Scheduling Problem in Call Centers with Uncertain Arrival Rates Forecasts

A Distributionally Robust Approach

Mathilde Excoffier[1], Céline Gicquel[1], Oualid Jouini[2] and Abdel Lisser[1]

[1]*Laboratoire de Recherche en Informatique - LRI, Université Paris-Sud, Orsay, France*

[2]*Ecole Centrale Paris - ECP, Châtenay-Malabry, France*

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Abstract: We focus on the staffing and shift-scheduling problem in call centers. We consider that the call arrival rates are subject to uncertainty and are following unknown continuous probability distributions. We assume that we only know the first and second moments of the distribution. We propose to model this stochastic optimization problem as a distributionally robust program with joint chance constraints. We consider a dynamic sharing out of the risk throughout the entire scheduling horizon. We propose a deterministic equivalent of the problem and solve linear approximations of the program to provide upper and lower bounds of the optimal solution. We applied our approach on a real-life instance and give numerical results.

1 INTRODUCTION

Call centers are the main interface between the firms and their clients: in the U.S. in 2002, call centers represent 70% of all business interactions (Brown et al., 2005). Whether it be for emergencies call centers or travel companies for example, the clients are to be answered within a very limited time. The Quality of Service is of prime importance in the management of call centers. In addition, the staff agents cost in call centers represents 60% to 80% of the total operating budget (Aksin et al., 2007). Thus firms have to propose a satisfying service while controling the cost of the manpower. The importance of this sector in the service economy and the practical inherent constraints of the scheduling problem make this problem a topical issue in Operations Research.

Practically, scheduling call centers consists in deciding how many agents handling the phone calls should be assigned to work in the forthcoming days or weeks. The goal is to minimize the manpower cost while respecting a chosen Quality of Service (QoS). In call centers, we usually consider the expected waiting time before being served, or the expected number of clients hanging up before being served, i.e. the abandonment rate, as a relevant measure of Quality of Service.

The standard model for this problem is based on forecasts of expected call arrival rates. These forecasts are computed from historical data giving the numbers of calls for the working time horizon. Since the quantity of calls vary strongly in time, the working horizon is split in small periods of time, usually 30-minute periods. Thus we obtain for each period an expected call arrival rate. Then we are able to compute the staff requirements for each period from the forecasts and an objective service level which represents the chosen Quality of Service. This computation is done with the well-known Erlang C model. Finally, the numbers of agents required for the whole working horizon are determined through an optimization program, using the previous period-by-period results.

The shift-scheduling problem presents some characteristics: first, we need to split the horizon into small periods of time in order to be able to represent the variation of rate with the best precision possible. This leads to an increasing number of variables. Second, since we are considering human agents we have to respect several manpower constraints. Thus, agents have to follow established shifts and can not work only for a few hours. Moreover, the solution of the problem represents humans, so it has to be integers. Finally, call arrival rates are forecasts and thus subject to uncertainty. Thus, the final numbers of agents computed is subject to uncertainty as well.

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This should be considered in order to propose a valid model.

Typical call centers models consider a queuing system for which the arrival process is Poisson with known mean arrival rates (Gans et al., 2003). Since the data of the problem are forecasts of arrival rates, the accuracy of this deterministic approach is limited. Indeed, these estimations of mean arrival rates may differ from the reality. Uncertainty is taken into account in several papers, with various approaches. Several published works consider that input parameters of the optimization program follow known distributions. Some deal with continuous distributions (Excoffier et al., 2014), discrete distributions (Luedtke et al., 2007) or discretizations of a continuous distribution into several possible scenarios (Robbins and Harrison, 2010), (Liao et al., 2012) or (Gans et al., 2012). However it can be difficult to estimate which distribution is appropriate. (Liao et al., 2013) for call centers and (Calafiore and El Ghaoui, 2006) for general problems consider a distributionally robust approach. The problem deals with minimizing the final cost considering the most unfavorable distribution of a family of distributions whose parameters are the given mean and variance. In (Liao et al., 2013), the χ^2 statistic is used to build the class of possibles discrete distributions, with a confidence set around the estimated values. (Calafiore and El Ghaoui, 2006) consider the set of radial distributions to characterise the uncertainty region, but do not solve the final optimization program for this set. Moreover they do not focus on a specific problem and do not consider integer variables.

