The Air Distribution Network Design Problem
A Complex Non-linear Combinatorial Optimization Problem

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Abstract: The objective of the air distribution network design optimization problem is to find the material and dimensions of each duct and fan in an air distribution network so that the total cost is minimized without violating aerolic constraints. Since the 1960s much research has been dedicated to the simulation and optimization of air distribution networks and numerous methods have been developed to solve this optimization problem. This paper aims to outline the current state-of-the-art in air distribution network design optimization and highlights the main shortcomings. Additionally, previous research is extended by presenting a model that integrates the network layout decisions into the optimization problem. In this problem, called the air distribution network design optimization problem the location of the fans and ducts in the network are determined so that the total cost of the network is minimized. This novel combinatorial optimization problem is characterized by discrete decision variables, and non-linear constraints. This paper also motivates the need for benchmark instances to evaluate the performance of existing or new developed optimization methods and advance future research in the field op air distribution network design optimization. A software tool is developed in this PhD research to generate such instances.

1 RESEARCH PROBLEM

1.1 Air Distribution Network Design (ADND)

One of the most energy-consuming and cost expensive (up to 35% in Belgium) parts of a heating, ventilation and air conditioning (HVAC) system is the air distribution system. Both the energy and material costs can be reduced significantly if air distribution systems or networks are designed properly. The quality of their design largely determines the effectiveness, energy-efficiency and comfort of the building's HVAC system.

Air distribution systems in non-residential buildings can be seen as large tree-networks of supply air ducts that convey conditioned air from one or more resource nodes, e.g. air handling units (AHU) or fans, out through the building to multiple demand nodes (terminal units). Usually, the air is returned back to the AHU to be conditioned again or exhausted from the building by the extraction and exhaust air ductwork respectively. It is the design engineer's responsibility to design the air distribution system in such a way that each demand point is provided with the required airflow at adequate pressure. The energy needed to distribute the air and overcome all the pressure losses of the various components in the network (e.g. fittings, silencers, dampers) is delivered by one or more fans.

The design process of air distribution systems can be subdivided in different phases. First, the ductwork's layout needs to be determined, i.e., the route that the branched ductwork follows starting from the resource node to the demand nodes (terminal units) in the building. Second, all duct types (i.e., size and material) and fan(s) are selected. Last, dampers for the different branches in the system are calculated to balance the system and ensure that every demand point receives the correct airflow.

1.2 State of the Art

Since the 1960s, much research has been dedicated to the simulation and optimization of air distribution networks (ADN) (Bouwman, 1982),(Kim, 2001),(Tsal and Behls, 1986),(Tsal et al., 1988),(Wang, 1986). Numerous design methods have been developed to support the design engineer in the second phase of the design process, namely the duct sizing and fan selection, starting from a given ductwork layout. Generally, current duct design methods can be subdivided in two main categories.

The first category consists of non-optimizing methods that rely on simple heuristics which do not explicitly take into account prevailing local economic conditions. Instead of optimizing an objective function, these methods only use assumptions for variables such as the air flow velocity and friction losses, which are based on rules of thumbs and the designer's experiences (Asiedu, 2000),(Mitchel and Braun, 2012). The obtained designs are workable, but not necessarily optimal.

The Equal Friction and Static Regain method are the two most commonly used methods in this category (ISSO, 1994),(Mitchel and Braun, 2012). In the first method, the frictional pressure drop per unit length of the duct (Pa/m), i.e. the friction rate, is maintained constant throughout the duct system, where the frictional pressure drop is associated with the duct wall friction. This method is straightforward but involves judgement in the selection of the friction rate, since there is a wide range of possible values for the friction rate. The objective of the static regain method is to obtain the same static pressure at diverging flow junctions and before each terminal unit by changing downstream duct sizes (Figure 1). This method of duct sizing is based on Bernoulli's equation, which states that a reduction of velocities results in a conversion of dynamic pressure into static pressure. The velocity for the root section is an arbitrary parameter and depends on the design engineer's experience.

