Laplacian Unitary Domain for Texture Morphing

Antoni Gurguí[1], Debora Gil[1] and Enric Martí[2]
[1]*Computer Vision Center of Catalunya, Universitat Autònoma de Barcelona, Bellaterra , Barcelona, Spain*
[2]*Department of Computing Sciences, Universitat Autònoma de Barcelona, Bellaterra , Barcelona, Spain*

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Abstract: Deformation of expressive textures is the gateway to realistic computer synthesis of expressions. By their good mathematical properties and flexible formulation on irregular meshes, most texture mappings rely on solutions to the Laplacian in the cartesian space. In the context of facial expression morphing, this approximation can be seen from the opposite point of view by neglecting the metric. In this paper, we use the properties of the Laplacian in manifolds to present a novel approach to warping expressive facial images in order to generate a morphing between them.

1 INTRODUCTION

Image morphing is the process of obtaining the sequence of intermediate images that transforms an image into another one. Morphing techniques create powerful visual effects that are widely used in the entertainment industry and have applications in medical visualization and industrial design (Smythe, 1990; Zell and Botsch, 2013).

The morphing sequence is obtained as an interpolation between two or more images and it is essentially based on two steps (Wolberg, 1998):

- Warping: Deformation of an image A to another B and vice versa according to a percentage between two symmetric deformations (40% and 60%, 70% and 30%) generating two images.
- Cross Dissolve: Merging two images to get the final one.

As there may be infinite transformations from one image to another, the desired transformation is computed from a correspondence established between pairs of key line segments, points or meshes which specify image features or landmarks. This feature correspondence is used to compute warps that interpolate the positions of such features across the morph sequence and extend it to all image pixels. The definition of the warping at pixel level is a main challenge, especially in case of facial images morphing which has to address the large variability across individuals facial shape and the elasticity associated to emotions.

There exists several image morphing methods, depending on how the correspondence is established and how the images are warped. Feature-based morphing (Beier and Neely,) uses line segments to establish the correspondence between the two images. To generate the sequence at pixel level, the influence of each line is blended to interpolate line correspondence to the whole image warp. A main limitation (Wu and Liu, 2012) is the identification of the key lines describing the geometry of the image objects that have to be matched, especially for objects with curved borders (like faces). Another approximation is to employ scattered-data interpolation methods (like radial basis function or thin plate splines (Lee et al., 1996)) to extend landmark correspondence to the whole image. In this case, the warp between images is obtained by constructing two surfaces (one per image) interpolating the set of scattered key points. Though fast and intuitive, scattered-data interpolation are prone to introduce distortion artifacts near object boundaries (Lipman et al., 2008) and blurring and ghosting effects across in-between images (Wu and Liu, 2012). Recent methods for reducing the impact of ghosting and blurring (Wu and Liu, 2012) are complex strategies that require the minimization of an energy using the alpha-expansion algorithm (Boykov et al., 2001).

Differential geometry representations are commonly used for texture mapping and three dimensional meshes remeshing (Yoshizawa et al., 2004; Floater and Hormann, 2005; Desbrun et al., 2002). The idea is to map the 3D mesh onto the 2D image plane and assign the mapped vertices coordinates to the texture coordinates of original vertices. This is

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equivalent to finding a suitable parametric map between the 3D surface and the 2D texture domain (Spivak, 1965). Many methods (Yoshizawa et al., 2004; Lévy et al., 2002; Desbrun et al., 2002) define such texture coordinates by using the solution to the Laplacian to compute each parametric coordinate function. By the mean value theorem (Evans, 1998), solutions to the Laplacian can be computed on irregular meshes as solutions to a sparse linear system that assigns weights to adjacent vertices. Such weights determine the deformation that the texture will undergo when mapped back to the 3D surface.

Existing mappings depend on the approximation considered to define the weights of the Laplacian system: barycentric coordinates (Tutte, 1963), mean value coordinates (Floater and Hormann, 2005), or angle-based coordinates (either conformal (Eck et al.,) or area-preserving (Desbrun et al., 2002)). In any case, all of them approximate the Laplacian in the cartesian domain which implicitly assumes an identity metric for computing deformations. A main limitation for a realistic texture deformation is that they might introduce areas of high local stretch (Yoshizawa et al., 2004) that visually distort textured patterns. Another concern in the context of face expression synthesis is that none of them guarantees that the assigned texture coordinates are consistent across different subjects and expressions. Although, such registration condition could be forced by using Dirichlet constrains at some anatomical locations (Xu et al., 2009; Vera et al., 2014), such constrains could introduce folds in the final parametrization in case of large deformations across cases.

