

Identifying Strong Statistical Bias in the Local Structure of Metabolic Networks

*The Metabolic Network of *Saccharomyces Cerevisiae* as a Test Case*

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Abstract: The detection of strong statistical bias in metabolic networks is of much interest for highlighting potential selective preferences. However, previous approaches to this problem have relied on ambiguous representations of the coupling among chemical reactions or in physically unrealizable null models, which raise interpretation problems. Here we present an approach that avoids these problems. It relies in a bipartite-graph representation of chemical reactions, and it prompts a near-comprehensive examination of statistical bias in the relative frequencies of topologically related metabolic structures within a predefined scope. It also lends naturally to a comprehensive visualization of such statistical relationships. The approach was applied to the metabolic network of *Saccharomyces cerevisiae*, where it highlighted a preference for sparse local structures and flagged strong context-dependences of the reversibility of reactions and of the presence/absence of some types of reactions.

1 INTRODUCTION

The detection of over- or under-represented local structures (motifs and anti-motifs, respectively) in biological networks has attracted much interest as a way of detecting potential selective constraints (Milo et al., 2002, Aittokallio and Schwikowski, 2006, Barabasi and Oltvai, 2004). The implicit rationale is that over-representation with respect to expectation from a prescribed null model is likely to be a consequence of natural selection for maintenance of the motif, driven by functional advantages provided by its dynamic properties. However, it is hard to conceive physically realizable and biologically plausible null models of metabolic networks. Null models generated by the randomization procedures considered in previous publications (Shellman et al., 2013) are not physically realizable, as they violate atom conservation. The meaningfulness of statistical over-representation with respect to such unrealistic null models is therefore questionable (Artzy-Randrup et al., 2004). As a more reasonable alternative, Milo et

al. (Noor et al., 2010) consider randomly generated networks such that all the allowed reactions could in principle be catalyzed by known enzyme activities considered at the third level of the E.C. classification. However, the extent to which networks generated in this way are legitimate “no-selection” null models is debatable. Enzymes are products of natural selection and expensive to maintain. So, the fact that an enzyme exists to catalyze a given reaction is already an indication that such a reaction has selective advantages for at least some organisms. Further, this procedure is liable to knowledge bias.

The representation of metabolic connectivity structures in motif analysis can also be problematic. Previous analyses (Shellman et al., 2013) defined structures as graphs where each metabolite A is connected to another one B by a directed edge if there is a reaction converting A to B. However, such metabolite connectivity structures are ambiguous and difficult to interpret and relate to dynamics and function. This is so because the same structure may correspond to very different reaction structures.

Here we present an approach to systematically highlight and visualize strong statistic preferences in local metabolic connectivity structures without requiring an artificial null model. In order to avoid the ambiguity problems discussed in the previous paragraph, structures are represented as directed bipartite graphs, with reaction and metabolite nodes (Aittokallio and Schwikowski, 2006). The approach consists in first constructing a topological hierarchy of structures of increasing complexity. Then, comparisons among the frequencies of structures differing by a single structural element (one reactant or product of one reaction, a reaction connecting two metabolites, or the reversibility of one reaction) can be properly performed using such framework. We illustrate its application to the set of structures formed by 3 metabolites and 2 or 3 reactions in the metabolic network of *Saccharomyces cerevisiae*.

2 METHODS

2.1 Definitions

We denote by “topological structure”, abbreviatedly “structure”, a specific pattern of interconnections between the two types of nodes of the metabolic network: metabolites and reactions. Isomorphic representations obtained from each other by relabeling metabolites and/or reactions are considered as the same structure.

Two structures are said to be directly related if the most complex one can be obtained from the simplest one by addition of just one of the following structural components: (i) a reactant or product to a reaction; (ii) the reverse of a reaction; (iii) a third reaction converting one metabolite into one other.

The simplest of two directly related structures is denoted as “parent”, and the other, derived from it by the addition of a structural element, is referred as its “child”. Each structure may have multiple parents and children. We denote by “offspring” of a structure the set of all of its children, grand-children, etc.

We denote by “instance” any concrete realization of a given structure in a metabolic network, i.e., an actual set of metabolites and reactions connected as defined in the structure.

