Measuring Distance by Angular Domain Filtering

Wei-Jun Chen
Systemtechnik, Carl Zeiss Meditec AG, Göschwitzer Straße 51–52, 07745, Jena, Germany

Keywords: Angular Domain filtering, Afocal System, Transform, Distance measurement, Bi-lateral telecentric optics.

Abstract: In this paper a paraxial imaging system with *incoherent* illumination is interpreted as a signal processing system in which a thin lens performs a forward angular transform on light rays traveling through it. Inverse angular transform exists and could be performed by another thin lens which coincides its focus plane with the first. The common focus plane acts as an angular domain, on which filtering is possible by placing aperture stops. A symmetrically angular filtering results in a direct correspondence between a recorded signal and its object distance. Such a "transform–filtering–inverse transform" system could be understood as a modified tele–centric system, with which a novel concept for measuring a distance without focusing on the target is suggested.

1 INTRODUCTION

Although an imaging system is generally considered as the data supplier of a signal processing system, there is no limit to extend processing concepts covering data acquisition. In *Fourier optics* (Goodman, 1968), concepts including *Fourier* transform, frequency domain filtering, point spread function (PSF), etc. are employed to get deep understanding of a thin–lens system. Such an understanding plays a key role in improving image quality in a wide range of imaging fields like bio–medical imaging, astronomy, and also electron microscopy.

On another hand, the meaning of high image quality is much more than just amusing human eyes. In photometry high image quality often means enhanced landmarks and ignorable environment, while in medical diagnosis high image quality normally asks for high signal–to–noise ratio for disease symptoms where misleading message is not acceptable. To satisfy various image quality requirements, many specified optical designs have been proposed in past decades. As an example, tele–centric optics (Lenhardt, 2001), is specially designed for high precision and distance invariant measuring system.

Illuminated with incoherent light, an imaging process could be generally described as a ray traveling process: light rays are emitted from object points; afterward they freely travel in air; thin lenses collect such rays and change their directions targeting a digital sensor; finally the sensor records the energy carried by these rays as pixel intensities.

In the same direction of *Fourier optics*, this paper tries to interpret such an incoherence ray journey by signal processing terms, where a transform, namely as **angular transform**, and its inverse are defined to describe the ray mapping between two focus planes of a thin lens. All the rays with the same traveling direction from a thin lens' anterior focus plane, will travel through an unique position on its posterior focus plane, and vice versa. If two thin lenses are so placed that the second lens coincides its anterior focus plane with the first lens' posterior focus plane, like a *4–f* system in *Fourier optics* or an *afocal* system in telescopy, selecting rays by their directions could be fulfilled by placing filters on the angular domain, i.e., the common focus plane. Moreover, when a single–hole aperture stop is placed at the angular domain center, a standard bi–lateral tele–centric system is built.

It is asked, that if the aperture stop is not placed at the domain center, i.e., performing an off–axis angular filtering, what result should be obtained? Section 4 gives out an answer that by off–axis angular filtering, object distance could be measured by single snapshot imaging.

The rest of this paper is organized as following: in next section some further readings are suggested for more background information; Section 3 describes the angular transform, its inverse, and the angular domain in details; Section 4 introduces the concept of measuring distance by angular domain filtering, which is further demonstrated by an imaging experi-

Chen W..
Measuring Distance by Angular Domain Filtering.
DOI: 10.5220/0005336400850090
In *Proceedings of the 3rd International Conference on Photonics, Optics and Laser Technology* (PHOTOPTICS-2015), pages 85-90
ISBN: 978-989-758-092-5

ment in Section 5; Afterward Section 6 provides some conceptual discussions; Finally Section 7 concludes this paper with a brief discussion on future works and open questions.

2 FURTHER READINGS

Along with the rapid development of digital sensor techniques, kinds of optical systems have been designed for digital imaging based machine vision applications (Zeuch, 2000) (Jaehne and ecker, 2000), where tele–centric optics plays an important role for focus analysis (Watanabe and Nayar, 1997), non–contact velocity sensing (Berger, 2002) and 3D imaging (Djidel et al., 2006) (Kim and Kanade, 2011). Geometric optics, which provides the design disciplines for imaging optics, is normally based on the paraxial approximation, which is often described by the matrix method (Gerrard and Burch, 2012) in a simple but powerful way.

