Impact on Bayesian Networks Classifiers When Learning from Imbalanced Datasets

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Abstract: In this paper we present a study on the behaviour of some representative Bayesian Networks Classifiers (BNCs), when the dataset they are learned from presents imbalanced data, that is, there are far fewer cases labelled with a particular class value than with the other ones (assuming binary classification problems). This is a typical source of trouble in some datasets, and the development of more robust techniques is currently very important. In this study, we have selected a benchmark of 129 imbalanced datasets, and performed an analytical approach focusing on BNCs. Our results show good performance of these classifiers, that outperform decision trees (C4.5). Finally, an algorithm to improve the performance of any BNC is also given. We have carried out an experimentation where we show how the using of oversampling of the minority class to achieve the desired value for the imbalance ratio (IR), which is the division of the number of cases for the majority class by the cases of the minority class. From this work we can conclude that BNCs show a very good performance for imbalanced datasets, and that our proposal enhance their results for those datasets that provided poor results.

1 INTRODUCTION

Supervised learning construct models from externally supplied instances to produce general hypotheses, which then make predictions about future instances. In other words, the goal of supervised learning is to build a concise *function* of the distribution of class labels in terms of predictive attributes features. There exist many families for these models, known as classifiers, for example decision trees, support vector machines, artificial neural networks or rule systems. Any kind of classifier is always used to assign class labels to a set of testing instances where the values of the predictor features are known, but the value of the class label is unknown. In fact, a classifier is needed to provide the correct class label for future or unseen cases, for example determining if a manufacture piece is defective based on a set of features extracted from visual information (an image taken by a camera), or determine if an e-mail is spam or not based on those attributes that can be extracted from the corresponding information (subject, sender, etc...).

Most classifier learning algorithms assume a relatively balanced distribution (Breiman, 1998), so when there is an under-represented class, this poses a serious problem for them, and they generally have poor generalisation performance on the minority class. In real applications, the imbalance class scenery appears more frequently than expected. For example, in medical diagnosis the disease cases are usually quite rare in global populations (where most of the people do not suffer from the related disease). However, the key task here is to detect which people have this disease. In this case, a classifier that provides a higher identification rate on the disease category would provide better performance. Other fields of application where imbalanced data naturally arise are fraud detection, monitoring, text categorization, risk management, etc... It happens in so many real problems that it can be considered one of the top problems in data mining today (Lopez et al., 2013).

Bayesian Network classifiers (BNCs) are Bayesian Network (BN) models (Korb and Nicholson, 2010) specifically tailored for classification tasks. There is a wide range of existing models that vary in complexity and efficiency. All of them have in common the ability to deal with uncertainty in a very natural way, at the same time providing a descriptive environment. In this work, we will focus on the family of semi-naive (Kononenko, 1991) Bayesian classifiers (Naive Bayes, AODE, TAN, kDB, etc.). The capability of a BN to *express* relationships, dependencies and independences between variables (features in this case) is given by its associated graph

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(qualitative part), and these relationships are also modelled with a second element, quantitative, that forms a BN: probability distributions. In our opinion, there are two main characteristics that have made BNCs so popular: they provide predictions in terms of probabilities (that could be interpreted as weights) and they are easily and intuitively interpreted by non-experts (thanks to the underlying graph). Empirically, BNCs have also been successfully in many application areas (Flores et al., 2012) such as Computing, Robotics, Medicine, Healthcare, Finance, Banking and Environmental Science.

This paper is organized as follows. In Section 2, we review some previous work related to the current one, and discuss the novelty of our approach. In Section 3, we present the first part of our experiments, where we analyse the behaviour of some BNCs in a benchmark of imbalanced datasets. Section 4 presents the final experimentation of the paper, from which we conclude which is the algorithm to apply. Some conclusions from these results are also given. Finally, Section 5 provides a general discussion and future research lines.

2 RELATED WORK

In (Lopez et al., 2013), authors present a very interesting study where they identify which are the intrinsic characteristics that affect when applying supervised classification models on imbalanced datasets. In this work authors pre-select a set of 66 datasets (subset of those in Table 1). This work proves that Imbalance Ratio (IR) is, of course, a very important factor when working with imbalanced datasets, however, the performance of classifiers can not be obtained for a simple (linear) function with respect to this measure. They see how other aspects can also influence, such as the presence of small disjoints, the lack of density in the training data, the overlapping between classes, the identification of noisy data, the significance of the borderline instances, and the dataset shift between the training and the test distributions. The problem with these other measures is that obtaining them is not an easy task, and when it can be done with approximate values, they suppose a high computing cost. Besides, these depend on the particular problem to solve, while we are interested in developing general techniques applicable to any dataset. That is why, we will firstly work on the behaviour with respect to the IR value, in combination with other graphical tools and plots.

