 1015	110 / 1282	47 / 1419
Ø / SD	112 / 98	241 / 85	134 / 80

The next step was to test the performance of the learning rules with different topologies and different intervals of the generated weight values. Table 2 shows the results.

According to our experiments ERS and ERS2 need in general a larger number of neurons in the hidden layer to reach better results than the BP networks. It might be not by chance that Widrow et al. (loc. cit.) report the same result when comparing their algorithm to BP.

[2] Viktor Schäfer has implemented the Tool, which enables the comparison between the rules.

Table 2. Learning performance with 10 patterns and the same topologies.

Topology	10—30—10	10—40—10	10—50—10
Weight intervals	(-0.5 , 0.5)	(-0.5 , 0.5)	(-0.3 , 0.3)
BP	**952**	**939**	**962**
LR /AF	0.9 / logistic	0.9 / logistic	0.9 / logistic
Min / Max	304 / 4787	290 / 6809	208 / 4769
Ø / SD	885 / 622	866 / 639	684 / 496
ERS	**934**	**982**	**998**
LR /AF	0.2 / TanH	0.2 / TanH	0.2 / TanH
Min / Max	53 / 4746	61 / 5857	147 / 9255
Ø / SD	310 / 468	205 / 240	778 / 857
ERS2	**998**	**1000**	**989**
LR /AF	0.4 / TanH	0.4 / TanH	0.4 / TanH
Min / Max	36 / 2798	38 / 660	98 / 9942
Ø / SD	140 / 160	127 / 71	625 / 1093

For the following experiments, we decided to make a compromise with respect to the number of neurons in the hidden layer, i.e. the acceptable according number for each learning rule, as a result of several samples.

The next figure shows the results for the learning processes with 20, 30, and 50 patterns. The two chief results are:

a) The speed, i.e. the number of learning steps, is approximately the same with all learning rules. In some experiments BP was the fastest learning rule, in others ERS and ERS2. Yet the differences are not significant in the sense that there is a general trend, which rule is the best one.

b) In all experiments ERS and ERS2 are more reliable than BP, i.e. their number of successfully completed learning processes is always larger than that of BP. Hence, although the new learning rules ERS and ERS2 are not significantly faster than BP they are obviously more reliable – an interesting result in particular for practical applications of neural networks (see below the real data experiments).

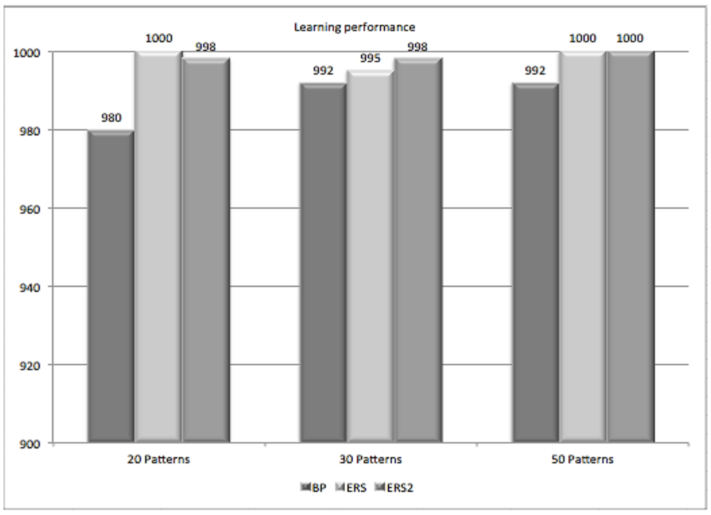


Fig. 1. Learning performance with 20, 30, and 50 patterns.

Details can be seen in the following table:

Table 3. Learning performance: details.

Topology Weight intervals	20—60—20 (-0.5 , 0.5)	30—80—30 (-1.0 , 1.0)	50—100—50 (-1.0 , 1.0)
BP	**980**	**992**	**992**
LR /AF	0.9 / logistic	0.9 / logistic	0.9 / logistic
Min / Max	344 / 4527	337 / 3222	467 / 3529
Ø / SD	849.5 / 578.5	614 / 339	802 / 321
ERS	**1000**	**995**	**1000**
LR /AF	0.2 / TanH	0.1 / TanH	0.03 / TanH
Min / Max	251 / 6068	230 / 3878	351 / 4114
Ø / SD	780 / 494.5	556.5 / 201	978 / 492.5
ERS2	**998**	**998**	**1000**
LR /AF	0.3 / TanH	0.2 / TanH	0.04 / TanH
Min / Max	254 / 9970	460/ 4468	361 / 3274
Ø / SD	1969.5 / 1414	1296.5 / 935	666 / 287

For comparison purposes we repeated all experiments with an error rate of 0.000 for each pattern number and each topology. ERS and ERS2 were successful but only with approximately twice iterations than in the previous experiments. The used standard version of BP did not succeed at all, i.e. it only seldom reached the learning goal. Additional parameters like momentum or decay would probably have been necessary; it will be interesting if such parameters can also improve the performance of ERS and ERS2.

