Inconsistency-based Ranking of Knowledge Bases

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Abstract: Inconsistencies are a usually undesirable feature of many kinds of data and knowledge. Measuring inconsistency is potentially useful to determine which parts of the data or of the knowledge base are conflicting. Several measures have been proposed to quantify such inconsistencies. However, one of the main problems lies in the difficulty to compare their underlying quality. Indeed, a highly inconsistent knowledge base with respect to a given inconsistency measure can be considered less inconsistent using another one. In this paper, we propose a new framework allowing us to partition a set of knowledge bases as a sequence of subsets according to a set of inconsistency measures, where the first element of the partition corresponds to the most inconsistent one. Then we discuss how finer ranking between knowledge bases can be derived from an original combination of existing measures. Finally, we extend our framework to provide some inconsistency measures obtained by combining existing ones.

1 INTRODUCTION

There is considerable evidence that conflicts are often inevitable in groups and organizations. Indeed, analyzing inconsistency is central to many domains of computer science and Artificial Intelligence (Bertossi et al., 2005). It has been widely studied due to its significant importance in many applications, including network intrusion detection (McAreavey et al., 2011), software specifications (Martinez et al., 2004), e-commerce protocols (Chen et al., 2004), belief merging (Qi et al., 2005), news reports (Hunter, 2006), integrity constraints (Grant and Hunter, 2006), requirements engineering (Martinez et al., 2004), databases (Martinez et al., 2007; Grant and Hunter, 2013), semantic web (Zhou et al., 2009), and multi-agents systems (Hunter et al., 2014).

In recent years, several logic-based models have been proposed to evaluate conflicts, including the maximal η-consistency (Knight, 2002), measures based on variables or via multi-valued models (Grant, 1978; Oller, 2004; Hunter, 2006; Ma et al., 2010; Xiao et al., 2010; Ma et al., 2011), n-consistency and n-probability (Doder et al., 2010), minimal inconsistent subsets based inconsistency measures (Hunter and Konieczny, 2008; Mu et al., 2011; Mu et al., 2012; Xiao and Ma, 2012), Shapley inconsistency value (Hunter and Konieczny, 2010), minimal proof based inconsistency measurement (Jabbour and Raddaoui, 2013), inconsistency degree based on partitioning knowledge bases (Jabbour et al., 2014a), and more recently inconsistency characterization using prime implicates (Jabbour et al., 2014c; Jabbour et al., 2014b).

There are different ways to categorize the proposed approaches. One way is to distinguish between syntax and semantics inconsistency measures. Semantic based approaches aim to compute the proportion of the language that is affected by the inconsistency, via for example paraconsistent semantics. Whilst, syntax based approaches are concerned with the minimal number of formulae that cause inconsistencies, often through minimal inconsistent subsets. Additionally, some basic properties (Hunter and Konieczny, 2010) such as *Monotony* (i.e., adding formulae to a knowledge base must not decrease its inconsistency value), are proposed to evaluate the quality of inconsistency measures.

However, using these different metrics, the inconsistency degree associated to a single formula or to the whole knowledge base, is evaluated differently. In other words, on the same set of inconsistent knowledge bases, syntax and semantics based approaches might lead to opposite conclusions. This observation suggests that finding a general inconsistency measure suitable for any knowledge base is clearly a hard and challenging task. Each measure allows us to derive some useful information about inconsistency. Usu-

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ally such information present some complementarities that can be exploited to finely derive a better inconsistency measure. As a summary, measuring inconsistency is clearly a multi-criteria based evaluation process.

Our main goal in the present paper is to exploit the strength and the complementarities of some proposed measures to better understand and quantify inconsistency. Our contribution includes a Pareto-based approach to decide which knowledge bases are dominating given a set of knowledge bases with respect to inconsistency. Then we discuss how finer ranking between knowledge bases can be obtained by combining existing measures.

The remainder of this paper is organized as follows. Some preliminary definitions and notations are given in the next section. In the section 3, we present our approach for comparing different knowledge bases using a well known social welfare measure, namely Pareto optimality. To our knowledge, this is the first attempt to evaluate knowledge bases in terms of its inconsistency degrees. Section 4 presents a new family of inconsistency measures, before concluding with some perspectives.

2 FORMAL DEFINITIONS

We assume, through this paper, a propositional language $\mathcal{L}$ built from a finite set of atoms $\mathcal{P}$ using classical logical connectives $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$. We will use letters such as p and q to denote propositional variables, and Greek letters like α and β to denote propositional formulae. The symbols $\top$ and $\bot$ denote tautology and contradiction, respectively.