In the optimization program, we need to take into account and manage the risk of not respecting the objective service level. (Liao et al., 2012) and (Robbins and Harrison, 2010) choose to penalize the non respect of the objective service level with a penalty cost in the objective function of the optimization program. (Gurvich et al., 2010) and (Excoffier et al., 2014) use a chance-constrained model, in which the constraints are probabilities to be respected with the given risk level. (Gurvich et al., 2010) focus on the staffing problem but not the scheduling problem, and consider only one period of time.

The contributions of this paper are the following: first we model our problem with uncertain mean arrival rates and a joint chance-constrained mixed-integer linear program. This approach corresponds well with the real requirements of the scheduling problem in call centers. Indeed, forecasts are a useful indication of what can happen in reality but can not be considered as enough. This approach is in contrast with most previous publications whose risk management rely on a penalty cost. This penality can be difficult to estimate.

Second we consider the risk level on the whole horizon of study instead of period by period with joint chance constraints. It enables to control the Quality of Service on the whole horizon of study, which is a critical benefit. Managers demand to have a weekly vision of the call center, and not only for short periods of time. Moreover we propose a flexible sharing out of the risk through the periods in order to guarantee minimization of the costs. As far as we know, this consideration is only used in (Excoffier et al., 2014) for the staffing and scheduling problem in call centers.

Finally we focus on a distributionally robust approach, considering that we only know the first two moments of the continuous probability distributions. Since we do not know in reality what is the adequate distribution, we investigate a way of solving the problem for unknown distributions. Unlike other proposed distributionally robust approaches ((Liao et al., 2013) in particular), we consider continuous distributions instead of discrete distributions. This allows to a better representation of the reality. Moreover, (Liao et al., 2013) focus on the uncertainty on the parameters of a known gamma distribution whereas we focus on the uncertainty of the distribution with known parameters.

The rest of the paper is organized as follows. In Section 2 we present the formulation of the problem. At first, we propose the staffing model used for computing the useful data of the scheduling problem. Then we introduce the distributionally robust chance-constrained approach. In Section 3 we propose computations leading to the deterministic equivalent of the distributionally robust program. We also present the piecewise linear approximations leading to the final programs whose solutions are lower and upper bounds of the initial optimal solution. Section 4 gives an illustrative example of our approach. Finally in Section 5 we give numerical results.

2 PROBLEM FORMULATION

2.1 Staffing Model

The shift-scheduling problem is induced by the fact that we consider whole number of human agents working according to manpower constraints. We have to consider that agents can not come and work for only a few hours and need to follow working shifts of full-time or part-time jobs. These shifts are made up of working hours and breaks, for lunch for example. The problem is then to decide how many working agents need to be assigned to each shift in the call center in order to respect a choosen objective service level. This computation uses data of calls arrival rates.

As previously explained, since arrival rates vary strongly in time, the horizon is split into T small periods, typically 15 or 30 minutes. For each small period of time t, forecasts are computed from historical data of numbers of calls. Based on these forecasts of number of incoming calls, we can compute the agents requirements at each period of time t.

In that goal we use the Erlang C model, (Gans et al., 2003). At each period of time t we consider the call center as a queuing system in stationary state (Gross et al., 2008). This is a $M_t/M/N_t$ queue, where the customer arrival process is Poisson with rate λ and the services times are independent and exponentially distributed with rate μ. The number of servers, i.e. number of agents of our problem, is denoted by N_t for the period t. The queue is assumed to have an infinite capacity, with a First Come-First Served (FCFS) discipline of service.

In our problem we consider the average waiting time as the Quality of Service. The Erlang C model gives the function of Average Speed of Answer (ASA). This function gives the expected waiting time according to the parameters of the queue: the service rate μ, the arrival rate λ and the number of servers N. The ASA function is the following (see (Gans et al., 2003) or (Brown et al., 2005)):

$$ASA(N,\lambda,\mu) = \mathbb{E}[Wait] \tag{1}$$

$$= P\{Wait > 0\}\mathbb{E}[Wait|Wait > 0] \tag{2}$$

$$= \frac{1}{N*\mu*(1-\frac{\lambda}{N*\mu})\left(1+(1-\frac{\lambda}{N*\mu})\sum_{m=0}^{N-1}\frac{N!}{m!}(\frac{\mu}{\lambda})^{N-m}\right)}$$

Note In this relation λ and μ are real numbers whereas N is an integer. In the studied problem, the objective service level is a maximum ASA value. We denote ASA^* this value. As in (Excoffier et al., 2014), we will introduce a function of λ, μ and ASA^* giving the required number of agents, which will be here considered as a real value.