4 A Case Study: Predictions of Parcel Delivering Times

A severe problem for all delivery services is to inform their customers about the probable time the parcels and other products will be delivered to the customers. In cooperation with DHL, the delivery service of the *Deutsche Post*, one of the biggest logistic firms worldwide, we obtained real delivery data for several districts of Essen, a large city in the West of Germany. The districts consist altogether of 20 streets. As far as we know we were the first to attempt predictions with neural networks for the problem of prognosticating different time intervals or time windows.

Our research question was two-fold, namely on the one hand to analyze if successful predictions for delivery times in different streets are possible at all and on the other hand which one of the three learning rules, compared in the experiments above, reached the best prediction results.

In all three cases we used the same network topology, namely 34—45—34. The number of input and output neurons is due to the number of time intervals each day was divided into. Hence each input and output neuron represents a certain time interval. The training phase was 20 weeks; each day in a week was mapped to the according day in in the next week. Hence we obtained for each day in a week a time series of twenty components. For BP we used again the logistic function and a learning rate of 0.9; for ERS we used a so-called logarithmic-linear function that was newly developed by us (cf. [2]):

$$A_j = \sum_{i=1}^{n} \begin{cases} \lg_3(A_i + 1) * w_{ij} & , \quad A_i \geq 0 \\ \lg_3(|A_i - 1|) * -w_{ij}, & A_i < 0 \end{cases} \tag{9}$$

The learning rate was 0.04; ERS2 had the tanh-function and a learning rate of 0.08.

The following figure shows the performance of BP for Tuesday in week 31 (KW); the numbers below the figure show how many predictions are characterized by a specific time difference with respect to the factual delivery. For example, 2 predictions differed just ± 15 minutes from the factual time (top row), 4 predictions differed ± 45 minutes (second row) and so on. The number of predictions is the according mean of the 20 streets.

The next 2 figures show the performances of ERS and ERS2 for the same day:

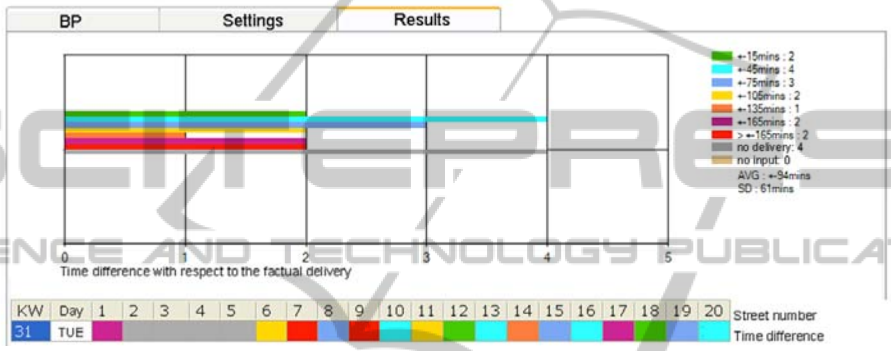


Fig. 2. Prediction performance of BP for Tuesday.

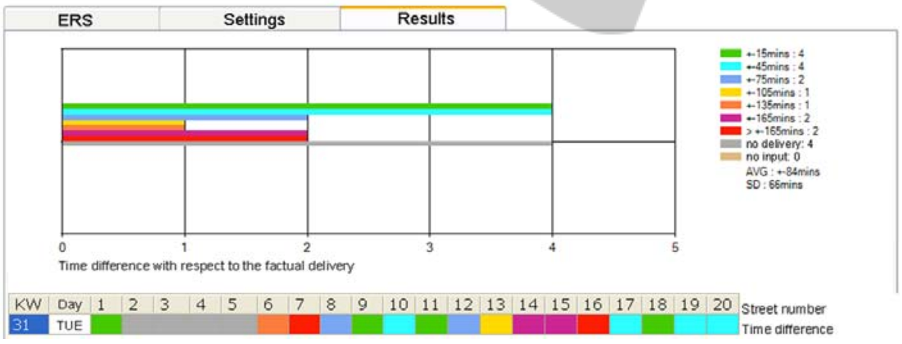
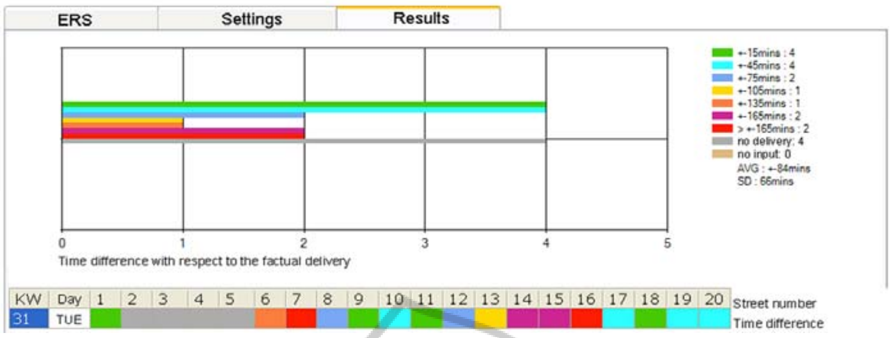