A knowledge base K consists of a finite set of propositional formulae over $\mathcal{L}$. We denote by $Var(K)$ the set of variables occurring in K. For a set S, $|S|$ denotes its cardinality. Moreover, a knowledge base K is inconsistent if there is a formula α such that $K \vdash \alpha$ and $K \vdash \neg\alpha$, where $\vdash$ is the deduction in classical propositional logic.

If the knowledge base K is inconsistent, then one can define a minimal inconsistent subset of K as (1) an inconsistent subset M of K, such that (2) all of its proper subsets are satisfiable.

Definition 1 (MUS). *Let K be a knowledge base and $M \subseteq K$. M is a minimal unsatisfiable (inconsistent) subset (MUS) of K iff:*

1. $M \vdash \bot$
2. $\forall M' \subsetneq M$, $M' \nvdash \bot$

The set of all minimal unsatisfiable subsets of K is denoted $MUSes(K)$.

When a *MUS* is singleton, the single formula in it, is called a *self-contradictory formula*.

Let us now, define a dual concept of MUS, called maximal satisfiable subset (in short MSS).

Definition 2 (MSS). *Let K be a knowledge base and $M \subseteq K$. M is a maximal satisfiable (or consistent) subset (MSS) of K iff:*

1. $M \nvdash \bot$
2. $\forall \alpha \in K \setminus M$, $M \cup \{\alpha\} \vdash \bot$

For a given inconsistent knowledge base K, and $M \subset K$ an MSS, the subset $K \setminus M$ is usually called a minimal correction subset (in short MCS). Indeed, an MCS gives us the minimal subset of the knowledge base that when removed, we recover the consistency. There exists a strong relationships between MUSes and MCSs (Bailey and Stuckey, 2005; Liffiton and Sakallah, 2008). This relationship can be stated simply: The set of MUSes of a knowledge base K and the set of MCSes of K are "hitting set duals" of one another. Note that any MCS is the complement of some MSS, and vice versa.

When a knowledge base K is inconsistent the classical inference relation is trivialized, i.e., any formula and its negation can be inferred from K. To address this problem, several techniques have been developed to analyse inconsistency in terms of minimal inconsistent subsets and maximal consistent subsets. Indeed, an inconsistency measure assigns a nonnegative number to every knowledge base as its conflicting degree. In the same vein, many logic-based frameworks of inconsistency measures have been proposed. For instance, I_{MI} measure counts the number of minimal inconsistent subsets of the knowledge base. Also, I_C value counts the sum of the number of maximal consistent subsets together with the number of self-contradictory formulae, but 1 must be subtracted to make the measure equal to zero when the knowledge base is consistent. Furthermore, the I_P inconsistency measure counts the number of formulae in minimal inconsistent subsets of the knowledge base.

For semantic-based inconsistency measures, one can cite the inconsistency degrees defined using paraconsistent semantics. The set of truth values for 4-valued semantics (Arieli and Avron, 1998) contains four elements: *true, false, unknown* and *both*, written by t, f, N, B, respectively. The truth value N allows to express incompleteness of information. The four truth values together with the ordering $\preceq$ defined below form a lattice $\mathsf{FOUR} = (\{t, f, B, N\}, \preceq)$: $f \preceq N \preceq t, f \preceq B \preceq t, N \npreceq B, B \npreceq N$. The 4-valued semantics of connectives $\vee, \wedge$ are defined according to the upper and lower bounds of two elements based on the ordering $\preceq$, respectively, and the operator $\neg$ is defined as $\neg t = f, \neg f = t, \neg B = B$, and $\neg N = N$.

A 4-valued interpretation ρ is a *4-model* of a knowledge base K, denoted $\rho \models_4 K$, if for each formula $\phi \in K$, $\phi^\rho \in \{t, B\}$. Let ρ be an interpretation under 4-semantics. Then $\mathit{Conflict}(K,\rho) = \{p \in Var(K) \mid p^\rho = B\}$ is called the *conflicting set* of ρ with respect to K. Intuitively, in terms of size-wise minimality, the larger the size of the conflicting set in 4-models of K, the more inconsistent K is.

Definition 3 (ID_4). *The 4-semantics based inconsistency degrees are defined as:*

$$ID_4(K) = \min_{\rho \models_4 K} \frac{|Conflict(K,\rho)|}{|Var(K)|}.$$

Example 1. *Let* $K = \{p, \neg p \vee q, \neg q \vee r, \neg r, s \vee u\}$. *Consider two 4-valued models* ρ_1, *and* ρ_2 *of* K *defined as:* $p^{\rho_1} = t$, $q^{\rho_1} = B$, $r^{\rho_1} = f$, $s^{\rho_1} = t$, $u^{\rho_1} = N$; $p^{\rho_2} = B$, $q^{\rho_2} = B$, $r^{\rho_2} = B$, $s^{\rho_2} = t$, $u^{\rho_2} = N$. *So* $ID_4(K) = 1/5$.