In this paper, we use the theoretical properties of solutions to the Laplacian to define texture coordinate changes suitable for the synthesis of facial expression images morphing. In particular, we present the use of an identity metric with fixed boundary conditions to define a unitary texture mapping that allows morphing of texture expressions by straight interpolation of texture values in the unitary domain. We have applied our method to the morphing of real frontal face textures from the BosphorusDB (Savran and Sankur, 2008) public database. Several transitions illustrate the accuracy and flexibility of unitary coordinates for expression and identity morphing.

2 EXPRESSIVE TEXTURE MORPHING IN AN UNITARY LAPLACIAN DOMAIN

A facial expression is given by the geometry of the expressive face and a function defining the color and texture of the face. Therefore, expression morphing should interpolate face vertex positions and its correspondent texture values. We use the solutions to the Laplacian to define a transformation that maps a given facial geometry to a common unitary domain for the interpolation of texture values.

Mathematically, a face is a 2D surface, S, that admits an embedding in $\mathbb{R}^3$. Thus, it exists a parametric map, ϕ, from a regular domain, $\mathcal{D} \in \mathbb{R}^2$ to S:

$$\begin{gathered}\mathbb{R}^2 \supset \mathcal{D} \xrightarrow{\phi} S \subset \mathbb{R}^3 \\ \mathbf{u} = (u_1, u_2) \longrightarrow (x_1(\mathbf{u}), x_2(\mathbf{u}), x_3(\mathbf{u}))\end{gathered}$$

The map ϕ should be a differentiable bijection between the surface and $\mathcal{D}$, so that the mapping of the level curves of the coordinates (u_1, u_2) define a mesh on the surface (Spivak, 1965). In the case of face parametrization (Floater and Hormann, 2005), $\mathcal{D}$ is usually a common unitary domain (either a square or a circle), which boundary, $\delta\mathcal{D}$, is mapped to the face surface boundary, δS. Surface geometric properties (Spivak, 1965) are encoded in the 2×2 matrix of the first fundamental form or metric, $g = (g_{ij})_{i,j}$ defined from ϕ first partial derivatives (Jacobian).

In this context, face texture is a function from the parametric domain to the real numbers:

$$I = I(u_1, u_2) : S \longrightarrow \mathbb{R}^n \tag{1}$$

where $n = 1$ in case of grey level textures and $n = 3$ in case of color images. Any transformation on the surface is given by a suitable coordinate change in $\mathcal{D}$, $\varphi(u_1, u_2) = (v_1(\mathbf{u}), v_2(\mathbf{u}))$. We will use the solutions to the Laplacian to compute a coordinate change that maps any mesh to common unitary coordinates.

Laplacian operators (Evans, 1998) are a powerful mathematical tool to compute differentiable functions on manifolds with values fixed at some locations. Let $g = (g_{ij})_{i,j}$ denote the manifold's first fundamental form and $g^{-1} = (g^{ij})_{i,j}$ its inverse matrix. The Laplacian is defined (Davies, 1989) as the divergence operator associated to that metric and a function is harmonic if the divergence of its gradient cancels. A function components v_k, $k = 1, 2$ are computed as:

$$\begin{aligned}\Delta_g v_k &= div_g(\nabla_g v_k) = \\ &= \frac{1}{det(g)} \sum_i \partial_{u_i} \left(det(g) (\sum_j g^{ij} \partial_{u_j} v_k) \right) = 0 \\ &\text{with } v_k|_{\delta\mathcal{D}} = \delta v_k \end{aligned} \tag{2}$$

for det denoting the determinant of a matrix and δv_k a differentiable function defined on the boundary of the domain, $\delta\mathcal{D}$. In case that $(\delta v_1, \delta v_2) \in \delta\mathcal{D}$, by the maximum principle for harmonic functions (Evans, 1998), we have that $\varphi = (v_1, v_2)$ defines a unique map from $\mathcal{D}$ onto itself.