2.2 Scope

All the structures considered in the present paper are formed by three metabolites connected through two or three (possibly reversible) reactions. Further, the

third reaction must connect a reactant-product pair not connected by any of the other reactions (Figure 1). Reactions in *instances* of each structure may have additional reactants or products not represented in the structure, but reversibility in the structure implies reversibility of a corresponding reaction in the instances.

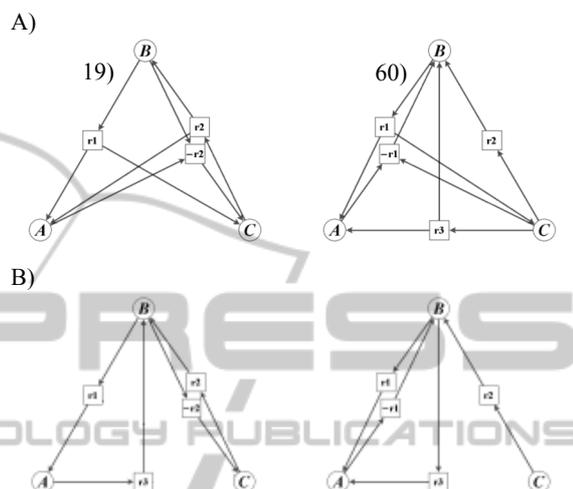


Figure 1: Examples of structures of three metabolites and two or three reactions: (A) considered in this work, (B) excluded.

2.3 Structure Enumeration

The three simplest structures include two reactions, each with a reactant and a product. All other structures were iteratively constructed from these three seed structures through the stepwise addition of any of the following three structural components: (i) a new reactant or product (from the structure) to a pre-existing reaction; (ii) the reverse of a pre-existing reaction; (iii) a third reaction converting one metabolite in the structure into another. At each step, structures were checked for isomorphism and inclusion in the structural scope defined in section 2.2.

The procedure above generates a layered network of topological relationships among structures (Figure 2). The nodes in this network are the 149 non-isomorphic topological structures, arranged in 8 generations, from the sparsest (*i.e.*, with the fewest structural components) structures to the densest. The edges join directly related structures as defined in section 2.1.

Importantly, the topology of this network is uniquely determined by the topological relationships of the structures and independent of the properties of concrete metabolic networks. It therefore provides a

convenient and sound reference to represent the local-topology characteristics of the metabolism of different microorganisms and assess their affinities and particularities through topological oriented statistical descriptors.

2.4 Instance Counting in a Metabolic Network

The approach was applied to a reconstruction (Forster et al., 2003) of the *S. cerevisiae* metabolic network, which includes 1058 reactions (335 reversible and 723 irreversible) and 991 metabolites. Each reversible reaction was replaced by a pair of unidirectional reactions, making a total of 1393 unidirectional reactions.

We determined the following statistics in this metabolic network.

2.4.1 Naked Frequency

We define the naked frequency of structure i , $f_N(i)$, as the number of instances matching the structure but not any of its offspring.

2.4.2 Embedded Frequency

We define the embedded frequency of structure i , $f_E(i)$, as the total number of instances of the structure, whether or not embedded in any of its offspring. The value of $f_E(i)$ is obtained by adding $f_N(i)$ and the naked frequencies of all of structure i 's offspring.

The embedding ratio of structure i into its child j , $E(i, j) = f_E(j)/f_E(i)$ measures the propensity of structure i to be embedded into its child j and respective offspring.

The independence ratio $R(i) = f_N(i)/f_E(i)$ measures the tendency of structure i to appear in its "naked" form, rather than embedded in any of its offspring structures.

2.4.3 Metabolite Coverage

We define the metabolite coverage of structure i , $C_{Met}(i)$, as the fraction of metabolites in the network that appear as nodes in all instances of structure i , considering only exact matches as for the computation of naked frequencies.

2.4.4 Children Information

We define the children information of structure n , $\Delta H(n)$, as (eq. 1):

$$\begin{aligned} \Delta H(n) &= H_{max}(n) - H(n) = \\ &= -\log_2\left(\frac{1}{N_{Ch}(n)}\right) \\ &- \left(-\sum_i f_N(i) \log_2 f_N(i)\right), \quad \forall i \in Ch(n) \end{aligned} \quad (1)$$

where $Ch(n)$ is the set of all the children of structure n . The value of $\Delta H(n)$ is null if all children are equally frequent, and high if the distribution of children frequencies is very uneven.