Recently, in (Xiao and Ma, 2012) the authors prove that ID_4 is closed to the cardinality of minimal hitting sets H of the MUSes of K. Formally, $ID_4(K) =$

$$\frac{\min_H\{|H| \mid \forall U \in MUSes(K),\ Var(U) \cap H \neq \emptyset\}}{|Var(K)|}$$

3 TOWARDS INCONSISTENCY-BASED RANKING OF KNOWLEDGE BASES

There exists many contradictory evaluation of the inconsistency degree between existing inconsistency measures (Grant and Hunter, 2011). Indeed, as an example, by applying the two measures I_{MI} and ID_4 on the knowledge base $K = \{\neg x_1, x_1 \wedge x_2, x_1 \wedge x_3, \ldots, x_1 \wedge x_n\}$, we have $I_{MI}(K) = (n-1)$ and $ID_4(K) = \frac{1}{n}$. For large values of n, K is considered strongly inconsistent according to I_{MI} while with ID_4 this base is viewed as marginally inconsistent.

This example illustrates the problem behind finding a relevant measure to evaluate efficiently the inconsistency of a knowledge base. The answer to this issue appears challenging. One can find arguments in favor of both contradictory conclusions. Indeed, it is justified to attribute $\frac{1}{n}$ as an inconsistency value since all the conflicts involve a single variable. On the other hand, there exists $(n-1)$ minimal conflicting subsets in K which justify the $(n-1)$ value as an inconsistency estimation.

Beyond these conflicting conclusions, both measures are complementary as they show different aspects of the inconsistency. More generally, this illustrating example also raises the problem of comparing the inconsistency degree of several knowledge bases. In other words, how one can identify a relevant ordering of different knowledge bases according to their inconsistency degrees. The disagreement between I_{MI} and ID_4 measures tends to suggest that such ordering does not exist. However, all known measures agree on the fact that a knowledge base with sparse MUSes hypergraph induces a high inconsistency value. This observation allows us to claim that given a set of MUSes, the maximal inconsistency value is obtained when the MUSes are pairwise disjoint.

In the sequel, we face the problem of finding an inconsistency ordering over a finite set $S_{\mathcal{K}}$ of knowledge bases according to a finite set of inconsistency measures S_I.

As discussed before, two measures $(I_1, I_2) \in S_I \times S_I$ can lead to conflicting points of view about the inconsistency of a given knowledge base. Consequently, comparing the inconsistency degree of two knowledge bases deserve to be further investigated. This comparison is easier when the different knowledge bases present some structure-based similarities, for example, when all their associated MUSes are disconnected. However, for knowledge bases presenting different structures, such comparison is clearly problematic.

Next, we define a proposal to characterize the knowledge bases from $S_{\mathcal{K}}$ that dominate others and propose a combination to reach a better compromise between them.

Definition 4 (Dominance). *Let* K_1 *and* K_2 *be two knowledge bases and* I *an inconsistency measure. We say that* K_1 *dominates* K_2 *w.r.t.* I *(in short* $K_1 \succeq_I K_2$*) iff* $I(K_1) \geq I(K_2)$.

Let us now introduce the notions of pareto dominance and pareto optimality.

Definition 5 (Pareto Dominance). *Let* $S_{\mathcal{K}}$ *be a set of knowledge bases and* S_I *a set of inconsistency measures. A knowledge base* $K_1 \in S_{\mathcal{K}}$ *Pareto Dominates a knowledge base* $K_2 \in S_{\mathcal{K}}$ *(in short* $K_1 \succ_{S_I} K_2$*) with* $K_1 \neq K_2$ *iff* $\forall\, I \in S_I$, $K_1 \succeq_I K_2$ *and* $\exists\, J \in S_I$, $K_1 \succ_J K_2$.

A knowledge base is Pareto optimal if it is not Pareto dominated by any other knowledge base w.r.t. a given set of measures. Formally:

Definition 6 (Pareto Optimality). *A knowledge base* $K_1 \in S_{\mathcal{K}}$ *is Pareto optimal (or Pareto efficient) w.r.t.* $S_{\mathcal{K}}$ *and* S_I *if there is no other knowledge base* $K_2 \in S_{\mathcal{K}}$ *s.t.* $K_1 \succ_{S_I} K_2$.

The figure 1 illustrates the Pareto front, when $S_I = \{I_{MI}, ID_4\}$. Each dot represents a knowledge base.