Fig. 3. Prediction performance of ERS.

An additional remark is necessary: The figures show just the results for 16 time intervals. This due to the fact that frequently no parcels were delivered at all; hence it would be quite useless to compute a difference for such days between the predicted time and the factually non-existent time. Therefore the prediction list is an adjusted one, which is certainly not the fault of the different networks.

The results for the other days are nearly the same as those for Tuesday; the shown results are in this way representative.



Legend	
🟩	+-15mins : 4
🟦	+-45mins : 4
🔷	+-75mins : 2
🟨	+-105mins : 1
🟧	+-135mins : 1
🟪	+-165mins : 2
🟥	> +-165mins : 2
▨	no delivery: 4
🟫	no input: 0
	AVG : +-84mins
	SD : 66mins

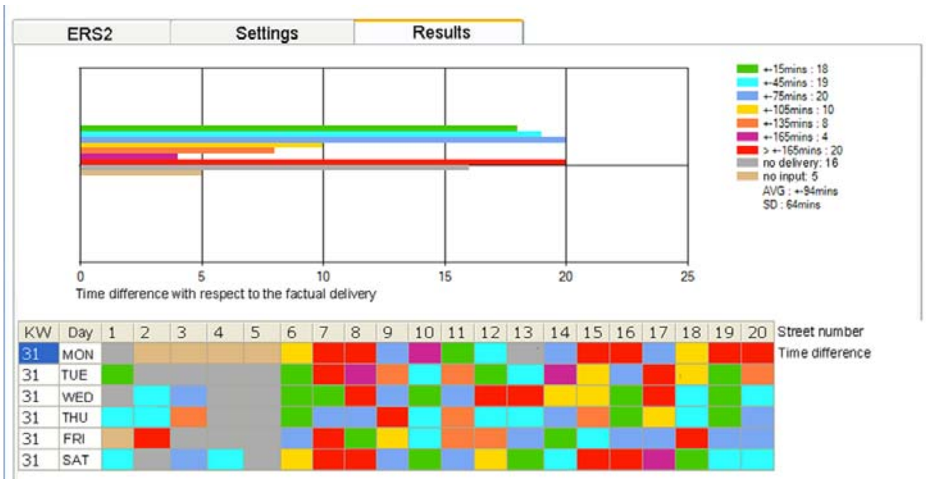
KW	Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Street number
31	TUE	🟩	▨	▨	▨	▨	🟧	🟥	🔷	🟩	🟦	🟩	🔷	🟨	🟪	🟪	🟥	🟦	🟩	🟦	🟦	Time difference

Fig. 4. Prediction performance of ERS2.

With respect to the first questions if such predictions are possible at all, the answer is “to a certain degree”. The delivery experts at DHL commented on our results that indeed delivery times permanently vary because of the changing of drivers, choosing different routes by the drivers, possibility of traffic jams and road works, differences between customers who frequently and those who only seldom receive parcels, and other factors. The opinion of the experts in general was that our results could probably still be improved but that these results are significantly better than their own estimates. Hence further cooperation will be arranged.

The answer to the second question generally confirms the results of the experiments described above. All three learning rules perform with a comparable success. BP and ERS2 are rather similar in their prediction successes; ERS is more successful, i.e. it generated slightly more very good predictions. Hence this case study is another indicator that at least ERS is a bit more reliable than BP.