3 RESULTS AND DISCUSSION

Some of the statistics above were graphically represented in Figures 2 and 3. The analysis of these statistics highlights the following remarkable features about the local structure of the metabolic network:

1. Although most structures have instances in the network, 22 of the 149 structures do not (e.g., 76, 92, 110). The latter are usually very dense (Figure 2).
2. Sparse structures tend to be more frequent than denser ones (Figures 2, 3). This is expected for *embedded* frequencies because simpler structures can *a priori* be matched by more instances than more complex structures for combinatorial reasons. However, sparser structures also tend to have higher *naked* frequencies, which may highlight a selective preference. The avoidance of denser structures may stem from the fact that maintaining multiple enzymes for converting among similar substrates (other than metabolic currencies) would spend cellular biosynthetic resources without bringing significant advantages.

Most structures that have high R values are dense. This is in part because these structures can have few denser offspring. However, some sparse structures have high R (e.g., $B \rightarrow C + A$, $B \rightarrow A + C$), and some relatively dense structures (e.g., $A \Phi C + B$, $B \Phi C$, $C + B \Phi A$) have very low R . Although there is a significant negative correlation between R and the number of offspring of the structures ($\rho_{\text{Spearman}} = -0.20$, $p < 0.02$) this correlation is low. Therefore, the number of offspring explains only a minor portion of the variation in R .

High values of f_E may stem from the following two distinct situations. First, there may be many disjoint instances of the structure. Second, there may