The novelty of the current work is that it is uniquely focused on the behaviour on BNCs, since most of the works related to imbalanced datasets are devoted to other kind of models, for example (Lopez et al., 2013) uses Decision trees (C4.5), Support vector machines (SVMs) and the k-Nearest Neighbours (kNN) model, which goes into the family of *Instance based learning*. On the other hand, (Sun et al., 2007) applies two kinds of systems: again C4.5 decision tree and an associative classification system called HPWR (High-order Pattern and Weight-of- evidence Rule based classifier). We considered that there is an open research line in the study of the behaviour of BNCs with imbalanced data and performing a study of how to approach this problem is the main aim of this paper.

Another related work, where BNCs are used is (Wasikowski and wen Chen, 2010), but this is not applicable to the available datasets, since the number of attributes in our problems are too low, and the number of instances is comparatively few, so it does not have sense to apply feature subset selection. In the referred work they use datasets which are not originally imbalanced, and we wanted to apply BNCs to real imbalanced datasets.

3 ANALYTICAL EXPERIMENTS

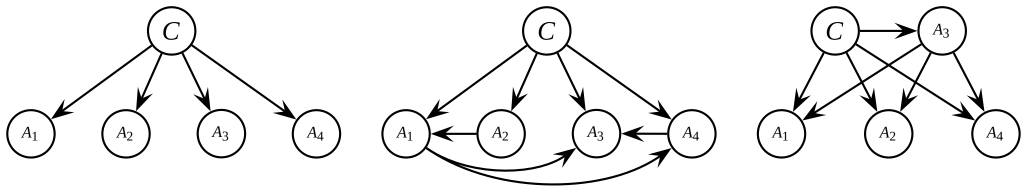
Here we will show the basis for our study and an initial experimental set-up, analysing their results.

3.1 Selected BNCs

The classification task consists of assigning one category c_i or value of the class variable $C = \{c_1, \ldots, c_k\}$ to a new object $\vec{e}$, which is defined by the assignment of a set of values, $\vec{e} = \{a_1, a_2, \cdots, a_n\}$, to the attributes $A_1, \ldots, A_n$. In the probabilistic case, this task can be accomplished in an exact way by the application of the Bayes theorem (Equation 1). However, since working with joint probability distributions is usually unmanageable, simpler models based on factorizations are normally used for this problem. In this work we apply four computationally efficient paradigms Naive Bayes (NB), KDB, TAN and AODE. Our experiments will also show that imbalanced problems does not benefit from more complex BNCs.

$$p(c|\vec{e}) = \frac{p(c)p(\vec{e}|c)}{p(\vec{e})}. \quad (1)$$

These models, as any BN, are represented by a Directed Acyclic Graph, whose nodes indicate variables and the presence/absence of edges imply their relationships, that can be obtained, with the d-separation concept (Korb and Nicholson, 2010). For example, from the structure of NB (Figure 1.(a)), when



(a) Naive Bayes (b) TAN/KDB1 (c) AODE

Figure 1: Examples of 4-nodes network structure for the BN classifiers: NB, TAN and KDB1, AODE.

the value of the class is known (observed), all the other variables (called attributes or features) remain independent. When we work with general BN any variable can be linked to any other (as long as the graph remains acyclic). Even though there are algorithms for learning BNs (Korb and Nicholson, 2010, Part II), they are much slower and complex that those for semi-Naive classifiers, where the structure is fixed or at least constrained. Notice that any learning algorithm needs to discover the graph structure first and then perform a parametrical learning for the probability distribution, for any variable X it stores $P(X|pa(X)$, where pa means parents. In particular, kDB allows k parents apart from the class, TAN learns a tree and then the class is linked to all the features (Figure 1.(b)), and AODE uses one model per every variable as the one in Figure 1.(c) and averages.

Most of these models require discrete data, so when the values are numeric a previous discretization task has to be performed. In the case of Naive Bayes, the continuous model assumes conditional Gaussian distributions.