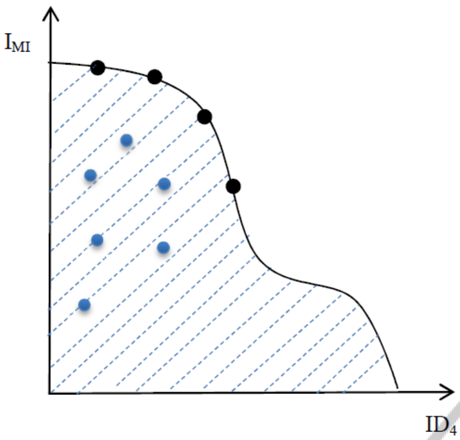

Figure 1: Pareto Optimality.

From the above definition, a set of knowledge bases $S_{\mathcal{K}}$ might contain several Pareto optimal knowledge bases with respect to a set of measures S_I.

One way to compute an ordering over the set of knowledge bases $S_{\mathcal{K}}$, consists in iterating the process of computing the pareto optimality to build a sequence $\mathcal{P} = \langle S^1_{\mathcal{K}}, \ldots, S^n_{\mathcal{K}} \rangle$, where each $S^i_{\mathcal{K}}$ corresponds to the set of Pareto optimal knowledge bases of $S_{\mathcal{K}} \setminus (S^1_{\mathcal{K}} \cup \ldots \cup S^{i-1}_{\mathcal{K}})$. This process allows us to compute a partition $\mathcal{P}$ of the set of knowledge bases $S_{\mathcal{K}}$. However, this approach does not allow us to derive a total ordering over all the individual knowledge base of $S_{\mathcal{K}}$, since the elements of $S^i_{\mathcal{K}}$ are not comparable. This can be seen as a form of stratification of the set of knowledge bases.

Nevertheless, considering all the measures S_I at the same time, does not allow us to derive a total ordering of the individual knowledge bases of $S_{\mathcal{K}}$. To allow a finer discrimination between all the individual knowledge bases and particularly on those belonging to the same element of the partition $S^i_{\mathcal{K}}$, we propose in the second step to combine several measures to better discriminate between them.

Indeed, imagine we have a set $S_{\mathcal{K}} = \{K_1, \ldots, K_m\}$ of knowledge bases and two inconsistency measures I and I'. According to I and I', suppose that $S_{\mathcal{K}}$ can be ordered as $\sigma(K_1) \preceq \ldots \preceq \sigma(K_m)$ and $\sigma'(K_1) \preceq \ldots \preceq \sigma'(K_m)$ where σ and σ' are bijections from $S_{\mathcal{K}}$ to $S_{\mathcal{K}}$. As we discussed above, the different measures are usually complementary as each one identifies some aspects of the inconsistency. An issue to benefit from such complementarities is to combine such measures to obtain a more relevant and more discriminating measure.

Indeed, a new combined measure can be defined by aggregating the different measures of $S_{\mathcal{K}}$. One can consider the average or their product as an aggregate function. Such derived measure can be used to further refine the partition. It can be seen as a way to tie-break between the knowledge bases belonging to the same element of the partition $\mathcal{P}$.

4 COMBINING INCONSISTENCY MEASURES

As shown before, while different inconsistency measures, based on semantics or syntax, provide an important way of evaluating inconsistency, they do have some limitations. Indeed, several measures show different aspects of the inconsistency of an inconsistent knowledge base.

By taking into account these different aspects, it is possible to provide more fine grained criteria for evaluating the conflict of knowledge bases.

Let us explore such possible combination of existing inconsistency degrees as follows.

Definition 7. *Let K be a knowledge base. We define* $I_{comb1}(K) = |H_{min}(K)| \times |MUSes(K)|$

Definition 7 define a new measure that aggregate the ones considering either the minimal hitting set and the one based on number of MUSes.

Proposition 1. *I_{comb1} is monotonic.*

Proof. direct since a knowledge base augmented with a new formula have a higher minimal hitting set and a higher number of MuSes. □

While fixing the number of MUSes, the knowledge bases are compared to the size of their minimal hitting set.

The second alternative measure can be defined by substituting the multiplication operator in Definition 7 with the sum operator leading to Definition 8.

Definition 8. *Let K be a knowledge base. We define* $I_{comb2}(K) = \frac{(|H_{min}(K)| + |MUSes(K)|)}{2}$

I_{comb2} consider the average between the minimal hitting set and the number of MUSes.

The last alternative is based only on the hitting sets. This is stated in Definition 9.