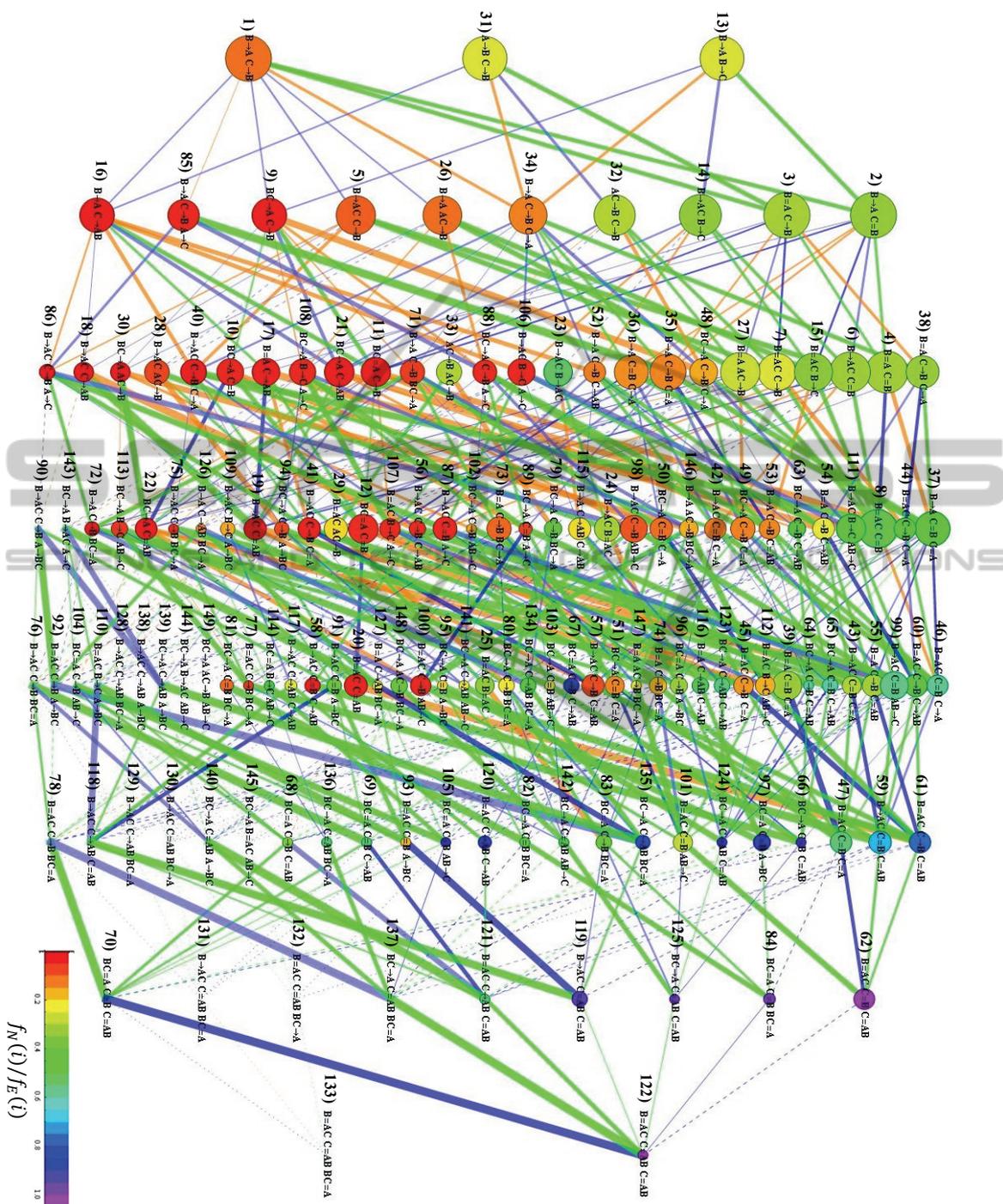


Figure 2: Network of topological relationships among the structures, and statistics for the metabolic network of *S. cerevisiae*. Nodes at each generation are ordered by naked frequency. Node diameter is proportional to $\text{Log}(C_{Met}(i))$. Node hue represents $R(i)$ (scale at the bottom right corner). Edge colours represent relationships: blue, addition of reactant/product to unidirectional (dark) or bidirectional (light) reaction; orange, addition of reaction; green, addition of reverse reaction. Edge thicknesses are proportional to $E(i, j)$. Dashed edges: $(i, j) < 0.05$. Dotted edges connect to structures that lack instances in the metabolic network.

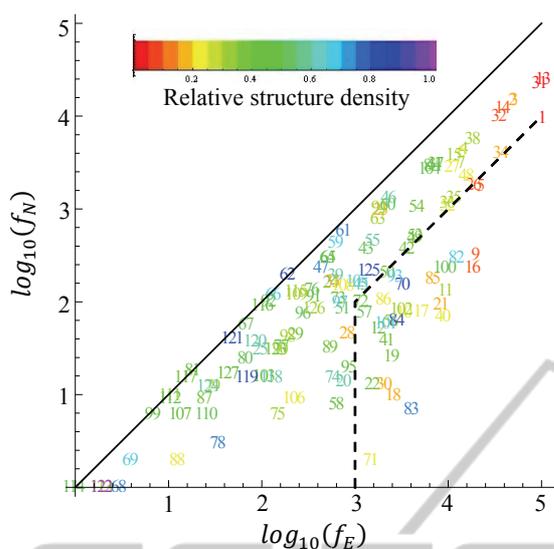


Figure 3: Relationship between naked frequency, embedded frequency and relative structure density. Relative structure density was computed as $\frac{N_{Edges} - 4}{14}$, with N_{Edges} the number of edges connecting metabolite and reaction nodes. Structures to the left of the dashed line have a ratio $R(i) \leq 0.1$ and $f_E(i) > 1000$. These structures are frequent and occur preferentially embedded in offspring structures.

be a combinatorial explosion such that part of the metabolites and reactions forming the structure are shared among many of the instances and the rest of the structure can be matched by many other metabolites and reactions in the metabolic network. The very high f_E values of most instances are due to the second factor. Thus, when only overlaps by up to one metabolite (and no reactions) are allowed, structure 16 ($B \rightarrow A, C \rightarrow A + B$), which ranks 12th in terms of f_E , becomes the one with highest naked frequency (12). However, the embedded frequencies with overlaps restricted to no more than one metabolite are strongly correlated to f_E ($\rho_{\text{Spearman}} = 0.90$), which is simpler to compute.

There is also a strong correlation between f_E and C_{Met} ($\rho_{\text{Spearman}} = 0.91$). But remarkably, although seed structures 13 ($B \rightarrow A, B \rightarrow C$) and 31 ($A \rightarrow B, C \rightarrow B$) have higher embedded frequency, their respective children 2 and 3, resulting from addition of one reverse reaction have higher metabolite coverage. This is explained by a greater overlap among instances for structures 13 and 31 than for structures 2 and 3.