3.2 Datasets for the Experimentation

We have used KEEL-dataset repository. KEEL stands for Knowledge Extraction based on Evolutionary Learning, and it is an open source Java software tool developed by six Spanish Research Groups. In particular we have worked with those datasets in the imbalanced category, which can be found at http://sci2s.ugr.es/keel/imbalanced.php. Among them, we have omitted those which did not have binary class, this is because multi-valued classes make the problem different (Sun, 2007), and we want to first provide a serious study for the binary case. In fact, any problem could be transformed into a binary one, using a *minority class vs. all* approach. This can be tackled in a future research. Table 1 shows the names of the datasets we used in our experiments, and their basic information:

- number of instances (#Inst)
- number of features or attributes (#att)

Actual	Predicted as	Predicted as
+	+ (TP)	- (FN)
-	+ (FP)	- (TN)

Figure 2: Confusion matrix.

- imbalance ratio (IR), being $\frac{\#M}{\#m}$, where $\#M$ represents the number of instances belonging to the majority class and $\#m$ those belonging to the minority class. See that $\#M + \#m = \#\text{Inst}$.

3.3 Evaluating Models

We must indicate that we will use the measure Area Under a ROC Curve (AUC) for evaluating the classifiers in all our experiments. We selected this measure because it has been shown that it can assess the performance when the instances are imbalanced with respect to the class labels. The area under a ROC curve (AUC) provides a single measure of a classifier's performance. For other applications, where the datasets is supposed to be better distributed in terms of the class labels, accuracy is a standard measure. By accuracy we mean the percentage of correctly classified instances. However, when classifying with the class imbalance problem, accuracy is no longer a proper measure since the rare class has very little impact on accuracy as compared to the majority class. For instance, if the minority class has only presence of 1% in the training data, a simple strategy can be to predict the prevalent class label for every example. It can achieve a high accuracy of 99%. However, this measurement is useless if the main concern deals with the identification of the rare cases.

Most measures for evaluating classification performance can be derived from the confusion matrix, as seen in Figure 2. In cells, T stands for True, F stands for False, P stands for Positive, and N stands for Negative, so we have the four possible combinations. To better grasp their meaning, suppose for example FN, this represents False Negative cases, that is, those cases classified as negative but whose actual value had to be positive, that is why they are called false, because they imply an error.

From this table is easy to find accuracy (in %),

Table 1: Imbalanced datasets used in this work (taken from KEEL repository).