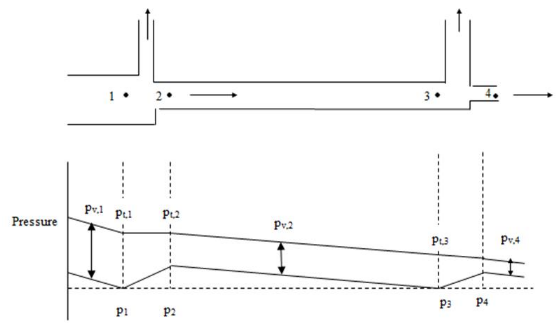

Figure 1: Schematic of pressure distribution for static regain design, where pt = total pressure, p = static pressure and pv = velocity or dynamic pressure (Mitchel and Braun, 2012).

The second category consists of optimization methods. Their main goal is to determine duct sizes according to optimal pressure losses and select a fan according to the optimal fan pressure that minimizes life cycle costs (LCC) (Asiedu, 2000),(Taecheol et al., 2002),(Tsal et al., 1988). The Reduced Gradient (Arkin and Shitzer, 1979), Quadratic Search and the Modified Lagrange Multipliers methods (Tsal and Adler, 1987) are some of the many computer-aided numerical optimization methods used for network optimization. These methods are all continuous methods and thus, they are not adequate to deal with discrete parameters such as nominal duct sizes. In 1968 Tsal and Chechic developed a method based on Bellman's dynamic programming method (1957). Unfortunately, when exact methods such as dynamic programming are used for large combinatorial optimization problems (i.e. NP hard problems) like ADN, combinatorial explosions occur, resulting in excessively long computation times (Sörensen and Glover, 2013).

The most widely known optimization method is the T-method (Tsal et al., 1988), which is also based on dynamic programming (Tsal and Behls, 1986). The method's objective is to find duct sizes and select a fan so that the system's life-cycle cost is minimized. The calculation procedure of the T-method consists of three main steps. First the entire duct system is condensed into a single duct section for finding the ratios of optimal pressure losses using sectional aerolic characteristics (= system condensing). After calculating the optimal system pressure loss in the second step, a fan is selected. Last, the system pressure is distributed throughout the system sections (= system expansion). Although this method is recommended by ASHRAE (American Society of Heating, Refrigeration and Air Conditioning Engineers) (ASHRAE, 2009), it is hardly used in practice. Yaw Asiedu (Asiedu, 2000) and Huan-Ruei Shiu (Shiu et al., 2003) list the main shortcomings of the T-method for large complex ADN. Yaw Asiedu, for example, states that metaheuristic techniques such as evolutionary metaheuristics are needed to tackle large complex network designs and proposes a Segregated Genetic Algorithm (Asiedu, 2000). Contrary to exact optimization algorithms, metaheuristics do not guarantee the absolute optimality of the obtained solutions. However, they provide solutions that are "good enough" in an "acceptable" computing time. Other (meta)heuristics that were used to deal partly or completely with the duct optimization problems are for example Simulated Annealing (Wang, 1986) and the Nelder and Mead downhill simplex method (Kim, 2001). Although recent papers have been published (Fong et al., 2010),(Kashyap, 2013),(Vitooraporn and Kritmaitree, 2003), these mainly reiterate the same ideas of previous research (Asiedu, 2000),(ISSO, 1994),(Tsal et al., 1988), i.e. they focus only on the duct sizing and fan selection and, more important, the objective function of the ADN optimization problem is largely the same as the objective functions defined in previous research.

Previously developed methods are often tested solely on two or three test networks, including the ASHRAE benchmark network (Figure 2). This network, however, does not reflect a realistic ADN in non-residential buildings. On the supply side, the ASHRAE network contains only one fan (resource node) which provides six terminal units (demand nodes) of air. Realistic networks in hospitals or large office buildings can have hundreds of terminal units and multiple fans.

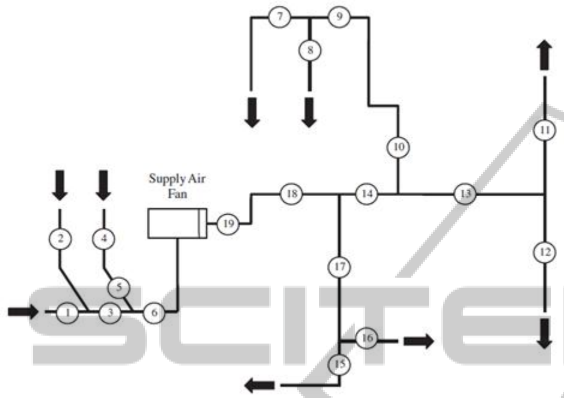

Figure 2: ASHRAE duct system example (ASHRAE, 2009).