2.2 Computation of Staffing Requirements

The previous ASA (Average Speed of Answer) function is used in an algorithm to compute the minimum number of agents required to reach the targeted ASA^*, given λ and μ.

The procedure is the following:

- We compute $ASA(N,\lambda,\mu)$ and $ASA(N+1,\lambda,\mu)$ such that

  $ASA(N,\lambda,\mu) \geqslant ASA^*$ and $ASA(N+1,\lambda,\mu) < ASA^*$

  We denote $ASA(N,\lambda,\mu)$ as $ASA_{N,\lambda}$.
- The real value of N is computed by a linearization in the $[ASA_{N,\lambda};ASA_{N+1,\lambda}]$ segment. The affine function is:

  $$\begin{aligned} ASA^* =&(ASA_{N+1,\lambda} - ASA_{N,\lambda}) * b \\ &+ (N+1) * ASA_{N,\lambda} - N * ASA_{N+1,\lambda} \end{aligned}$$

  and b is the real value of required agents we are looking for. □

For each period, this algorithm gives us the requirement value b as a function of λ.

$$b = \frac{ASA^* + N * ASA_{N+1,\lambda} - (N+1) * ASA_{N,\lambda}}{ASA_{N+1,\lambda} - ASA_{N,\lambda}} \tag{3}$$

Note For a simpler reading we chose the $ASA_{N,\lambda}$ notation instead of $ASA_{N,\lambda,\mu}$.

Finally we are able to compute the number of agents b required to respect the objective service level ASA^* when the clients arrive at the rate λ and they are served at the rate μ.

The values of b obtained represent estimations of agents requirements. Since our computed results are subject to uncertainty, we consider that they are in fact the means of random variables of requirements. By considering real values rather than integers through the previous algorithm, we ensure a better precision in the uncertainty management. We assume that these variables are independent.

In next section, we present the distributionally robust optimization program for solving the shift-scheduling problem, considering the agents numbers as random variables.

2.3 Distributionally Robust Model

We consider the following chance-constrained shift-scheduling problem:

$$\begin{aligned} &\min c^t x \\ &\text{s.t. } P\{Ax \geqslant b\} \geqslant 1-\varepsilon \\ &\quad x \in (\mathbb{Z}^+)^S, \varepsilon \in]0;1] \end{aligned} \tag{4}$$

where c is the cost vector, x is the agents vector, b is the vector of agents requirements b_t and A is the shifts matrix. The matrix $A = (a_{i,j})_{[\![1;T]\!]\times[\![1;S]\!]}$ is the matrix of S shifts of T periods. The term $a_{i,j}$ is equal to 1 if agents are working during period i according to shift j and 0 otherwise. The agents vector x is composed of S variables ; x_i is the number of agents assigned to the shift i. Thus there are T constraints, each for one period of time, and the product Ax represents the number of assigned agents for each of these periods. Finally, ε is the risk we allow us to take. Then $1-\varepsilon$ is the confidence interval.

This program minimizes the manpower cost of working agents while respecting the chosen objective service level for the horizon time under the risk level ε. The objective service level is the value ASA^* described in previous section. Thus we want to guarantee a maximum expected waiting time for the client while controlling the costs.

The chance constraints approach is chosen in order to deal with random variables. We want to guarantee that the probability that we staffed enough agents is higher than the given proportion $1-\varepsilon$. Then, our program deals with joint chance constraints. Indeed, instead of considering individual constraints and one risk level for each period, we set the risk for the whole horizon time.

We assume that we do not know exactly what distributions the random variables b_t are following, but we know the means $\bar{b}_t$ and the variances σ_t^2. We focus here on the distributionally robust approach: we do not know which distribution is the correct distribution but we want to optimize our problem for all the possible distributions and thus the most unfavourable distribution with known expected value and variance. We note $b \sim (\bar{b}, \sigma^2)$ the vector of variables b_t, with means $\bar{b}_t$ and variances σ_t^2.

Then, we consider the following program:

$$\begin{aligned} &\min c^t x \\ &\text{s.t.} \inf_{b\sim(\bar{b},\sigma^2)} P\{Ax \geqslant b\} \geqslant 1-\varepsilon \\ &\quad x \in (\mathbb{Z}^+)^S, \varepsilon \in]0;1] \end{aligned} \tag{5}$$

Since we assume that the random variables are independent, we can split the constraint into T independent constraints. We propose here to dynamically share out the risk through the periods. Indeed, instead of choosing how to share out the risk through the periods before the optimization process, we decide that the proportion for each period will be a variable of the optimization program. This flexibility leads to cheaper solutions and are still satisfactory in term of robustness (Excoffier et al., 2014).