nr	dataset	#Inst	#att	IR	nr	dataset	#Inst	#att	IR
1	ecoli-0_vs_1	220	8	1.86	2	ecoli1	336	8	3.36
3	ecoli2	336	8	5.46	4	ecoli3	336	8	8.6
5	glass-0-1-2-3_vs_4-5-6	214	10	3.2	6	glass0	214	10	2.06
7	glass1	214	10	1.82	8	glass6	214	10	6.38
9	haberman	306	4	2.78	10	iris0	150	5	2.0
11	new-thyroid1	215	6	5.14	12	new-thyroid2	215	6	5.14
13	page-blocks0	5472	11	8.79	14	pima	768	9	1.87
15	segment0	2308	20	6.02	16	vehicle0	846	19	3.25
17	vehicle1	846	19	2.9	18	vehicle2	846	19	2.88
19	vehicle3	846	19	2.99	20	wisconsin	683	10	1.86
21	yeast1	1484	9	2.46	22	yeast3	1484	9	8.1
23	abalone19	4174	9	129.44	24	abalone9-18	731	9	16.4
25	ecoli-0-1-3-7_vs_2-6	281	8	39.14	26	ecoli4	336	8	15.8
27	glass-0-1-6_vs_2	192	10	10.29	28	glass-0-1-6_vs_5	184	10	19.44
29	glass2	214	10	11.59	30	glass4	214	10	15.46
31	glass5	214	10	22.78	32	page-blocks-1-3_vs_4	472	11	15.86
33	shuttle-c0-vs-c4	1829	10	13.87	34	shuttle-c2-vs-c4	129	10	20.5
35	vowel0	988	14	9.98	36	yeast-0-5-6-7-9_vs_4	528	9	9.35
37	yeast-1-2-8-9_vs_7	947	9	30.57	38	yeast-1-4-5-8_vs_7	693	9	22.1
39	yeast-1_vs_7	459	8	14.3	40	yeast-2_vs_4	514	9	9.08
41	yeast-2_vs_8	482	9	23.1	42	yeast4	1484	9	28.1
43	yeast5	1484	9	32.73	44	yeast6	1484	9	41.4
45	ecoli-0-1-4-6_vs_5	280	7	13.0	46	ecoli-0-1-4-7_vs_2-3-5-6	336	8	10.59
47	ecoli-0-1-4-7_vs_5-6	332	7	12.28	48	ecoli-0-1_vs_2-3-5	244	8	9.17
49	ecoli-0-1_vs_5	240	7	11.0	50	ecoli-0-2-3-4_vs_5	202	8	9.1
51	ecoli-0-2-6-7_vs_3-5	224	8	9.18	52	ecoli-0-3-4-6_vs_5	205	8	9.25
53	ecoli-0-3-4-7_vs_5-6	257	8	9.28	54	ecoli-0-3-4_vs_5	200	8	9.0
55	ecoli-0-4-6_vs_5	203	7	9.15	56	ecoli-0-6-7_vs_3-5	222	8	9.09
57	ecoli-0-6-7_vs_5	220	7	10.0	58	glass-0-1-4-6_vs_2	205	10	11.06
59	glass-0-1-5_vs_2	172	10	9.12	60	glass-0-4_vs_5	92	10	9.22
61	glass-0-6_vs_5	108	10	11.0	62	led7digit-0-2-4-5-6-7-8-9_vs_1	443	8	10.97
63	yeast-0-2-5-6_vs_3-7-8-9	1004	9	9.14	64	yeast-0-2-5-7-9_vs_3-6-8	1004	9	9.14
65	yeast-0-3-5-9_vs_7-8	506	9	9.12	66	abalone-17_vs_7-8-9-10	2338	9	39.31
67	abalone-19_vs_10-11-12-13	1622	9	49.69	68	abalone-20_vs_8-9-10	1916	9	72.69
69	abalone-21_vs_8	581	9	40.5	70	abalone-3_vs_11	502	9	32.47
71	car-good	1728	7	24.04	72	car-vgood	1728	7	25.58
73	dermatology-6	358	35	16.9	74	flare-F	1066	12	23.79
75	kddcup-buffer_overflow_vs_back	2233	42	73.43	76	kddcup-guess_passwd_vs_satan	1642	42	29.98
77	kddcup-land_vs_portsweep	1061	42	49.52	78	kddcup-land_vs_satan	1610	42	75.67
79	kddcup-rootkit-imap_vs_back	2225	42	100.14	80	kr-vs-k-one_vs_fifteen	2244	7	27.77
81	kr-vs-k-three_vs_eleven	2935	7	35.23	82	kr-vs-k-zero-one_vs_draw	2901	7	26.63
83	kr-vs-k-zero_vs_eight	1460	7	53.07	84	kr-vs-k-zero_vs_fifteen	2193	7	80.22
85	lymphography-normal-fibrosis	148	19	23.67	86	poker-8-9_vs_5	2075	11	82.0
87	poker-8-9_vs_6	1485	11	58.4	88	poker-8_vs_6	1477	11	85.88
89	poker-9_vs_7	244	11	29.5	90	shuttle-2_vs_5	3316	10	66.67
91	shuttle-6_vs_2-3	230	10	22.0	92	winequality-red-3_vs_5	691	12	68.1
93	winequality-red-4	1599	12	29.17	94	winequality-red-8_vs_6	656	12	35.44
95	winequality-red-8_vs_6-7	855	12	46.5	96	winequality-white-3-9_vs_5	1482	12	58.28
97	winequality-white-3_vs_7	900	12	44.0	98	winequality-white-9_vs_4	168	12	32.6
99	zoo-3	101	17	19.2	100	03subcl5-600-5-0-BI	600	3	5.0
101	03subcl5-600-5-30-BI	600	3	5.0	102	03subcl5-600-5-50-BI	600	3	5.0
103	03subcl5-600-5-60-BI	600	3	5.0	104	03subcl5-600-5-70-BI	600	3	5.0
105	03subcl5-800-7-0-BI	800	3	7.0	106	03subcl5-800-7-30-BI	800	3	7.0
107	03subcl5-800-7-50-BI	800	3	7.0	108	03subcl5-800-7-60-BI	800	3	7.0
109	03subcl5-800-7-70-BI	800	3	7.0	110	04clover5z-600-5-0-BI	600	3	5.0
111	04clover5z-600-5-30-BI	600	3	5.0	112	04clover5z-600-5-50-BI	600	3	5.0
113	04clover5z-600-5-60-BI	600	3	5.0	114	04clover5z-600-5-70-BI	600	3	5.0
115	04clover5z-800-7-0-BI	800	3	7.0	116	04clover5z-800-7-30-BI	800	3	7.0
117	04clover5z-800-7-50-BI	800	3	7.0	118	04clover5z-800-7-60-BI	800	3	7.0
119	04clover5z-800-7-70-BI	800	3	7.0	120	paw02a-600-5-0-BI	600	3	5.0
121	paw02a-600-5-30-BI	600	3	5.0	122	paw02a-600-5-50-BI	600	3	5.0
123	paw02a-600-5-60-BI	600	3	5.0	124	paw02a-600-5-70-BI	600	3	5.0
125	paw02a-800-7-0-BI	800	3	7.0	126	paw02a-800-7-30-BI	800	3	7.0
127	paw02a-800-7-50-BI	800	3	7.0	128	paw02a-800-7-60-BI	800	3	7.0
129	paw02a-800-7-70-BI	800	3	7.0					