By setting $g = Id$, for Id the 2×2 identity matrix, and $\delta v_k = u_k|_{\delta\mathcal{D}}$, we obtain the Laplacian in $\mathbb{R}^2$ and the coordinate change defines a uniformly distributed mesh. Further, by setting boundary conditions at equal fixed values, the solution to (2) with identity metric defines a coordinate change that maps any facial surface to equal unitary coordinates.

In the case of interpolation of textured values, surfaces correspond to the image domain and the unitary coordinates can be taken in $\mathcal{D} = [0,1] \times [0,1]$. Given two different image surfaces, S_1, S_2, and their texture intensities, I_1, I_2, the following diagram:

$$(S_1, Id) \xleftarrow{\phi_1^{-1}} (\mathcal{D}, Id) \xrightarrow{\phi_2^{-1}} (S_2, Id)$$

induces a composition of the texture intensities $I_1^* := I_1 \circ \phi_1^{-1} = I_1(\phi_1^{-1})$, $I_2^* := I_2 \circ \phi_2^{-1} = I_2(\phi_2^{-1})$ defined at the same coordinate values. Therefore, they can be linearly interpolated to define a transformation between the two textures in the unitary domain:

$$I_t^* := tI_1^* + (1-t)I_2^* \tag{3}$$

for $t \in [0,1]$ the texture morphing sequence parameter.

2.1 Implementation in a Discrete Domain

Solutions to the Laplacian in manifolds (Davies, 1989) satisfy an interesting property for implementation of expression morphing. By the mean value theorem (Evans, 1998), the function v_k, $k = 1,2$, can be defined by its values in a ball centered at each point as the average given by:

$$v_k(\mathbf{u}) = \frac{1}{||B_\mathbf{u}||_g} \int_{\delta B_\mathbf{u}} v_k(\tilde{\mathbf{u}})\, d\tilde{\mathbf{u}} \tag{4}$$

for $||B_\mathbf{u}||_g$ the ball area given by the metric g and $d\tilde{\mathbf{u}}$ the area element of the ball boundary, $\delta B_\mathbf{u}$.
The mean value formulation (4) can be easily implemented for irregular surface meshes. Let $\mathcal{M} = \{\mathbf{V}_i, T\}$ be a triangular mesh on $\mathcal{D}$, where $\mathbf{V}_i = \mathbf{u_i} = (u_{1i}, u_{2i})$ is a discrete sampling on $\mathcal{D}$ and T is a triangulation defining vertex connectivity. Then eq. (4) is approximated by the weighted sum:

$$\begin{aligned} w_k(\mathbf{u_i}) &= \frac{1}{\sum (detg)_j} \sum_{u_j \in N_i} (detg)_j w_k(\mathbf{u_j}) = \\ &= \frac{1}{\sum_j a_{ij}} \sum_{u_j \in N_i} a_{ij} w(\mathbf{u_j}) \end{aligned} \tag{5}$$

for N_i the 1-ring of each vertex in the discrete mesh, $\mathbf{u_i}$, and $(detg)_j$ the metric determinant evaluated at $\mathbf{u_j}$. It follows that solutions to the Laplacian are fast to compute in the form of a linear system:

$$Aw_k = b \tag{6}$$

The matrix A codifies the weighted average given by (5) and b is a vector that contains the boundary conditions. In case that we have the identity metric, i.e. $(detg)_i = 1\ \forall i$, the matrix A is simply the adjacency matrix of the triangulation and the map $\varphi = (w_1, w_2)$ defines a mesh uniformly distributed inside $\mathcal{D}$, provided that the adjacency is constant.

Texture interpolation can be easily implemented using the solutions to (6) as follows. Given two different meshes, $\mathcal{M}_1$, $\mathcal{M}_2$, and their texture images, I_1, I_2, their morphing sequence is obtained by interpolating vertex positions, as well as, their texture values.