We introduce the variables y_t which represent the proportion of risk allocated to each period t:

$$\begin{aligned} &\min c^t x \\ &\text{s.t.} \ \forall t \in [\![1;T]\!], \inf_{b_t\sim(\bar{b}_t,\sigma_t^2)} P\{A_t x \geqslant b_t\} \geqslant (1-\varepsilon)^{y_t} \\ &\quad \sum_{t=1}^{T} y_t = 1 \\ &\quad x \in (\mathbb{Z}^+)^S, \varepsilon \in]0;1], \ \forall t \in [\![1;T]\!], \ y_t \in]0;1] \end{aligned} \tag{6}$$

The sum of the variables y_t should be equal to 1 in order to reach the chosen risk level. In the next subsection, we give a deterministic equivalent of the chance constraints of the problem.

3 DETERMINISTIC EQUIVALENT PROBLEM

3.1 Dealing with the Constraints

Let us focus on the expression of one constraint. For a given period t, we have:

$$\inf_{b_t\sim(\bar{b}_t,\sigma_t^2)} P\{A_t x \geqslant b_t\} \geqslant (1-\varepsilon)^{y_t} \tag{7}$$

Using (Bertsimas and Popescu, 1998) (Prop.1), we obtain the following result:

$$\sup_{b_t\sim(\bar{b}_t,\sigma_t^2)} P\{A_t x < b_t\} = \begin{cases} \frac{\sigma_t^2}{\sigma_t^2+(A_t x-\bar{b}_t)^2} & \text{if } A_t x \geqslant \bar{b}_t \\ 1 & \text{otherwise} \end{cases} \tag{8}$$

Then, considering

$$\inf_{b_t\sim(\bar{b}_t,\sigma_t^2)} P\{A_t x \geqslant b_t\} = 1 - \sup_{b_t\sim(\bar{b}_t,\sigma_t^2)} P\{A_t x < b_t\}$$

The constraint (7) is respected if and only if

$$A_t x - \bar{b}_t \geqslant 0, \ \frac{(A_t x-\bar{b}_t)^2}{\sigma_t^2+(A_t x-\bar{b}_t)^2} \geqslant (1-\varepsilon)^{y_t}$$

Then we can give an equivalent to the constraint:

$$\begin{aligned} &\inf_{b_t\sim(\bar{b}_t,\sigma_t^2)} P\{A_t x \geqslant b_t\} \geqslant (1-\varepsilon)^{y_t} \\ &\Leftrightarrow \frac{(A_t x-\bar{b}_t)^2}{\sigma_t^2+(A_t x-\bar{b}_t)^2} \geqslant (1-\varepsilon)^{y_t} \\ &\Leftrightarrow \frac{(A_t x-\bar{b}_t)^2}{\sigma_t^2} \geqslant \frac{(1-\varepsilon)^{y_t}}{1-(1-\varepsilon)^{y_t}} \end{aligned}$$

We note $p = 1-\varepsilon$ and since $A_t x - \bar{b}_t \geqslant 0$, we have the result

$$\frac{A_t x - \bar{b}_t}{\sigma_t} \geqslant \sqrt{\frac{p^{y_t}}{1-p^{y_t}}}$$

We now have a deterministic equivalent of our distributionally robust chance constraints. Finally, we propose to linearize the Right-Hand Side of the constraints and obtain bounds of the optimal solution.

3.2 Linear Approximations

We focus here on giving an upper bound and a lower bound of the problem by considering linearizations of the Right-Hand Side (RHS). Let us consider the following function, with $\varepsilon \in]0;1]$ and $p = 1-\varepsilon$:

$$\begin{aligned} f :]0;1] &\to \mathbb{R}^+ \\ y &\mapsto \sqrt{\frac{p^y}{1-p^y}} \end{aligned} \tag{9}$$

By deriving this function twice, we prove that it is convex. The detail is in the Appendix. □

This result guarantees that the following linearizations are upper and lower bounds of the functions, that is to say, we propose linearizations that are always above or below the function's curve.