which is $\frac{TP+TN}{TP+FN+FP+TN}$. As we commented before, the problem of this measure is that it can not catch the differences between errors. One measure able to deal with this difference is F-measure, which combines precision ($\frac{TP}{TP+FP}$) and recall ($\frac{TP}{TP+FN}$).

AUC was the measure reported in the works we revised about imbalanced datasets, and it has been proved to work properly for measuring good performance for this kind of problems (Huang and Ling, 2005). When classifiers assign a probabilistic score to its prediction, class prediction can be changed by varying the score threshold. Each threshold value generates a pair of measurements of False Positive Rate (FPR) and True Positive Rate (TPR). By linking these measurements with FPR on the X-axis and TPR on the Y-axis, a ROC graph is plotted. This plot is called the ROC curve, and it gives a good summary of the performance of a classification model. AUC is the area under this curve, being 1 the best value and 0.5 the worst (given by a random classifier).

3.4 Impact of IR

The first natural experiment is to replicate those done in (Lopez et al., 2013) where authors present how IR impacts on the classifiers evaluation, we show the classifiers measuring AUC with 5fold stratified Cross Validation (CV), this CV is also used when discretization is previously performed. We used supervised discretization, in particular the one at (Fayyad and Irani, 1993). We discarded the use of distinct techniques, since it has been proved for BNCs, that the discretization method does not have an impact when the number of datasets is significantly large (Flores et al., 2011).

We have carried out a set of tests and the conclusion is similar: we cannot find a pattern of behaviour for any range of IR, and the results can be poor both for low and high imbalanced data. In Figure 3 we show a plot with the AUC obtained for train and test (average on 5-folds), only for NB (continuous and Discretized). This tendency is similar for the other BNCs we tested. This plot shows IR until 40, where most of the datasets are concentrated to see it better, since the behaviour for larger IR values is also similar, with ups and downs and non linear with respect to IR. Notice that it could happen that two datasets have the same IR value, as datasets nr. 59 and 65, for example, in those cases the AUC shown is the average of the obtained measures.

From Figure 3 we can also see how the test and train values are close, so we can corroborate we are not producing high overfitting thanks probably to CV. Initially NB seems to perform better in its continuous version, but this is not always true, for example when

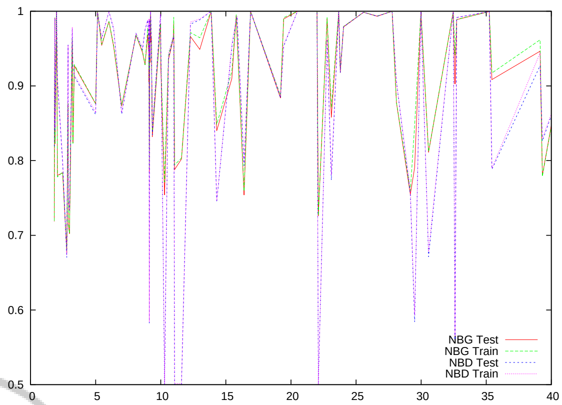


Figure 3: x axis shows IR, y axis show AUC value for test and train, Naive Bayes G(aussian) and (D)iscrete.