Intermediate vertex positions, $\mathcal{M}_t = \{\mathbf{V}_t, T\}$, can be directly obtained by a linear interpolation:

$$\mathbf{V}_t = t\mathbf{V}_1 + (1-t)\mathbf{V}_2$$

for the sequence parameter $t \in [0,1]$. In order to interpolate intensity values for all pixels between two different texture images, I_1, I_2, we should deform images to obtain two warped versions, I_1^*, I_2^*, such that face structures are matched. In the context of Laplacian coordinate changes, this can be achieved by transforming mesh vertices, $\mathcal{M}_1$, $\mathcal{M}_2$, to common unitary coordinates in $\mathcal{D}$ that take equal values for points belonging to the same face structure. Such a transformation is computed using the solution to the Laplacian with identity metric.

Intermediate texture values, are computed by evaluating formula (3) at the intermediate vertex positions $\mathbf{V}_t$ using the composition with the coordinate map, ϕ_t, that maps $\mathcal{M}_t$ to the unitary coordinates:

$$\begin{aligned} I_t &:= I_t^* \circ \phi_t = I_t^*(\phi_t) = \\ &= (tI_1(\phi_1^{-1}(\phi_t)) + (1-t)I_2(\phi_2^{-1}(\phi_t)) \end{aligned}$$

for $t \in [0,1]$.

Figure 1 illustrates the whole strategy for texture morphing using interpolation in Laplacian unitary coordinates. We show meshes (in blue) and the corresponding texture images for: the original space (left column), the Laplacian unitary domain (central column) and the final textured meshes using the interpolated images in the Laplacian unitary domain (right column). Top row shows the source data, $\mathcal{M}_1, I_1$, bottom row the target data, $\mathcal{M}_2, I_2$, and the central row an intermediate step of the deformation between source and target. In the original domain, vertex positions can be interpolated from source to target as they are from corresponding points on same face structures. However, texture values can not be directly interpolated since images are not registered. After transforming to Laplacian unitary coordinates, vertex positions