3.2.1 Piecewise Tangent Approximation

We give here a lower bound of $f : y \mapsto \sqrt{\frac{p^y}{1-p^y}}$. Since we proved the convexity of the function, we know that the piecewise tangent approximation is a lower bound.

Let us choose n points $y_j \in]0;1]$, $j \in [\![1;n]\!]$ be n points such that $y_1 < y_2 < \ldots < y_n$.

We denote $\hat{f}_{l,j}$ the piecewise tangent approximation around the point y_j (the subscript l stands for *lower*). We compute this linearization with a first-order Taylor series expansion around the n points:

$$\forall j \in [\![1;n]\!],$$

$$\hat{f}_{l,j}(y) = f(y_j) + (y - y_j) f'(y_j) \tag{10}$$

$$= f(y_j) + (y - y_j) f(y_j) \frac{\ln p}{2(1 - p^{y_j})}$$

$$= \delta_{l,j} * y + \alpha_{l,j} \tag{11}$$

Since we proved the convexity of the function, we describe the new constraint as

$$\hat{f}_l(y) = \max_{j \in [\![1;n]\!]} \{\hat{f}_{l,j}(y)\} \tag{12}$$

In the program, this condition is expressed in each period with one constraint for each approximation point. Finally, we give a lower bound of the solution of the initial problem with the following program:

$$\begin{aligned} &\min c^t x \\ &\text{s.t. } \forall t \in [\![1;T]\!], \forall j \in [\![1;n]\!], \\ &\quad \frac{A_t x - b_t}{\sigma_t} \geqslant \delta_{l,j} y_t + \alpha_{l,j} \\ &\quad \sum_{t=1}^{T} y_t = 1 \\ &\quad x \in (\mathbb{Z}^+)^S, \varepsilon \in]0;1], \forall t \in [\![1;T]\!], y_t \in]0;1]b \end{aligned} \tag{13}$$

where S is the number of shifts and T the number of periods.

3.2.2 Piecewise Linear Approximation

Similarly, we give here an upper bound of the function with a piecewise linear approximation.

Let us choose n points $y_j \in]0;1]$, $j \in [\![1;n]\!]$ be n points such that $y_1 < y_2 < \ldots < y_n$ and interpolate linearly between them.

We denote $\hat{f}_{u,j}$ the piecewise linear approximation between the points y_j and y_{j+1} (the subscript u stands for *upper*):

$$\forall j \in [\![1;n-1]\!],$$

$$\begin{aligned} \hat{f}_{u,j}(y) &= f(y_j) \\ &\quad + \frac{y - y_j}{y_{j+1} - y_j} * (f(y_{j+1}) - f(y_j)) \\ &= \delta_{u,j} * y + \alpha_{u,j} \end{aligned} \tag{14}$$

Again our new program respects the constraint

$$\hat{f}_u(y) = \max_{j \in [\![1;n]\!]} \{\hat{f}_{u,j}(y)\} \tag{15}$$

Finally, the following program gives an upper bound of our problem:

$$\begin{aligned} &\min c^t x \\ &\text{s.t. } \forall t \in [\![1;T]\!], \forall j \in [\![1;n-1]\!], \\ &\quad A_t x - b_t \geqslant (\delta_{u,j} y_t + \alpha_{u,j}) \sigma_t \\ &\quad \sum_{t=1}^{T} y_t = 1 \\ &\quad x \in (\mathbb{Z}^+)^S, \varepsilon \in]0;1], \forall t \in [\![1;T]\!], y_t \in]0;1] \end{aligned} \tag{16}$$

where S is the number of shifts and T the number of periods.

In this section we first proposed a deterministic equivalent to the initial distributionally robust stochastic problem. Therefore, the optimal solution of the deterministic program is the optimal solution of the initial program. We had to deal with a mixed-integer nonlinear program. Second, we provided close upper and lower bounds of the optimal solution by introducing piecewise tangent and linear approximations. This was possible because of the convexity of the constraints. This led to two mixed-integer linear programs whose number of integer and binary variables are not increased compared to the initial formulation. These two programs are easily computed with an optimization software (CPLEX for example). This enables to give bounds of the optimal solution of the initial complex problem.

Next section gives an example of the method to solve a scheduling problem in call centers.

4 ILLUSTRATIVE EXAMPLE

We consider here we want to schedule one day of 10 hours. We propose a simple matrix of 8 shifts with 1-hour periods.