IR is between 5 and 8 (approx.) the discrete version outperforms the continuous model, the same happens in the interval [13,15]. These results are not conclusive, the comparison among classifiers will be done in the next subsection

3.5 Comparison Among Classifiers

In order to compare all classifiers, we have discarded the summary view of the previous analysis, where each IR point could concentrate several datasets. In this case we are going to use radial plots where each angle (from 0 to 2π radians) represents a dataset, and the length of the plotted point indicates the AUC obtained. To find the correspondence between datasets in Table 1 and these plots, we indicate that they are plotted anticlockwise (ACW), starting at *three*, and the circle is drawn ACW, finishing at dataset nr. 129.

Notice that AUC can value 1 at maximum, so this is the radius of this plot. From now on, we will just plot AUC values for the test (in fact, this is the average of the 5-folds done in CV), so that we can focus on the performance for all the classifiers. In Figure 4, we show a comparison among NB, AODE and C4.5.

We found this analysis quite informative for our work, and Figure 4 succeeds in summarising all the values at a single glance. Firstly, this figure justifies that BNCs perform much better than C4.5, which was one of the chosen models in previous relevant papers dealing with imbalanced datasets. It is remarkable how C4.5 reaches 0.5 value (the worst result) for so many datasets. In some cases (angles from 0 to $\frac{\pi}{4}$ radians), it has slightly better results, but the general comparison indicates this is clearly the worst model, since all the other circular lines mostly cover it around if we look at the whole graph. With respect to the BNCs here plotted, there is not a clear winner, in some areas NBG seems to win, but in other parts it is clearly im-

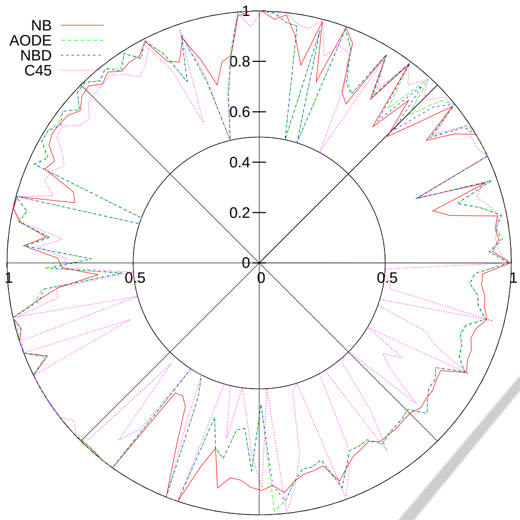


Figure 4: Radial plot, each angle represents a distinct dataset, from those listed in Table 1.

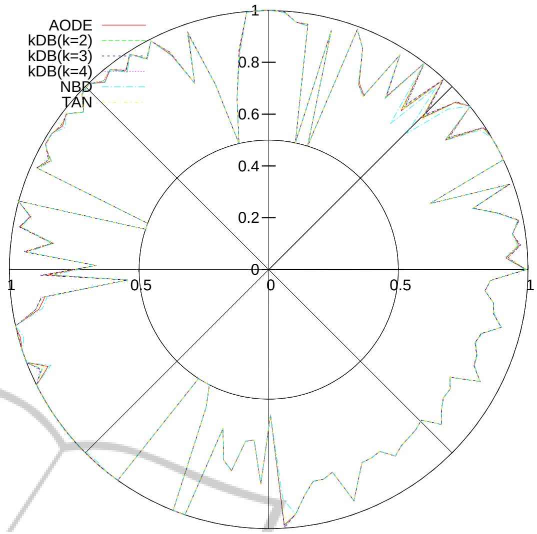


Figure 5: Radial plot comparing semi-Naive classifiers.

proved by AODE and its own discrete version, NBD.

We wanted to extend the comparison to more complex BNCs, that allow relationships among attributes. We originally chose AODE, as literature shows it as the best semi-Naive Bayes classifier (Webb et al., 2005). However, this test seemed to be usefulness to discard or not other models, which can catch more dependencies but which are also slower to learn. We can appreciate in Figure 6 that the differences when evaluating the chosen semi-Naive models (AODE, kDB with k $\in \{2,3,4\}$ and TAN) are almost insignificant. The only area where we can perceive some differences is in some datasets between π and $\frac{15\pi}{4}$, which means less than 6 datasets, 5% of the total sample. Which is the explanation for this result? To our view, including more complexity, in the form of more dependencies, does not provide better results because of the imbalance class problems. It has sense that simpler models will perform better, they provide similar AUC values but simple anf faster-to-learn models. That is the reason why we will select, for our experiments only AODE, and kDB with low values for k, 2 and 3. On the other hand, we can see how Naive Bayes is sometimes outperformed by all the other BNCs, this is due to the fact that the conditional independence *naively* assumed do not usually holds in real problems.