1.3 Problem Statements

In general, three main shortcomings can be identified that characterize previous research on air distribution network optimization:

First, existing optimization methods only focus on the second phase of the design process, i.e., they only determine the size of each duct and/or fan in the network and consider the network layout to be given. The layout itself is determined using rules of thumbs, which result in designs that are workable, but not necessarily optimal. Clearly, the layout of the network and the duct sizes are interrelated decisions that jointly influence the quality of the air distribution system.

Second, due to lack of benchmark instances, current optimization methods have not been adequately tested, and thus no strong conclusions can be drawn on their performance in realistic circumstances nor can their performances be compared to other existing methods. As a result, air distribution systems are generally largely designed manually, and rely for their performance on the knowledge and experience of the engineer in charge of the design (Mitchel and Braun, 2012). Clearly, the field of air distribution design would benefit greatly from models and methods that allow more advanced automation.

Last, no modern metaheuristic techniques have been used to solve the air distribution network design (ADND) optimization problem. Since the last innovative research conducted in the field of ADN optimization, many new metaheuristic techniques have been developed to handle the optimization of large combinatorial problems.

2 OUTLINE OF OBJECTIVES

The overarching aim of this PhD project is to tackle aforementioned shortcomings in five steps. The first two steps have already been performed and their outcome is described in this document.

1. Formulating the air distribution network design problem (ADND) as a single-objective, non-linear combinatorial optimization problem, in which both the layout decisions and the duct and fan type decisions are taken simultaneously (see Section 3.1);
2. Developing a software tool that generates a large database of realistic artificial ADN that can be used for testing purposes. These instances can be used to compare the performance of existing and future optimizations methods and evaluate their robustness (see Section 3.2);
3. Developing a simulation model that can simulate large air distribution networks of arbitrary complexity. The simulation method will be used during the optimization phase to calculate the objective functions and constraints of a given network configuration. Moreover, the model will be addressed during the validation phase of the optimization algorithm to determine the added value compared to the traditional methods;
4. Developing an efficient metaheuristic that is able to calculate the optimal air distribution network configuration. The simulation model will be responsible for the major part of the computation time, making it essential that the metaheuristic has to be able to generate candidate solutions intelligently, i.e. generate only high-quality solutions. Moreover, the metaheuristic needs to be flexible so that it can be extended to realistic networks;
5. Representing the ADND optimization problem as a multi-objective optimization problem with the minimization of the life cycle cost, energy consumption and initial material cost as conflicting objective functions (see Section 4.2).

3 METHODOLOGY

3.1 Formulation of the Air Distribution Network Design Optimization Problem

In this research an ADN is represented as a graph $G(N,E)$ with E being the set of edges that represent (potential) ducts and N the set of nodes representing junctions, points of demand (terminal units), and (potential) points of supply (fans). The possible locations of the fan(s), as well as the possible fan types, and all possible types of ducts between any pair of nodes are assumed to be known and given as input to the optimization algorithm. The required airflow at each terminal unit and thus the total airflow for the entire ADN is also assumed to be predetermined. The output of the optimization algorithm will be either a minimum cost spanning tree with one large fan or multiple subtrees where each subtree has its own fan (Figure 3).

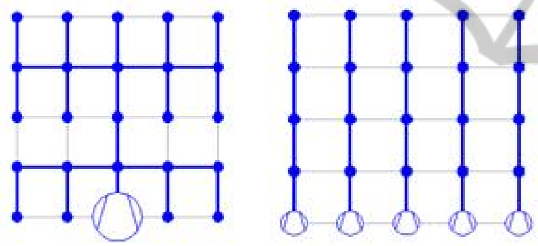

Figure 3: ADN with 1 fan (a) and 5 ADN with 5 fans (b).