Shift	1	2	3	4	5	6	7	8
08:00 - 09:00	1	0	0	1	0	0	1	0
09:00 - 10:00	1	1	0	1	1	0	1	1
10:00 - 11:00	1	1	1	1	1	1	1	1
11:00 - 12:00	1	1	1	0	1	1	0	1
12:00 - 13:00	0	1	1	1	0	1	0	0
13:00 - 14:00	1	0	1	1	1	0	1	0
14:00 - 15:00	1	1	0	1	1	1	1	1
15:00 - 16:00	1	1	1	1	1	1	1	1
16:00 - 17:00	0	1	1	0	1	1	1	1
17:00 - 18:00	0	0	1	0	0	1	0	0

Figure 1: Example of a simple shifts matrix.

For each period we consider a value of the call arrival rate. For one day, let us consider the following vector $\lambda = \{38;77;82;41;18;53;75;64;54;29\}$. The values of arrival rates for day follow a typical saisonnality.

The queue parameters are the following: the goal Average Speed of Answer $ASA^* = 1$ and the service rate $\mu = 1.5$.

Using the algorithm of Subsection 2.2, we are able to compute the number of staff requirements: $b = \{26;52;56;28;13;36;51;44;37;20\}$.

For each period, let us consider values of variances: $\sigma^2 = \{1;2;2;1;1;2;2;2;2;1\}$.

We then want to solve the distributionally robust program (6). For this example, we set the risk level at 10%: $\varepsilon = 0.10$. Previous section gives us upper and lower bounds of the problem. Then we consider the two programs (13) and (16).

We choose several interpolation points y_j. For each period of time, there is one constraint for each of the interpolation points.

Here is an illustration of the piecewise approximations of function f for :

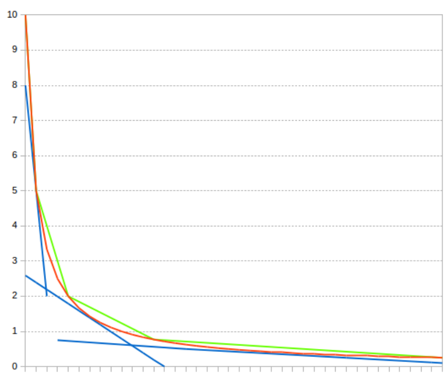

Figure 2: Piecewise linear approximations of function f.

The solutions of the programs give the staffings according to the possibles shifts. They are the given in Table 1.

Table 1: Result for upper and lower bounds.

Shift	Lower Bound	Upper Bound
1	3	3
2	0	0
3	18	13
4	15	15
5	0	0
6	7	13
7	21	20
Total Staff	84	85
Cost	81.14	82

The Cost Gap here is $CG = 0.011$. Finally we can compute Ax very easily if needed.

5 NUMERICAL EXPERIMENTS

5.1 Instance

In order to evaluate the quality and the robustness of our model, we applied our approach to instances based on data from a health insurance call center. This data provides forecasts for one week from Monday morning to Saturday midday (5 days of 10 hours and 1 half-day of 3.5 hours). The horizon is split in 30-minute periods. 24 differents shifts, from both full-time and part-time schedules, make up the shifts matrix. As we previously said in Section 2, we can standardize the service rate μ without loss of generality. We consider that all agents have the same hourly salary, thus the cost of one agent is proportional to the number of periods worked.

We computed the vectors of scheduled agents x_l and x_u for one week with the two programs (13) and (16) of the previous section, providing an upper bound and a lower bound of the optimal solution cost. We used 17 points for computing the piecewise tangent and linear approximations. We noticed that the order of magnitude of variables y_t is between 10^{-2} and 10^{-1}, thus we reduced the gap between the upper and lower bounds by gathering most of the points around this area.

We want to evaluate the quality of our solutions x_l and x_u. To this end we simulate possible realizations of arrival rates according to different distributions with the same data as previously. We consider different possible distributions: gamma distributions, uniform distribution, Pareto distribution, and varia-

Table 2: Results for different parameters.

Parameters				Results	
Variance range	λ range	ASA^*	Risk ε	Cost Gap	% Violations
$0.3-1$	$16-86$	1	15%	0.0045	$5-3$
$0.3-5$	$16-86$	1	15%	0.011	$1-0$
$0.1-1$	$4-20$	1	15%	0.027	$13-9$
$0.1-1$	$4-20$	1	10%	0.034	$7-4$
$2.5-9$	$4-20$	0.3	15%	0.052	$1-1$

tions of normal distributions (log-normal, folded normal).