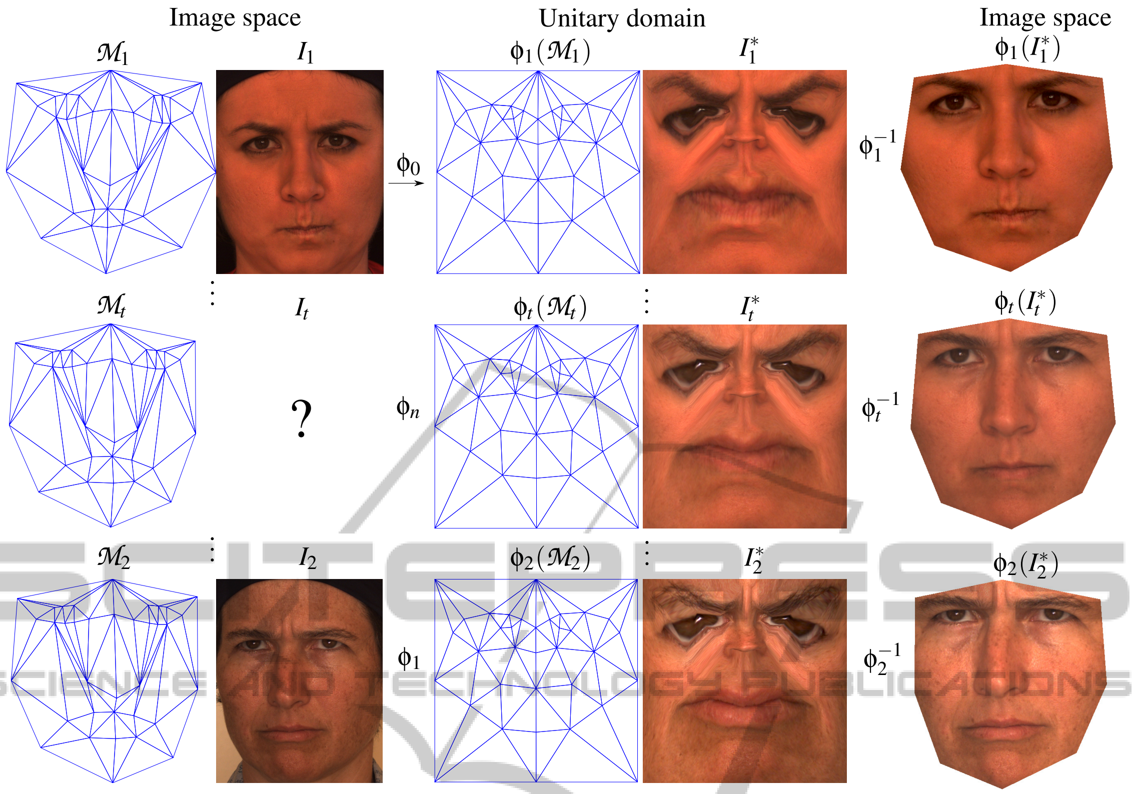

Figure 1: Mesh morphing and texture morphing through Unitary Metric parametrization.

are equal for all meshes and, thus, the transformed images are registered and can be interpolated to create the intermediate textures. In the right column, such textures are rendered on the interpolated meshes.

3 EXPERIMENTS

The goal of our experiments is to illustrate the potential of Laplacian unitary coordinates for the morphing of textures coming performing different facial expressions. In order to do so, the model described in Section 2.1 has been applied to deform 2D frontal face meshes and textures selected from the BosphorusDB (Savran and Sankur, 2008) public database[1].

We have considered 3 different kinds of morphing transitions between: different expressions from the same person, different persons performing the same expression and different persons performing different expressions. The first transition checks the compatibility/accuracy for expression morphing. The second transition checks identity morphing accuracy. The last one is the more general and complex case and illustrates the ghost effect (Wu and Liu, 2012).

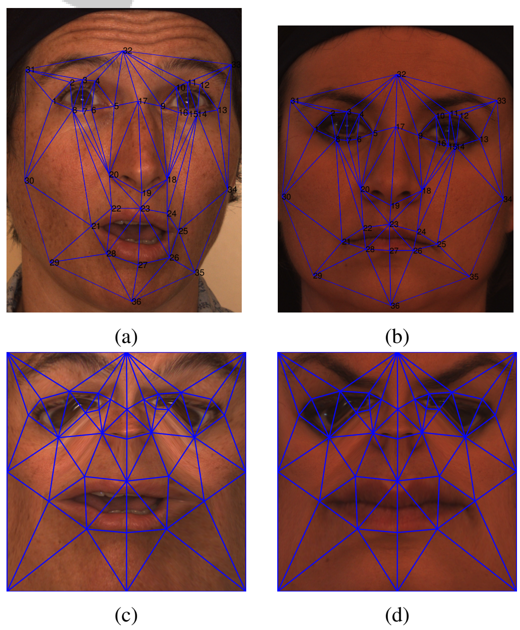

Figure 2: BosphorusDB Examples in the cartesian, (a), (b), and unitary Laplacian, (c),(d), domains.

Faces have been sampled using a sparse set of 36 landmarks that define the main facial structures (eyes, lips, nose and face boundary). Figure 2 shows two

[1]available at http://bosphorus.ee.boun.edu.tr/

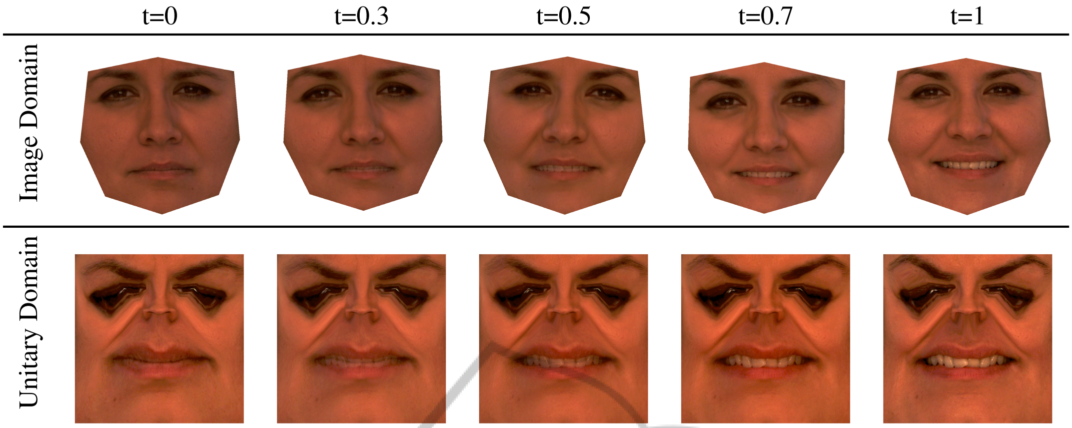

Figure 3: Morphing of different expressions of the same subject.