Although real-life ADN should be evaluated on multiple criteria (installation cost, life-cycle cost, noise levels, ...), minimization of the installation cost is generally seen as the most important objective. We therefore define the ADND problem as a single-objective optimization problem. The objective function is defined as the sum of the duct costs and the fan costs:

$$\text{minimize cost} = \sum_{d\in D}\sum_{t\in T} x_{td}\, C_{td}\, L_d \;+\; \sum_{f\in F}\sum_{s\in S} x_{sf}\, C_{sf} \tag{1}$$

In the equation x_{td} is a discrete decision variable that determines whether duct d of type t is selected ($x_{td} = 1$) or not ($x_{td} = 0$). The same applies to the fan selection, i.e. when a fan of type s is selected, x_{sf} equals 1 and when a fan of type s is not selected, x_{sf} equals 0. The first term of equation 1 represents the cost of the ductwork, which depends on both the total length L_d of each duct d, the type t selected for duct d, and the cost per unit of length for a duct of type t. Each duct type has a different nominal duct size (chosen from a list of commercially available types T) and specific material characteristics, resulting in a certain unit cost per meter C_{td}. The second term of the formula represents the material cost of the fans, where C_{sf} is the cost of a fan of type s. The type of a fan is amongst others determined by its size, fan performance or characteristic curves and its application field (centrifugal or axial fan).

Significant for the design of air distribution systems is the large number of constraints to which it is subjected. Generally these can be divided in two main categories: physical and external constraints. The physical constraints such as mass and pressure balancing are determined by the physical laws that act upon the ADN and are decisive for the proper functioning of the system. The external constraints are imposed by the fact that the ADN needs to be built in an environment that does not allow infinite degrees of freedom.

The mass balance or mass conservation law states that the mass of air (expressed in kg) flowing into a node in the network per unit of time (in s) equals the mass of air flowing out of this node and must be satisfied for each node $n \in N$:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \tag{2}$$

The total airflow rate in the entire air distribution system, i.e. the airflow that is delivered every second by the fans in the system, equals the sum of the desired airflow rates at each terminal unit. These airflow rates are assumed to be given.

The pressure balancing constraint requires that the pressure losses are the same for all duct paths in the network. If this constraint is not fulfilled, balancing dampers must be installed to balance the air flow in the system. Since every balancing damper induces extra pressure losses and thus an extra cost, the designer should aim to meet this constraint. The pressure drop (expressed in Pa) due to friction for a constant-area duct is given by the Darcy-Weisbach equation:

$$\Delta p = f \frac{L}{D_H} \frac{\rho v^2}{2} \tag{3}$$

where f is the friction factor (dimensionless), L the duct length (m), D_H the hydraulic diameter (m), ρ the density (kg/m^3) and v the average velocity (m/s). The last grouping of terms is also called the velocity or dynamic pressure. Parameter D_H is determined by the type of duct and is assumed to be given for each available type. The density ρ of the medium is given and considered to be constant for the whole air distribution system. Parameter v depends on the duct type that is selected and the pressure loss in that duct.

Lastly, the friction factor f depends on both the selected duct type and the velocity v.

The second category of constraints, i.e. external constraints can be further subdivided in mandatory (hard) and non-mandatory (soft) constraints, where the latter stands for the preferences from the designer or building owner that do not predominantly contribute to the proper functioning of the system (e.g., a preference for smaller ducts or a specific duct layout from an aesthetic point of view). Mandatory constraints, however, ensure that the obtained design is feasible (limited set of commercially available duct sizes, limited space, ...):

$$L_i \leq D_i \leq U_i \tag{4}$$

where:

- D_i = diameter of duct section i, where i = 1, 2,..., n and $D_i \in T$,
- n = the number of duct sections in the air distribution system (unitless),
- L_i and U_i = the lower and upper bounds of duct section i, due to velocity or geometric constraints,
- D_i, L_i and U_i are expressed in meter.

The evaluation of the physical and external constraints requires the simultaneous solution of a set of non-linear equations. These equations depend on the values chosen for the decision variables in the way mentioned before. Solving these equations can be done using software such as Dymola, i.e. an equation based software with domain specific knowledge.

The ADND optimization problem is therefore a complex combinatorial optimization problem. The evaluation of the constraints requires the simultaneous solution of a set of non-linear equations. It can be posited that such a problem is outside the realm of exact methods and can be best approached by metaheuristic techniques. Sörensen and Glover define metaheuristics as "high-level, problem-independent algorithmic frameworks that provide a set of guidelines or strategies to develop heuristic optimization algorithms" (Sörensen and Glover, 2013). Metaheuristics are also particularly well-suited in a simulation-optimization environment where either the objective function or the constraints (such as in this case) require a run from a simulation module to be evaluated. The development of such a simulation module or a metaheuristic for the ADND optimization problem, however, is outside the scope of this paper.