Definition 9. *Let K be a knowledge base. We define* $I_{comb3}(K) = \frac{(|H_{min}(K)| + |H_{max}(K)|)}{2}$

Let us motivate this last measure. Inconsistency are often linked to the cost of recovering consistency. Hitting set of minimum size often represents this cost. Clearly this vision is optimistic. In contract, the hitting set of maximum size $H_{max}(K)$ is the worst case allowing to recover consistency. I_{comb3} allows us to consider a compromise by summing up this two bounds and dividing by 2 to consider their average.

Contrary to I_{comb2} that combines $H_{min}(K)$ and $MUSes(K)$, I_{comb3} considers that new MUSes that do not involve the implication of new formulas in the inconsistency have the same inconsistency.

Proposition 2. *I_{comb1}, I_{comb2}, and I_{comb3} are monotonic*

Proof. Adding new formula to a knowledge base increases both its minimal hitting, its maximal one and the number of MUSes. □

The set of proposed measure can be ranked as follows:

$$I_{comb3} \leq I_{comb2} \leq I_{comb1}$$

Note that for all the considered measures and when the number of MUSes is fixed, knowledges bases with disjoint MUSes are associated with higher inconsistency value.

Example 2. *Let us reconsider again the knowledge base $K = \{\neg x_1, x_1 \wedge x_2, x_1 \wedge x_3, \ldots, x_1 \wedge x_n\}$. We have: $I_{comb1}(K) = n$, $I_{comb2}(K) = \frac{n+1}{2}$, and $I_{comb3} = \frac{n}{2}$*

Now in order to have a fair comparison, let us consider the case of the knowledge base $K = \{x_1, \neg x_1, x_2, \neg x_2, \ldots, x_n, \neg x_n\}$. We have : $I_{comb1}(K) = n^2$, $I_{comb2}(K) = n$, and $I_{comb3} = n$. Indeed, in this particular case, the size of the hitting set and the number of MUSes are the same. Suppose now we augmented K with the following set $K' = \{\neg x_1 \vee \neg x_2, \neg x_2 \vee \neg x_3, \ldots, \neg x_{n-1} \vee \neg x_n\}$. We have now $2n-1$ MUSes and the minimal hitting set remains the same i.e., breaking the MUSes of K allows in the same time to break the ones of K'.

We have: $I_{comb1}(K) = (2n-1) \times n$, $I_{comb2}(K) = \frac{3n}{2}$, and $I_{comb3} = n$

As we can see, I_{comb1}, and I_{comb2} are more discriminative compared to I_{comb3}.

Let us finally consider a case of combined semantic and syntactic measures. To show the intuition behind its introduction, let us consider the following knowledge base $K_1 = \{x_1 \vee \ldots \vee x_n, \neg x_1 \wedge y, \ldots \neg x_n \wedge y_1, \ldots, \neg x_1 \wedge y_n, \ldots, \neg x_n \wedge y_n\}$ and $K_2 = \{x_1 \vee \ldots \vee x_n, \neg x_1 \wedge y, \ldots \neg x_n \wedge y_1, \ldots, \neg x_1 \wedge y_n, \ldots, \neg x_n \wedge y_n\}$.

K_1 contains n^2+1 formulas leading to n^n MUSes. While n takes higher values, reasoning on MUSes becomes harder. Let us notice that this formula contains $(2n+1)$ variables. So, clearly from semantic point of view, conflicts concern the $(2n+1)$ variables. Then, combining semantic measures should be done with syntactic measures not leading to exponential value.

Definition 10. *Let K be a knowledge base. We define*

$$I_{comb4}(K) = \frac{ID_4(K) \times |Var(K)|}{max\{|M|,\, M \in MSSes(K)\}}$$

$I_{comb4}(K)$ considers a high inconsistent knowledge base as the one maximizing the number of variables linking the MUSes while minimizing the maximum MSS. A similar version of$I_{comb4}(K)$ can be rewritten as

$$ID_4(K) \times |Var(K)| \times |H_{max}(K)|.$$

5 CONCLUSION

Several measures have been proposed to evaluate the inconsistency degree of a given knowledge base. In this paper, we identified the difficulty to compare their relevance or quality. A highly inconsistent knowledge base with respect to a given measure can be considered less inconsistent using another one. This issue induces even more difficulty to compare the conflict of different knowledge bases. To deal with such problems, we first proposed a new framework allowing us to partition a set of knowledge bases as a sequence of subsets according to a set of inconsistency measures, where the first element of the partition corresponds to the most inconsistent ones. We also pointed out that such partition sequence can be refined thanks to a combination of existing measures. As future works, we plan to study what is the adequate aggregation operator and what is the most relevant set S_I that can be considered.

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