3 ANGULAR TRANSFORM

A light ray traveling in an imaging system could always be described by a 5–dimensional vector: (x, y, z, α, β), where (x, y, z) denotes a spatial position in a Cartesian coordinate system[1], and (α, β) denotes two angles describing the ray direction $\vec{r} = \vec{r}_x + \vec{r}_y + \vec{r}_z$, where $\vec{r}_x$, $\vec{r}_y$ and $\vec{r}_z$ denote three axis projections of $\vec{r}$. Without loss of generality, α denotes the angle between $\vec{r}$ and $\vec{r}_z$, and β denotes the angle between $\vec{r}_x$ and $\vec{r}_x + \vec{r}_y$, as shown in Fig. 1.

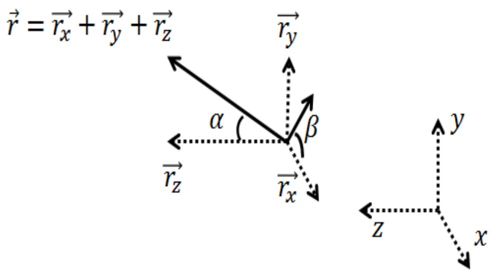


Figure 1: A traveling ray $\vec{r}$, its axis projections of $\vec{r}_x$, $\vec{r}_y$, $\vec{r}_z$, and its direction angles (α, β).

Placing a thin lens perpendicular to the z axis, a ray traveling through it will arrive at a spatial position on its posterior focus plane, as shown in Fig. 2.

The thin lens performs a transform on a given light ray (x, y, z, α, β). Defining $c = \tan\alpha\cos\beta$ and

[1]In this paper the right-hand rule is applied for defining a 3–dimensional Cartesian coordinate system.

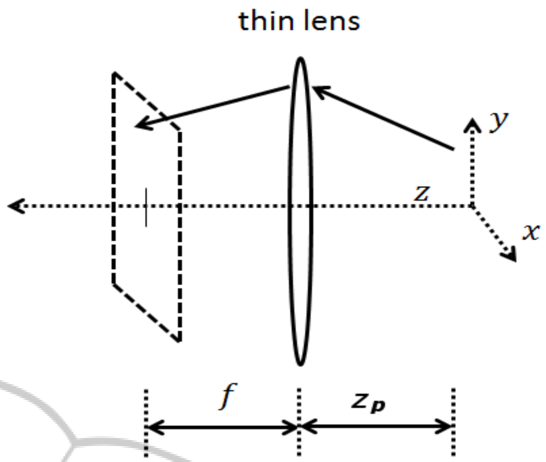


Figure 2: A light ray traveling through a thin lens, where f denotes the lens' focus length.

$s = \tan\alpha\sin\beta$, such a transform could be expressed as:

$$\begin{bmatrix} x_f \\ y_f \\ c_f \\ s_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & f & 0 \\ 0 & 0 & 0 & f \\ -\frac{1}{f} & 0 & 1 & 0 \\ 0 & -\frac{1}{f} & 0 & 1 \end{bmatrix} \begin{bmatrix} x + z_p c \\ y + z_p s \\ c \\ s \end{bmatrix}, \quad (1)$$

where z_p denotes the z–axis projection of a traveling path from its starting position z to the thin lens position z_{lens}: $z_p = z_{lens} - z$.

The 4×4 transform matrix in Eq. 1 is a combination of two element transform matrices (Gerrard and Burch, 2012): one for free space light propagation and another one for thin lens with focus length of f. The vector (x_f, y_f, c_f, s_f) defines a light ray passing through the spatial position $((x_f, y_f, z_f)$ on the back focus plane (i.e., $z_f = z_{lens} + f$), with its direction defined by (c_f, s_f). Since the following equation exists:

$$\begin{bmatrix} 1 & 0 & -f & 0 \\ 0 & 1 & 0 & -f \\ \frac{1}{f} & 0 & 0 & 0 \\ 0 & \frac{1}{f} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & f & 0 \\ 0 & 0 & 0 & f \\ -\frac{1}{f} & 0 & 1 & 0 \\ 0 & -\frac{1}{f} & 0 & 1 \end{bmatrix} = I, \quad (2)$$

an inverse transform could be defined:

$$\begin{bmatrix} x_v \\ y_v \\ c_v \\ s_v \end{bmatrix} = \begin{bmatrix} 1 & 0 & f_v & 0 \\ 0 & 1 & 0 & f_v \\ -\frac{1}{f_v} & 0 & 0 & 0 \\ 0 & -\frac{1}{f_v} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_f \\ y_f \\ c_f \\ s_f \end{bmatrix}, \quad (3)$$