Finally we show in addition the mean of the performance values for all streets and all days of the week that should be predicted. Fig. 5 contains the results of ERS:



Legend	
🟩	+-15mins : 18
🟦	+-45mins : 19
🔷	+-75mins : 20
🟨	+-105mins : 10
🟧	+-135mins : 8
🟪	+-165mins : 4
🟥	> +-165mins : 20
▨	no delivery: 16
🟫	no input: 5
	AVG : +-94mins
	SD : 64mins

KW	Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Street number
31	MON	▨	🟫	🟫	🟫	🟫	🟨	🟥	🟥	🔷	🟪	🟩	🟦	▨	🔷	🟥	🟥	🔷	🟨	🟥	🟥	Time difference
31	TUE	🟩	▨	▨	▨	▨	🟩	🟥	🟪	🟧	🟦	🟧	🟩	🟦	🟪	🟨	🔷	🟥	🟨	🟩	🟧	
31	WED	▨	🟦	🔷	▨	▨	🟩	🟩	🟥	🔷	🟩	🔷	🟥	🟥	🟨	🟨	🟩	🟥	🟦	🟩	🟦	
31	THU	🟦	🟦	🟧	▨	▨	🟩	🔷	🔷	🟥	🟦	🟧	🟦	🟦	🔷	🟧	🟩	🟨	🟦	🟩	🔷	
31	FRI	🟫	🟥	▨	▨	▨	🔷	🟥	🟩	🟨	🟦	🟧	🟧	🔷	🟩	🟦	🔷	🔷	🟥	🔷	🔷	
31	SAT	🟦	▨	🔷	🟦	▨	🟨	🟥	🟥	🔷	🟩	🔷	🟨	🟩	🟦	🟥	🟥	🟪	🟩	🟦	🟦	

Fig. 5. Mean values of ERS in detail.

The next table shows the results for all learning rules:

Table 4. Comparison of the mean values of the different rules.

Time difference:	±15	±45	±75	±105	±135	±165	>±165	No delivery	No input	AVG	SD
BP	16	16	25	11	9	6	16	16	5	± 94	60
ERS	18	19	20	10	8	4	20	16	5	± 94	64
ERS2	20	17	19	9	9	5	20	16	5	± 95	65

These results show that the trend demonstrated for Tuesday is indeed representative. The DHL experts again assured us that they were not able to produce comparable results.

5 CONCLUSION

To sum up, both ERS versions performed at least as well as BP. The general trend in all experiments and in the case study is unambiguous: ERS and ERS2 seem to be quite suited to solve complex learning problems at least as well as the established learning rule, namely BP. The main differences between the rules are on the one hand that ERS and ERS2 operate well with a small learning rate, which should be even decreased if the networks become larger. BP in contrast always needs a rather large learning rate. One has to take into account, though, that frequently the size of the learning rate depends on the selection of the activation function (e.g. [7]). In our experiments ERS and ERS2 performed best with tanh and BP with the logistic function; in the case study we chose the new logarithmic-linear function for ERS, which performed rather well, and which we already had successfully applied in our new self-organized learning network [2, 3].

On the other hand both ERS versions perform best with rather large hidden layers; BP in contrast usually needs only smaller ones. It is probably not by chance that Widrow [8] (loc. cit.), as remarked, report a similar result from their experiments with the No-Prop algorithm and BP; further investigations will possibly explain this remarkable concordance.

Of course several methodical *caveats* must be taken into consideration. These results confirm that ERS and ERS2 are able to solve different learning tasks as far as we have them analyzed. Yet the results are from samples with certain topologies and from a specific real data case study and they cannot claim to be representative in the sense that, e.g., ERS and ERS2 are *always* as fast as BP or for each problem more reliable than BP. After all, BP has been tested in its different versions for more than two decades and with the two versions of ERS we are just in the beginning. Other parameters like other activation functions, different topologies, e.g. short cuts, more levels, introduction of momentum and decay, or other versions of BP should be investigated too, which we shall undertake in the near future. In addition, the influence of parameters like momentum on the two ERS rules should and will be analyzed too.

It might well be, by the way, that there are classes of learning problems where BP is significantly better than ERS or ERS2, but it might also be that there are other problem classes where ERS or ERS2 should be preferred. Without doubt ERS and

ERS2 are comparatively simpler than BP and hence also simpler to program; in particular ERS and ERS2 do not need any external parameters like, e.g., momentum. According to our didactical experiences BP is usually rather difficult for students of computer science to understand. Teaching ERS and ERS2 as an alternative to BP will be an interesting didactical experience; Widrow [8] stress for the same reasons the comparative simplicity of the No-Prop algorithm as an advantage with respect to BP.

To emphasize it again, we do not mean that BP or the Delta rule become obsolete because of our new learning rules. These established learning rules have been analyzed and applied for many years and they will be always useful and necessary algorithms. ERS and ERS2 for three-layered or more-layered feed forward networks might be additional useful tools for users of neural networks – not more but also not less.

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