4 OUR PROPOSAL: IMPROVING IR WITH RE-SAMPLING METHODS

We propose a general method that could be applied to any dataset enhancing the overall performance of the classification task. It is clear that IR affects classification, yet not in a linear way, since the problems arrive when the number of instances for each class are clearly imbalanced. So, we try to re-balanced datasets and see how this affects.

4.1 Smoted Re-balance

When dealing with the imbalanced class problem, we can use data-level solutions or algorithm-level. We are going to use the most widely known method in the first direction, called generally re-sampling. One technique could be under-sampling the majority class instances and on the opposite side we can over-sample the minority class cases. We chose this second possibility, since the real datasets we have available do not present many cases.

4.2 Description of the Experiment

We performed an experiment with the purpose of analysing how re-balancing the dataset can improve classification. For every dataset, we have its initial IR, that we note as IR_0. Then, we apply SMOTE (Chawla et al., 2002) to get a better balance (then, smaller), using $IR' = IR_1, \cdots, IR_4$, since we have tested with values from 1 to 4. There is a key point here, when using SMOTE we use 5-CV, so that we only apply oversampling to those instances in the training set (the 4-folds that correspond), but we test on the original fold, where obviously SMOTE can not be applied so that we can report fair evaluation. When applying SMOTE we have selected the default value (5) for the number of nearest neighbours for interpolating values. Tests indicated this is the most appropriate value.

SMOTE also needs to know the percentage of samples belonging to the minority class to be created. Since we want to compensate IR' to produce distinct values, this "% of instances to-add" can be obtained as shown in Equation 2. For example, suppose we have a dataset with 1200 cases for the majority class (#*M*) and 100 for the minority class (#*m*), that yields $IR_0 = 12$. If we want to oversample the minority class cases so that we reach an $IR' = 4$, we obtain 100 $\times(\frac{1200}{4\times100} - 1)$ = 200. That will produce 300 (original 100 + generated 200) cases for the minority class, and $IR' = 1200/300 = 4$, which was our aimed *smoted* IR.

$$perc = 100 \times \left(\frac{\#M}{IR' \times \#m} - 1 \right) \tag{2}$$

So the experiment we have done in this stage can be summarised as below:

$IR_s[\,] \leftarrow \{1,2,3,4\}$
$Classifiers[\,] \leftarrow \{NBD, AODE, 2DB, 3DB\}$
$Data[\,] \leftarrow \{d_i, \cdots, d_n\}$ ▷ datasets in Table 1.
for $i \leftarrow 1,\cdots,4$ **do**
  $m \leftarrow Classifiers[i]$ ▷ *m* is a model
  for $j \leftarrow 1,\cdots,n$ **do**
    $d \leftarrow Data[j]$
    $auc[j][0] \leftarrow$ CVORIGINAL(m,d)
    for $k \leftarrow 1,\cdots,4$ **do**
      p ← OBTAINPERCFROMIR(IR_s[k])
      $auc[j][k] \leftarrow$ CVSMOTEDTRFOLDS(m,d,p)
    end for
  end for
end for

The output of this experiment is shown per classifier, just to see if this smoted re-balance produce better results and to which extend these results are relative to IR'.

In the light of the results (Figure 6), this smoted technique produce incredibly good results for all classifiers. For space problems, we do not show all the classifiers, but we remark that the same tendency repeats for every classifier. There seem to be a particular dataset where using IR' 3 and 4, performs slightly worse than using the original IR. This is less than 1% of our datasets. Furthermore, when IR' is 1 or 2, we obtain much better results in all the datasets.

4.3 Conclusions from the Results

From the previous results, it is evident that the use of smoted re-balancing provides incredibly good results for all datasets. Experimentation seems to recommend especially IR' as 1 or 2. The decision about this value can be taken depending on the importance of using less times (IR2 will create less new minority cases). However, time for small datasets is not important, on average (for all tested models), evaluating the four new IR values for one dataset takes around one third of a second, in a relatively old computer (Java on Intel(R) Core(TM)2 Duo CPU, 2.66GHz and 2GB for RAM), and AODE is faster than this average.