We elaborate a scenario as following: for each period of time we simulate a call arrival rate according to one of the given probability distributions. Then we compute the number of effective required agents for each period. A scenario covers requirements for the whole time horizon. Finally we compare these values of requirements with our solutions of the problem (lower solution x_l and upper solution x_u). A scenario is considered as violated if at least in one period the scheduled solution (by x_u or x_l is not enough in comparison of what the realization requires.

We computed between 100 and 500 scenarios for each probability distributions. The percentage of violations gives us an idea of the robustness of our approach for several chosen distributions. The cost of the solutions gives us an idea of the quality of the minimization.

5.2 Results

In Table 2, we give the percentage of violated scenarios for various ranges of values of means and variances, and risk level. The queue parameters μ was set to 1 as it simply represents a multiplicity factor. The first column gives the range of values of the variances through the day. The second column gives the range of values of the means through the day, following a typical seasonality.

The value Cost Gap (CG) of the 5th column is given by the relative difference between the cost of the upper bound solution and the cost of the lower bound solution: $CG = \frac{c^t x_u - c^t x_l}{c^t x_l}$.

The last column gives the number of violated scenarios for the lower bound and for the upper bound.

In Table 2 we can notice that both upper and lower bound solutions respect the set risk level. The variations of the parameters show that the bigger the variances, the better the model. The distributionally robust model deals very well with increasing of variances. We notice that even if we allow 15% risk, only a few scenarios are violated when the variances are higher (second and last lines of Table 2). In these cases the call center is over-staffed and the given solutions seem too conservative. But it is important to remember that all the observations are based on simulations of only a few examples of distributions. These very low percentages only show that if the arrival rates λs follow in reality one of the studied distributions, it may be over-staffed. However the distributionally robust model indeed consists in taking all possible distributions with given mean and variance into account. Thus it may be possible to reach the maximum risk level with other particular distributions.

These results show that our approach is robust, considering the numbers of violations never exceed the risk level we set. The values of Cost Gap show that the two bounds are close enough to propose a very close solution to optimal solution.

We can notice that even if the solutions costs are very close, the number of violations is different between the upper solution and the lower solution. This is due to the fact that the distribution of the agents through the different shifts is different according to the programs.

Table 3 focuses on comparing results for different risk levels. The simulations were made with these parameters:

- $\mu = 1$
- λ follows a daily seasonality, varying between 4 calls/min and 21 calls/min
- σ^2 varies through the periods, between 0.25 and 1.

These parameters show well the performance of the model. Table 3 gives the costs of the two bound solutions and the Cost Gap. Like previously we ran 100 simulations and evaluated the number of violated scenarios, which is given in the last column of the table.

The first two columns of Table 3 gives the chosen parameters. Columns 3 and 4 gives the solution costs of the two programs and column 5 gives the Cost Gap. Finally, the two last columns give the number of violated scenarios for the two solutions.

Unsurprisingly, the cost of the solution increases when the risk level decreases. The Cost Gap seems

Table 3: Results for different risk levels.

Parameters		Results				
ASA^*	*Risk* ε	Lower Cost	Upper Cost	Cost Gap	% Violations Upper	% Violations Lower
5	15%	27	27.75	0.028	5	3
5	10%	29	30	0.034	2	1
5	05%	33.81	35.31	0.044	0	0
1	15%	27.5	28.5	0.036	7	4
1	10%	29.44	30.5	0.036	3	2
1	05%	34.31	35.88	0.046	1	1
0.3	15%	28.88	29.94	0.037	12	7
0.3	10%	31.12	32.19	0.034	5	2
0.3	05%	35.69	37.56	0.052	0	0

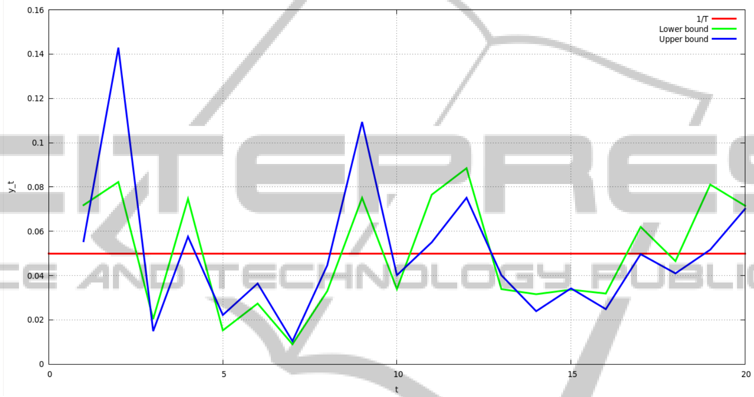

Figure 3: Sharing out of the risk through the day.

to remain in a small range, even if we notice a small increase of the gap when the risk is lowered.