where the sign difference in Eq. 2 and 3 indicates two different light traveling directions: in Eq. 2 a ray goes backward and in Eq. 3 a ray goes forward. Physically Eq. 3 is equivalent to placing another thin lens with focus length f_v after the first one, with a distance $f+f_v$ thus that the rear lens coincides its focus plane with the front lens. The above inverse transform results in a new ray just traveling through the second lens. Placing a sensor after the second lens with a distance w, as shown in Fig. 3, the light ray will finally be recorded at a 2D sensor position, where we have

$$\begin{bmatrix} x_i \\ y_i \\ c_i \\ s_i \end{bmatrix} = \begin{bmatrix} 1 & 0 & w & 0 \\ 0 & 1 & 0 & w \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ c_v \\ s_v \end{bmatrix}. \qquad (4)$$

Angular Transform: Above Eq. 1–4 describe a ray journey in which a signal is emitted from a spatial position (x,y,z) and finally is recorded by a 2D sensor at (x_i,y_i). Between signal emission and recording, a forward transform and its inverse are performed sequentially by two thin lenses. On the transform domain, each signal, i.e., each light ray, travels through a 2D spatial position (x_f,y_f) which only depends on the angular information of the signal and is independent to its z position. Such a transform is named as angular transform while each 2D position on the common focus plane, namely as the angular domain, is equivalent to a unique (α,β) pair describing the angular information of rays.

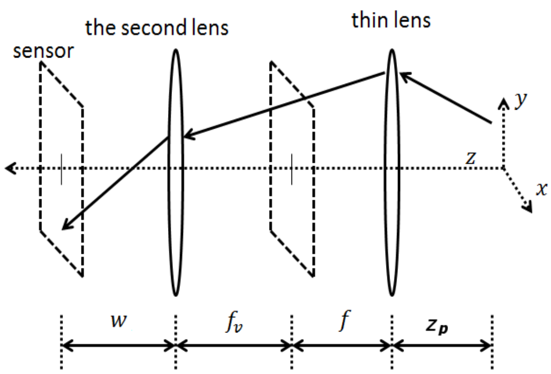


Figure 3: The second lens with focus length of f_v coincides its focus plane with the first lens, and a sensor after it with a distance of w.

4 DISTANCE FROM ANGULAR FILTERING

Although a light ray in Eq. 4 is finally recorded by a 2D sensor, the z information of its origin is still preserved. Defining $m=(f/f_v)$, we have:

$$z_p = m(f+f_v) - m^2 w - m \times x_i/c - x/c, \qquad (5)$$
$$z_p = m(f+f_v) - m^2 w - m \times y_i/s - y/s. \qquad (6)$$

Moreover, if two rays with symmetry angles, i.e., $(\pm\alpha,\beta)$ emitted from the same 3D source are recorded, the z information could be obtained by measuring a distance between two sensor positions:

$$z_p = m(f+f_v) - m^2 w - \tfrac{m}{2}(x_i^{\alpha} - x_i^{-\alpha})/c, \qquad (7)$$
$$z_p = m(f+f_v) - m^2 w - \tfrac{m}{2}(y_i^{\alpha} - y_i^{-\alpha})/s, \qquad (8)$$

where $(x_i^{\alpha}, y_i^{\alpha})$ denotes the sensor position of the light ray (α,β), as well as $(x_i^{-\alpha}, y_i^{-\alpha})$ for the light ray $(-\alpha,\beta)$.

The selective ray recording for Eq. 7 and 8 could be fulfilled by placing a filter on the angular domain. An example for such a filter is shown in Fig. 4(b), where two small holes are symmetrically defined with a distance h from the domain center, allowing light rays with angle of $(\alpha = \pm\tan^{-1}(h/f), \beta = 90^{\circ})$ to be finally recorded.

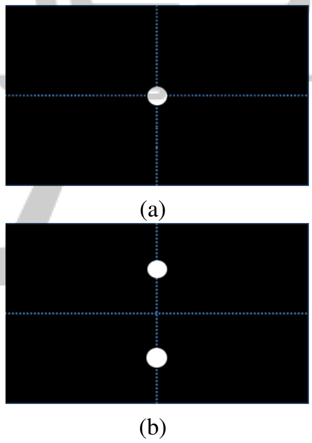


Figure 4: Two example filters for angular domain filtering: a), a near–zero–pass filter; b), a symmetrically off–axis filter. Both of them are thin (0.1 *mm* thickness) aluminum sheets with small holes ϕ 1.5 *mm*.