3.2 Generation of Artificial Air Distribution Networks (Benchmark Instances)

Currently a database of benchmark instances is lacking and thus the performance of existing and new ADN optimization methods cannot be evaluated and compared properly. The software tool developed in this paper attempts to address this shortcoming by generating realistic artificial ADN, based on insight into real-life building plans and typical ADN design procedures. By means of adjustable parameters, instances of arbitrary size and characteristics are generated. To this end, supply ADN in multi-storey office buildings and universities or school buildings can be simulated. Although these types of buildings differ in their demand pattern, they generally share a similar layout. Each building can be subdivided in different zones, depending on their heating, cooling and ventilation needs, where each zone contains several rooms, which are interconnected through hallways. The supply ductwork runs typically from the centralized AHU or fan(s), located in a technical room or on the roof, vertically through the shafts to the several floors of the building. From there the air ducts run horizontally above the false ceiling of the corridors to the different zones and rooms that need to be ventilated and/or conditioned. The dimensions of the shafts (height and width) and the false ceiling (height) influence both the sizing and the layout of the ductwork significantly. The network generator developed in this paper and written in C++ is based on this layout principle.

Basically the generation of the benchmark instances is carried out in two main phases. First, input is required from the user about the characteristics of the building, including the building type, number and dimensions of the shafts and the number and size of the different zones that need to be ventilated and/or conditioned. Second, the ADN are generated algorithmically whereby sequentially the nodes and edges are generated in the graph. The edges represent the air ducts in the network.

The second step is described in detail in the following subsections and shown graphically. Figures 4 to **??** illustrate step by step the generation of ADN for a multi-storey office building with four shafts and consisting of eight zones.

3.2.1 Generation of the Nodes in the Graph

Per fan, a rectangular grid is created, whereby only a percentage of the grid points will be allocated as hallway nodes, i.e. junctions. Both the size of the grid and the percentage are adjustable parameters.

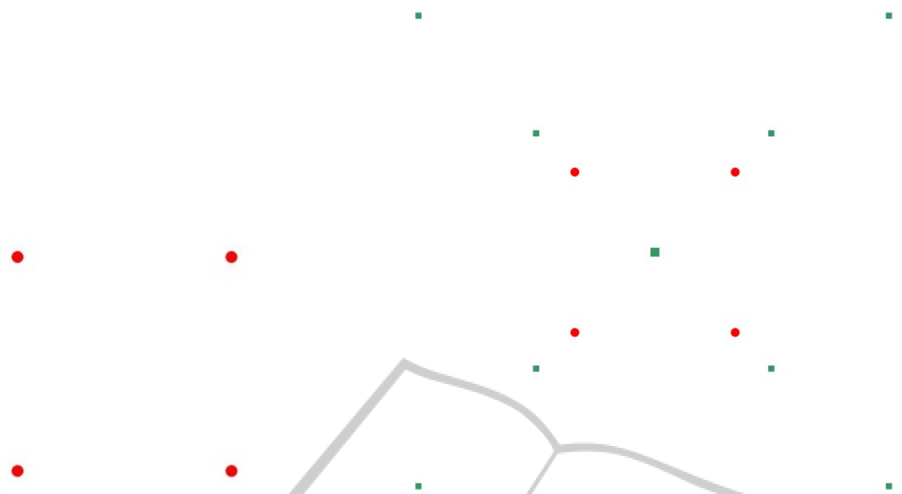

Figure 4: generation of shafts (left) and supply nodes (right).

Figure 5: generation of hallway nodes (left) and room nodes (right).

Generation of Shafts. Nodes of the type 'shaft' are characterized by a set of coordinates (x, y) and a demand that equals zero m^3/h. Additionally, maximum dimensions (height and width) are assigned to these types of nodes, which will determine the maximum permitted diameter of the incoming and outgoing ducts.

As can be seen in figure 4, the shafts are generated on the perimeter of a rectangle with the origin (0,0) as centre. Up to eight shafts can be generated per building.

Generation of All Potential Supply Nodes. In a second step, the (potential) fan locations are generated, including a discrete set of potential fan types that can be installed at the corresponding locations. Each fan type is represented by its performance curves. The first supply node, i.e. the primary fan, is generated in the origin (0,0) of the graph. Additionally, for every zone in the building, secondary supply nodes are generated, where each fan and thus zone is associated with a shaft. Besides the number of zones, the distances between the zones and the shafts can be adjusted as well by the user.