The most important conclusions from our exploration on BNCs in this representative (129) set of imbalanced datasets could be summarised as follows:

- All BNCs performed much better in imbalanced data than decision trees (C4.5), one of the tested models in previous papers.
- Among BNCs, Naive Bayes does not work bad, but the conditional independence assumption makes the model too simple and it provides lower performance for some datasets.
- AODE seems to provide the better results, since more complex semi-Naive Bayes, as kDB or TAN do not obtain better results, in terms of AUC.
- Our proposal, based on smoted re-balancing, outperforms the original results very significantly.

5 FINAL DISCUSSION

In this work we have performed a study on how Imbalance Ratio affects when using Bayesian Networks models in classifying, when the dataset from which the model is learned presents imbalance between classes. As seen in the introduction, this problem is quite frequent in certain fields. We have seen that this relation IR with performance is not trivial, and cannot be caught with a linear function, as many other factors, intrinsic in the dataset can affect, as discussed in (Lopez et al., 2013). However, working with this IR in combination with over-sampling techniques, as SMOTE, can produce an incredible gain in the classification assessment. So, the two main contributions of this paper is the use of BNCs for imbalanced datasets together with analysis of their performance in a public benchmark with 129 datasets, and the proposal of a new algorithm in which we recommend to use AODE as classifier and IR' as 1 or 2, but which could be parametrised with other BNC and value and will work properly depending on the user preferences.

As future work there are two possible lines: investigate sophisticated boosting techniques focused on imbalanced dataset that use cost functions (Sun et al., 2007), probably with some adaptation to BNCs, and also, the use of other oversampling methods distinct

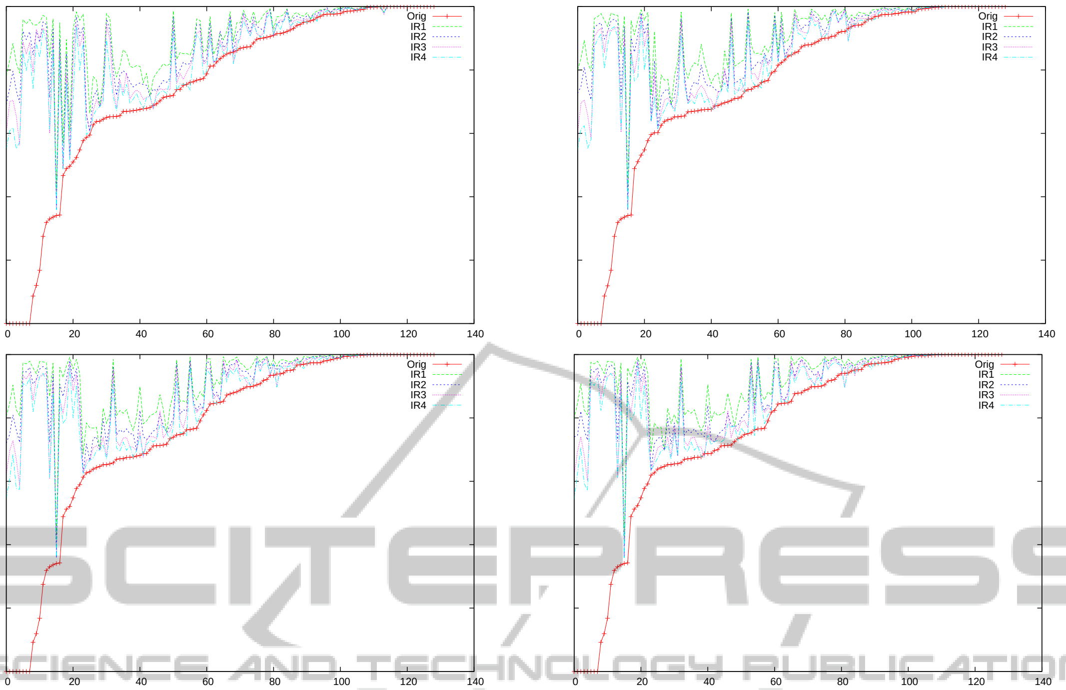


Figure 6: For each classifier, datasets are set in the x-axis in an ascending order with respect to original AUC value. Top left: NB Discrete; Top right: AODE; Bottom left: 2DB; Bottom right: 3DB. y-axis range is 0.5 to 1.

from SMOTE. Finally, we aim at studying algorithms to learn BNCs tailored for imbalanced data. The extension to multi-class problem could also be useful.

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