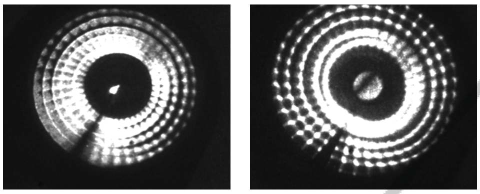
5 EXPERIMENT

An imaging experiment has been performed for a conceptual demonstration for above angular domain filtering. An eLED torch for kids (Fig. 5(a)), inside which a LED lamp is fixed at the center of a cone base, and the inner cone surface is tiled with small mirrors (Fig. 5(b)), was adopted as our object light source.

This object, a consumer CMOS sensor, and two thin lenses (with the same focus lenth of 75 *mm*) were placed in the way described in Fig. 3. Without surprise, for a fixed sensor position, roughly two focused object images could be observed with different object

(a) (b)

Figure 5: The object light source: a), an eLED torch for kids; b), its front view.



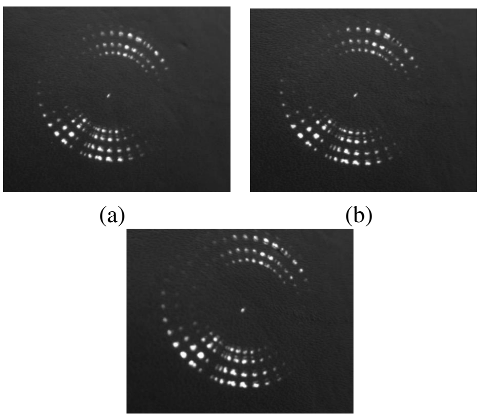
(a) (b)

Figure 6: Two conjugate images of the light source: a), the sharp lamp spot (center) and the surrounding blurred virtual lamps; b), the blurred lamp spot and sharp virtual lamps.

positions: one for the center LED lamp, and another one for the virtual lamp images generated from mirror reflection, as shown in Fig. 6

A near–zero–pass angular domain filtering has been performed in the experiment. Images were recorded from three different object positions: focusing on the virtual lamp, focusing on the center real LED lamp, and the third one further than these two focusing positions. As shown in Fig. 7, all the recorded images contain sharp signals and are similar to each other with the help of near–zero–pass angular filtering.

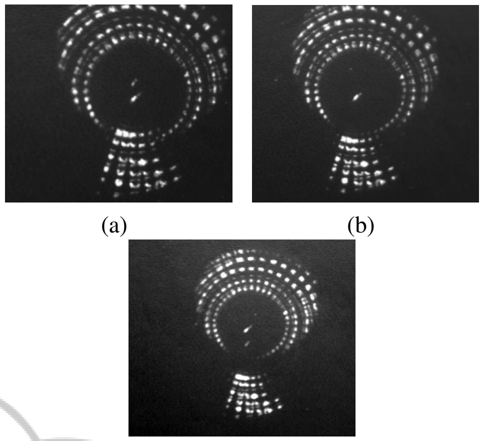
Applying the example filter described in Fig. 4(b), an off–axis angular domain filtering has also been



(a) (b)

(c)

Figure 7: Imaging results from a near–zero–pass angular filtering, at different object positions: a), focusing position of the virtual lamp signal ; b), focusing position of the LED lamp; c), an object position further than a) and b).



(a) (b)

(c)

Figure 8: Imaging results from an off–axis angular filtering, at different object positions: a), focusing position of the virtual lamp signal ; b), focusing position of the LED lamp; c), an object position further than a) and b).

performed. Images were recorded at the same three positions as above. When the object is placed at the focusing position of the center lamp, only one spot was recorded in the image center (Fig.8(b)); Otherwise two vertically separated spots were recorded (Fig.8(a)8(c)).

Based on the recorded images in Fig.8, basic digital image processing techniques were further employed to measure the corresponding object distances. The center LED signals were picked out and enhanced (Fig.9(a), 9(c) and 9(e)); Gravity centers of individual spots were detected from the enhanced images; Afterwards for each image, the lateral distance between two gravity centers of separated spots was measured. In above experiment, three lateral distances were measured: $dy > 0$ for Fig.9(b), $dy = 0$ for Fig.9(d) and $dy < 0$ for Fig.9(f), where dy denotes the vertical distance of $y_i^{\alpha} - y_i^{-\alpha}$; Finally the three object distances for this experiment were calculated from Eq.8 based on the measured lateral distances of dy, and the system design parameters including the sensor position w, the filter parameter s, and two lens parameters f and f_v.