Generation of Zones. As mentioned in section 3.2, a zone contains multiple rooms which are interconnected through hallways.

- **Hallway Nodes (Junctions).** Per fan, a rectangular grid is created, whereby only a percentage of the grid points will be allocated as hallway nodes, i.e. junctions. Both the size of the grid and the percentage are adjustable parameters.

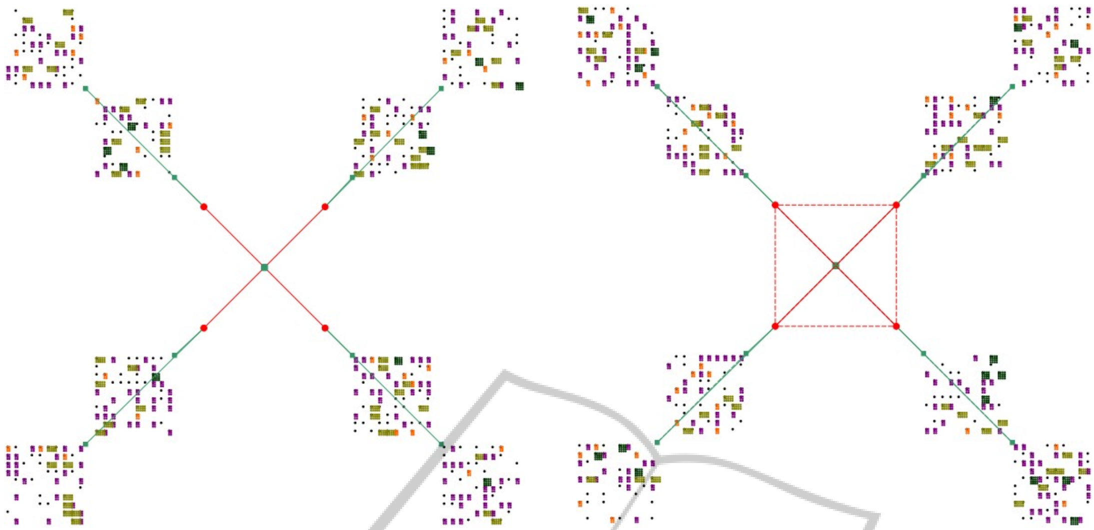

Figure 6: Generation of main ducts.

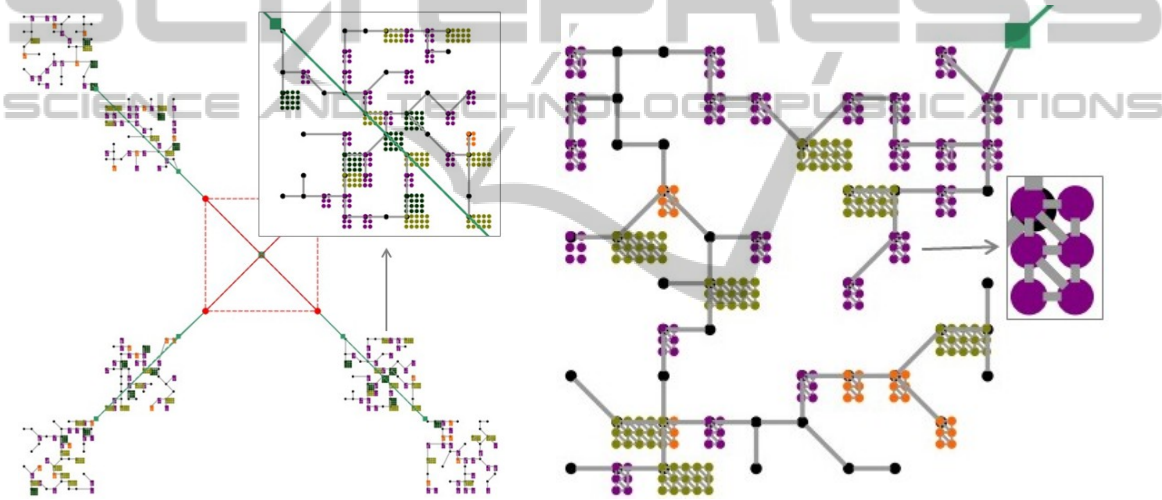

Figure 7: Generation of hallway ducts (left) and room ducts (right).