Structures with low R and high f_E are those that occur frequently when embedded in more complex structures but not in isolation. A more detailed analysis of the statistics for some of these structures

highlights the following remarkable features of the local structure of the metabolic network in point.

1. In 96% of the (total) instances of structure 18 ($B \rightarrow A + C, C \rightarrow A + B$) the latter reaction is reversible (structure 19 and offspring), and in 95% of the instances there is also a $C \rightarrow B$ reaction (structure 56 and offspring). On the other hand, structure 18 is rarely embedded in offspring that also contain a $B \rightarrow A$ or a $A \rightarrow B$ reaction.
2. In turn, in 96% of the instances of structure 56 the reaction $C \rightarrow A + B$ is reversible (structure 60 and offspring), but structure 56 is rarely embedded in offspring containing either a reaction of the form $C + A \rightarrow B$ or a second reaction of the form $C \rightarrow A + B$.
3. In 93% (51%) of its occurrences, structure 19 is associated to a $B \rightarrow C$ (respectively $C \rightarrow B$) reaction, but it is rarely associated to reactions $A \rightarrow B, B \rightarrow A, A \rightarrow C$ or $C \rightarrow A$.
4. 91% of the instances of structure 17 ($A \rightarrow B + C, B \Phi C$) contain also a $A \rightarrow B$ reaction.
5. In 93% of the occurrences of the cycle $A \rightarrow B \rightarrow C \rightarrow A$ (structure 85 and offspring) at least one of the steps is reversible.

The computation and ranking of children information provide another means of flagging unexpected bias. The analysis of the results allowed determining interesting features of the local structure of the metabolic network:

1. Structure 40 ($A \rightarrow B + C, C \rightarrow B, C \rightarrow A$) has $\Delta H = 2.0$ bit. The structure has seven children, three of which are obtained by adding a reversible reaction. The child with the first reaction reversible, structure 44, is clearly dominant, representing 85% of the instances of all children.
2. Structure 11 ($A \Phi B + C, C \rightarrow B$) has $\Delta H = 1.9$ bit. Of its seven children, four are obtained by adding a 3rd reaction: $C \rightarrow A, A \rightarrow C, A \rightarrow B$ or $B \rightarrow A$. Structure 44 is obtained by adding the first reaction and comprises 82% of the instances of all the children.
3. Structure 21 ($B + C \rightarrow A, C \rightarrow A$) has $\Delta H = 1.8$ bit. Of its 8 children, six are obtained by adding a new reaction. Adding the reaction $C \rightarrow B$ leads to structure 111, which comprises 72% of the children's instances.

4 CONCLUSIONS

The approach to characterize statistical trends in the

local structure of metabolic networks presented in this article shows several desirable features.

First, it represents local topological structures as bipartite and directed graphs, which permits an unambiguous description of the chemical reaction patterns involved.

Second, it presents a comprehensive enumeration of the structures within a prescribed scope and highlights the topological relationships among structures, facilitating a comprehensive examination of statistical trends.

Third, it prompts a comprehensive visualization of structure statistics and of statistic relationships among topologically related structures (e.g. Figure 2). This will facilitate direct comparisons among the local-topological characteristics of the metabolic networks of distinct organisms and organelles, which can be put in relation to their environments and functions.

Fourth, the approach to detecting potential selective constraints does not hinge on null models based on physically meaningless random networks. It emphasises not so much the absolute frequencies of the structures as the relative frequencies among topologically related structures. Therefore it puts in relief the local-context dependence of the presence/absence of additional reactions, reversibility, etc. We presented above several statistics that help detecting strong bias.

Applied to the *S. cerevisiae* metabolic network, the approach highlights a preference for sparse structures. It also highlights some very strong context-dependence of the reversibility of reactions and of the presence/absence of some types of reactions. The underpinnings of these trends deserve further investigation as a way to reveal functional (e.g. dynamic) properties underlying an evolutionary preference for some reaction-coupling configurations, with the potential to guide synthetic biology and metabolic engineering approaches.

Ongoing algorithmic developments include the expansion of the topological scope of the analysis and strategies to efficiently navigate the network of topologically related structures towards highly represented complex structures.

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