We can also see an increasing of the cost when ASA^* (the objective Average Speed of Answer) decreases.

Like previously, the violations results show that our model respects the initial risk conditions, for both upper and lower solutions.

Figure 3 show the values of y_t variables through the horizon for the upper bound (in blue) and the lower bound (in green). The red line shows the equal division of the risk through the day. This figure brings out the interest of dynamically sharing out the risk: optimization of the variable y_t shows their value are different from the simple equal division through the periods. Thus our approach is more complicated but leads to cheaper solutions than a simpler approach with fixed risk levels.

6 CONCLUSION

This paper presents a distributionally robust approach for the staffing and shift-scheduling problem arising in call center. We introduced the distributionally robust approach, considering that the call arrival rates are following an unknown continuous distributions. Moreover, instead of considering the risk level on a period-by-period basis, we decided to set this risk level for the whole horizon of study and thus consider a joint chance-constrained program. Then, we proposed a deterministic equivalent of the distributionally robust approach with a dynamic sharing out of the risk. We were thus able to propose solutions with reduced costs compared to other published approaches. Finally we gave lower and upper bound of the problem with piecewise linear approximations. Computational results show that both upper and lower solutions respect the objective risk level for a given set of continuous distributions. This shows that our approach proposes robust solutions. The Cost Gap was small enough to be able to bring out a valid solution for the initial problem, which is eventually useful for the managers.

In the simulations, we noticed that mainly the Pareto distribution and Gamma distribution are the ones with violated scenarios. The solutions of the model show that for other distributions, the call center may be over-staffed. Thus, we could study further the call center model in order to evaluate what are the interesting distributions to consider. This can lead, as an improvment for our work in the future, to the study of a given set of distributions, according to some conditions (in addition to the known mean and variance).

Moreover, we can focus on improving the queuing system model by considering another approach of the representation of the service level in order to have a closer representation to reality.

Another interesting future research would be to conduct a sensitivity analysis that accounts for the forecast bias.

Finally we made the assumption that periods of the day are independent. In reality, we can notice a daily correlation of the periods in a call center: busy periods may appear in an entire busy day and rarely alone. Conversely, light periods should lead to an entire light day. We can then consider that the effective arrival rates depend on a *busyness* factor, which represents this level of occupation of the day.

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APPENDIX

Here we give the proof of the convexity of

$$\begin{aligned} f :]0;1] \rightarrow & \quad \mathbb{R}^+ \\ y \mapsto & \quad \sqrt{\frac{p^y}{1-p^y}} \end{aligned} \tag{17}$$

with $p \in [0;1[$.

Function f is C^∞, so we can compute the second derivative of function f. We have first:

$$\begin{aligned} \frac{df}{dy} &= \frac{\frac{\ln p}{2} p^{\frac{y}{2}} (1-p^y)^{\frac{1}{2}} + \frac{\ln p}{2} p^y (1-p^y)^{-\frac{1}{2}} p^{\frac{y}{2}}}{1-p^y} \\ &= \frac{\ln(p)(1-p^y)^{-\frac{1}{2}} (p^{\frac{y}{2}}(1-p^y) + p^{\frac{3}{2}y})}{2(1-p^y)} \\ &= f(y) \frac{\ln p}{2(1-p^y)} \end{aligned}$$

Then,

$$\begin{aligned} \frac{d^2 f}{dy^2} &= \frac{\ln p}{2} \frac{f'(y)(1-p^y) + \ln(p) p^y f(y)}{(1-p^y)^2} \\ &= \frac{\ln^2(p)(1+2p^y)}{4(1-p^y)^2} f(y) \\ &= \frac{\ln^2(p)(1+2p^y)}{4(1-p^y)^2} \frac{p^{\frac{y}{2}}}{(1-p)^{\frac{y}{2}}} \end{aligned} \tag{18}$$

Since every term of the second derivative is positive, we conclude that $\frac{d^2 f}{dy^2}$ is positive and then, f is convex. $\square$