6 DISCUSSIONS

In this section, the concept of measuring distance by angular filtering will be discussed in three conceptual directions: 1), distance from an afocal system; 2), transform by a thin lens; 3), off–axis tele–centric system.

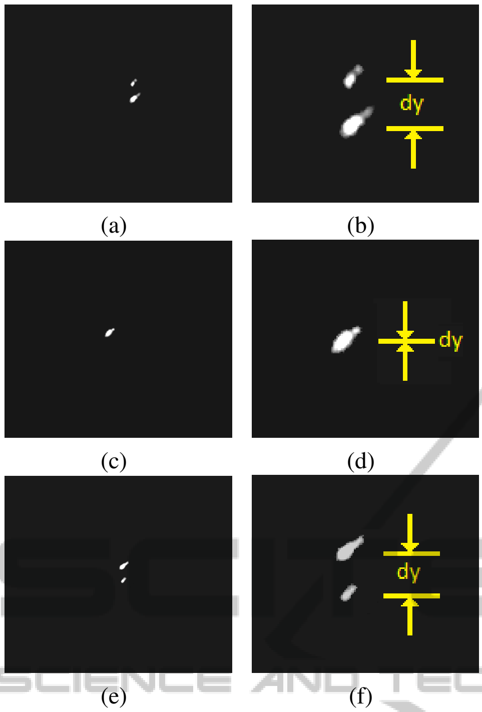


Figure 9: Measurements from an off–axis angular filtering, at different object positions: a) and b), focusing position of the virtual lamp signal ; c) and d), focusing position of the LED lamp; e) and f), an object position further than a) and b). The image b) is a magnified version for better visualization of a), as well as the image d) for c) and image f) for e).

6.1 Distance from an Afocal System

Retrieving object distance from camera parameters has been addressed for a long time. A direct way is based on the thin lens equation $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$, from which the object distance d_o could be directly calculated from the imaging distance d_i and the lens' focus length f, with the condition that the image is well focused.

Considering that the two thin lenses in Fig. 3 work actually as one thin lens with a compound focus length of ∞, say, an afocal lens (Greivenkamp, 2004), the distance relationship between the object and its conjugate image is simplified as:

$$z_p = m(f + f_v) - m^2 w, \tag{9}$$

from which angular separations, $x_i^{\alpha} - x_i^{-\alpha}$ in Eq. 7 and $y_i^{\alpha} - y_i^{-\alpha}$ in Eq. 8, are equal to zero.

Without finely searching for its corresponding conjugate of a measured object in either a focusing system, or an afocal system, this paper suggests to measure its axial distance, i.e., $z_p = z_{lens} - z$, by measuring its spatial separation defined by an angular domain filter, where the image distance w is fixed but could also be varied thus that introducing more flexibility for system design.

6.2 Transform by a Thin Lens

In spite of its wave optics nature, *Fourier* optics could also be explained by geometric optics (Jutamulia and Asakura, 2002). Physically an above afocal system has the same lens layout as a 4–f system in *Fourier* optics. Normal imaging systems are generally based on non–coherent illumination while *Fourier* optics is based on coherent light. With a thin lens, the superposition principle works on the real field resulting in an angular domain for non–coherent rays, while it works on the complex field resulting in a spatial frequency domain for coherent rays. In both situations the transform performed by a thin lens decomposes a source signal in a particular domain, on which filters select preferred components for further signal recording. As shown in Eq. 7 and 8, angular domain filtering provides an opportunity to measure an object's axial position by measuring one 2D distance on its image, which might be either in–focused or out–focused.

6.3 Off–axis Tele–centric Optics

If an aperture stop of a single hole is placed at the angular domain center, only the light rays with near–zero angles, i.e., $\forall\beta, |\alpha| \approx 0$, will arrive in the sensor. All the recorded light rays are parallel to the optical axis; Such an optical system is a bi–lateral tele–centric system.

It is widely understood that a tele–centric imaging system provides more accuracy and better repeatability for image based measurements, which are expected to be theoretically invariant to the object distance because of the parallel–to–axis nature of received signals. This paper further interprets a tele–centric system as a zero–angle–pass filtering system. Moreover, once the aperture stop is shifted from the domain center, signals with other angles of α, β will be selected, where the allowing angles are defined by the spatial 2D position of the aperture stop. Two symmetrically placed filtering holes make it possible to measure the axial position of an object with or without focusing on it.