- **Room Nodes.** All spaces in non-residential buildings can be classified into different types. Depending on the function of the building, some types of rooms may be more or less present. The ratio between the room types is given in table 1, where only the spaces with an airflow demand are taken into account. The percentages given in table 1 are assumed average values based on multiple real-life building plans. However, since these percentages can vary considerably from building to building, they are represented as adjustable parameters in the software tool.

  Every hallway node generated in the previous step, has a γ% chance to get assigned a room type, where γ is an adjustable parameter. Which room type that is assigned depends on the percentages given in table 1. For instance, in the case of an office building, a small office has a 65% chance of being generated, an open office and reception each 15% and a cafeteria or restaurant 5%.

  The characteristics of the various room types are given in table 2. The airflow rates and floor areas in are calculated based on the occupancy rate per room type, using to the European standard EN

Table 1: Occurrence of different room types in a building.

Room types	Office building	Universities
Small office	65%	20%
Open office	15%	/
Meeting room	15%	20%
Restaurant	5%	10%
Class room	/	30%
Auditorium	/	20%

Table 2: Characteristics of the different room types.

Room type	Max. occupancy (*no. of people*)	Floor area (m^2)	# nodes (-)	Airflow rate/node ($m^3/h/node$)	Total airflow rate (m^3/h)
Small office	10	150	6	50	300
Open office	20	300	15	40	600
Meeting room	16	56	6	80	480
Restaurant	165	250	16	300	4800
Class room	40	160	15	80	1200
Auditorium	200	800	29	200	5800

Table 3: Classification of indoor air quality or IAQ (typical range of outdoor air for a non-smoking area).

Category	Description	Rate of outdoor air ($m^3/h/person$)
IDA 1	High IAQ	> 54
IDA 2	Medium IAQ	36 - 54
IDA 3	Moderate IAQ	22 - 36
IDA 4	Low IAQ	< 22

13779, which applies to the design and implementation of ventilation and room conditioning systems for non-residential buildings subject to human occupancy. It focuses on the definitions of the various parameters that are relevant for such systems. The air rates used in this paper are those that are associated with an acceptable air quality IDA 3 in a non-smoking area (table 3).

3.2.2 Generation of the Edges in the Graph

In a second phase, the software tool generates all potential edges or ducts in the graph. Similar to the nodes, the ducts are characterized by a set of parameters such as a roughness coefficient, a start and end node, a length (i.e. the Euclidian distance between the start and end node) and a maximum (hydraulic) diameter. The last parameter implies that all smaller duct sizes can be installed as well at this location. Since supply ADN are subjected to a telescopic constraint, i.e. the diameter of an upstream duct must not been less than the diameter of a downstream duct, the size of the set of potential ducts will reduce when going downstream in the system.

Generation of Main Ducts. First, ducts are created which connect the shafts with the centralized supply node and the corresponding secondary supply nodes. Moreover the shafts and secondary fans are connected mutually as well. As mentioned in before, the maximum diameter of these ducts is determined by the dimensions of the shafts in the building.

Generation of Ducts in the Zones:

- **Hallway Ducts**
  For every set of hallway nodes generated, a minimum spanning tree, connecting all hallway nodes associated with one fan, is drawn, using Prim's algorithm. The begin-nodes and end-nodes of the edges or ducts are assigned to these edges while drawing the spanning tree. The edge weights or duct lengths equal the Euclidean distances between the begin-nodes and end-nodes.
- **Room Ducts**
  All demand nodes within a room are connected by triangulation. The begin-node, end-node and length of each duct are defined by the triangulation itself. The maximum diameter of the ducts, however, depends on the room type as every room type has a different demand (table 2). The characteristics of the artificial ADN that are generated by the software tool and described in the previous subsections are available in both Graph ML and text file format, which can serve as input file for optimizations methods developed in future research.

4 STAGE OF THE RESEARCH

4.1 Conclusions and Accomplishments

Since the 1960s, much research has been dedicated to the simulation and optimization of ADN. Numerous design methods have been developed to support the design engineer in the second phase of the design process, namely the duct sizing and fan selection, starting from a given ductwork layout. This paper provides a thorough critical review of previously developed methods for duct size optimization and proposes some recommendations for future research in the field of ADN optimization.

Moreover, two of the four objectives outlined in section 2, have already been accomplished. First, the air distribution network design (ADND) problem is represented as a non-linear combinatorial optimization problem, which can best be solved with metaheuristic optimization techniques. Previous research is extended by integrating the network layout into the formulation of the optimization problem. Additionally, the need for realistic artificial benchmark instances is motivated and a software tool to generate these kinds of instances is proposed. To this end, instances for multiple-floor office building and school or university buildings can be generated. In the very near future, the software tool will be extended to generate instances for other non-residential buildings such as sport complexes and industrial buildings. The tool will become freely available to foster research in the field ADND optimization and will be useful for testing purposes.

4.2 Pointers for Future Research

As mentioned in section 2 'Outline of Objectives', the next step in this research is developing a simulation model in Dymola that can simulate large ADN of arbitrary complexity. It will be used during the optimization phase to calculate the simplified objective function and the constraints of a given network configuration as formulated in section 3.1. Subsequently an efficient metaheuristic algorithm will be developed for this simplified ADND optimization problem.

Since the design of air distribution systems depends strongly on the requirements of the end user, the long term aim of this research is to represent the ADND optimization problem as a multi-objective optimization problem with the minimization of the life cycle cost, energy consumption and initial material cost as conflicting objective functions. Conflicting in the sense that, for example, a larger cross-section of the ductwork induces higher material costs, but lower energy consumption. A Pareto-set of non-dominated solutions will be generated by the optimization algorithm and it is up to the decision taker to make a trade-off between the different solutions.

5 EXPECTED OUTCOME

The final outcome of this research is an efficient optimization method that supports the decision-making of the contractor, engineering office or architect during the design phase of large, complex air distribution networks in non-residential buildings.

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REFERENCES

Arkin, H. and Shitzer, A. (1979). Computer aided optimal life-cycle design of rectangular air supply duct systems. *ASHRAE Transactions*, 85.

ASHRAE (2009). *ASHRAE Handbook - Fundamentals*, chapter 21: Duct design, pages 21.1–21.67. American Society of Heating, Refrigeration and Air-Conditioning Engineers.

Asiedu, Y. (2000). *Life-cycle cost analysis and probabilistic cost estimating in engineering design using an air duct design case study*. PhD thesis, University of Saskatchewan.

Bouwman, H. B. (1982). *Optimum air duct system design*. PhD thesis, Technische Hogeschool Delft.

Fong, K. F., Yuen, S. Y., Chow, C. K., and W., L. S. (2010). Energy management and design of centralized air-conditioning systems through the non-revisiting strategy for heuristic optimization methods. *Applied Energy*, 87:3494–3506.

ISSO (1994). *Luchtkanalen drukverlies-, thermische en akoestische berekeningen*. ISSO.

Kashyap, M. (2013). Carbon fiber composites for duct design based on vav network in hvacs system. *International Journal of Emerging Trends in Engineering and Development*, 2:67–73.

Kim, T. (2001). *Optimum duct design for variable air volume systems*. PhD thesis, Oklahoma State University.

Mitchel, J. W. and Braun, J. (2012). *Heating, ventilation and air conditioning in buildings*. Wiley.

Shiu, H., Ou, F., and Chen, S. (2003). Optimization design of exhaust duct system in semiconductor factory using dynamic programming method. *Building and Environment*, 38:533–542.

Sörensen, K. and Glover, F. (2013). Metaheuristics. In Gass, S. and Fu, M., editors, *Encyclopedia of OR/MS, 3rd edition*, New York. Springer.

Taecheol, K., Spitler, J. D., and Delahoussaye, R. D. (2002). Optimum duct design for variable air volume systems - part 1: problem domain analysis of vav duct systems and part 2: optimization of vav duct systems. *ASHRAE Tran*, 108:96–127.

Tsal, R. J. and Adler, M. S. (1987). Evaluation of numerical methods for ductwork and pipeline optimization. *ASHRAE Transactions*, 93.

Tsal, R. J. and Behls, H. F. (1986). Evaluation of duct design methods. *ASHRAE transactions*, 92.

Tsal, R. J., Behls, H. F., and Mangel, R. (1988). T-method duct design - part 1: optimization theory and part 2: calculation procedure and economic analysis. *ASHRAE transactions*, 94:90–151.

Vitooraporn, C. and Kritmaitree, P. (2003). Modified t-method duct design for use in thailand. *ASHRAE Journal*, pages 10–24.

Wang, F. (1986). Computer aided optimal design of duct system using simulated annealing. Master's thesis, Oklahoma State University.