7 CONCLUSION AND FUTURE WORKS

This paper is based on a general optical system, which appears as the afocal system in telescope (Greivenkamp, 2004), the bi–lateral tele–centric imaging system in photometry (Lenhardt, 2001), as well as the *4–f* correlation system in *Fourier optics* (Goodman, 1968). In principle all these systems use the same lens layout: two thin lenses with a common focus plane. Such a lens layout appears also in many specified applications like *Schlieren photography* (Settles, 2001) in which the concept of angular domain filtering is implicitly employed.

It is well known that with coherence light illumination, the common focus plane of the above two–lens–layout, is a *Fourier* domain (Goodman, 1968) (Jutamulia and Asakura, 2002). This paper further explicitly points out that, for *incoherent light*, the common focus plane is NOT Fourier domain any more, but an angular domain. In this paper a set of 4×4 matrix equations (Eq. 1–4) explicitly describe the forward transform, the domain, as well as the inverse transform.

Based on the above transform interpretation of thin lenses, it is possible to select and exclude information according to its incoming angle. Particularly selecting the near–zero–angle lights, i.e., placing a small–aperture stop at the center of the common focus plane (Fig. 4(a)), the system is a bilateral tele–centric system.

This paper further points out that, shifting the small aperture stop away from the domain center results in non–zero angular domain filtering. Especially, if two symmetrically angular filtering holes are used (Fig.4b), the object distance could be measured from an one–shot image, based on Eq. 7, 8, and 9.

It is not a surprise that an afocal image with finite opening mask contains information about the object distance. But to retrieval such distance information often needs to analyze blurred 2D images. In this paper the distance information is directly calculated from lateral separation of sharp signals, rather than analyzing blur information from input data.

Some questions are still open for future works. At first, the experiment described in this paper is a qualitative one. More experiments should be performed for quantitative evaluations like the object defocus amplitude, measurement resolution, as well as the system error function; Second, this paper is limited to the same scale of geometric optics based on the paraxial approximation. The measuring accuracy of an object distance in Eq. 7 and 8 should be further evaluated since there is a gap between the physical reality and mathematical models. For instance, the zero–angle–pass filter in a tele–centric system has a finite aperture size so that recorded signals are “parallel enough” to the optical axis rather than strictly “parallel” to it. Such a gap should also be further addressed for accuracy evaluation; Third, a filter placed in the angular domain does not only select and exclude signals according to its domain position as well as its aperture shape, but also heavily reduce the energy projected on the sensor. Digital signal enhancement and object detection from a weak energy image should be also taken into account for designing angular domain filtering systems.

ACKNOWLEDGEMENT

The author would like to thank Dr. Hexin Wang, Dr. Christopher Weth, Dr. Matthias Reich, Mr. Ralf Ebersbach, and other anonymous reviewers for their valuable suggestions.

REFERENCES

Berger, C. (2002). Design of telecentric imaging systems for noncontact velocity sensors. *Optical Engineering*, 41(10):2599–2606.

Djidel, S., Gansel, J. K., Campbell, H. I., and Greenaway, A. H. (2006). High-speed, 3-dimensional, telecentric imaging. *Optical Express*, 14(18):8269–8277.

Gerrard, A. and Burch, J. M. (2012). *Introduction to Matrix Methods in Optics*. Dover Publications, Incorporated.

Goodman, J. W. (1968). *Introduction to Fourier Optics*. McGraw-Hill, New York.

Greivenkamp, J. E. (2004). *Field guide to geometrical optics*. SPIE Press, Bellingham, WA.

Jaehne, E. and ecker, H. H. (2000). *Computer Vision and Applications: A Guide for Students and Practitioners*. Academic Press.

Jutamulia, S. and Asakura, T. (2002). Optical fouier-transform theory based on geometrical optics. *Optical Engineering*, 41(01):13–16.

Kim, J.-S. and Kanade, T. (2011). Multiaperture telecentric lens for 3d reconstruction. *Optics Letters*, 36(7):1050–1052.

Lenhardt, K. (2001). Optical measurement techniques with telecentric lenses. In *Know How: Technical and Scientific Contributions*, pages 1–61. Schneider Kreuznach.

Settles, G. S. (2001). *Schlieren and shadowgraph techniques: Visualizing phenomena in transparent media*. Springer-Verlag, Berlin.

Watanabe, M. and Nayar, S. K. (1997). Telecentric optics for focus analysis. *IEEE Trans. on PAMI*, 19(12):1360–1365.

Zeuch, N. (2000). *Understanding and Applying Machine Vision, Second Edition, Revised and Expanded*. CRC Press.