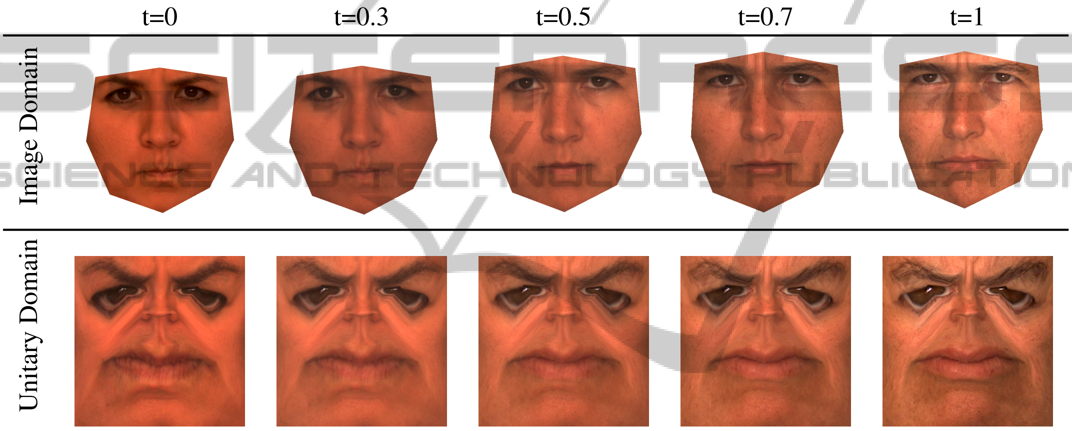

Figure 4: Morphing of the same expression from different subjects.

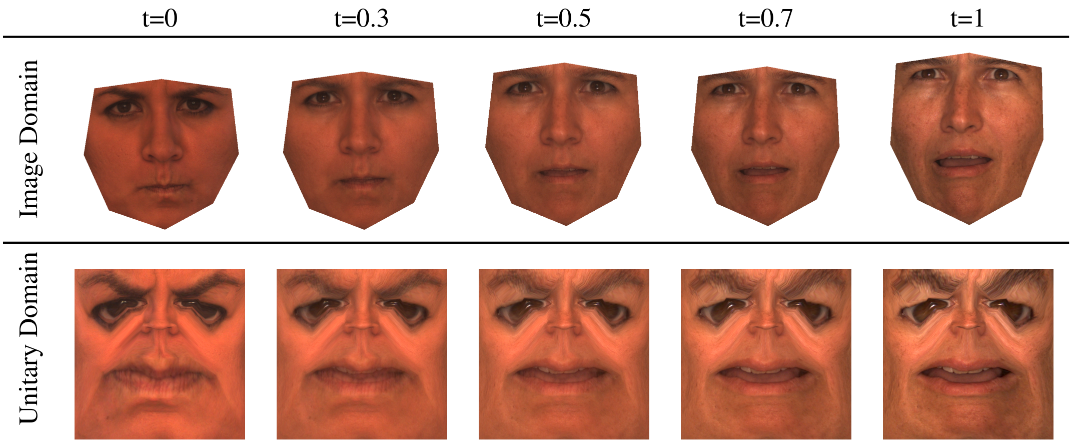

Figure 5: Morphing of different expressions from different subjects.

examples of the BosphorusDB faces from two different subjects, one expressing surprise (fig.2(a)) and the other one with a neutral expression (fig.2(b)). The triangulation defined by the sparse set of selected landmarks (numbered from 1 to 36) is shown in blue lines. Such landmarks have been used to compute the unitary Laplacian coordinates.

Bottom images in fig.2 show the transformation of texture coordinates and image to the unitary Laplacian domain computed over the mesh of landmarks.

We observe that texture coordinates (blue lines) coincide for, both, expressions and subjects. Thus, individuals and expressions are registered in these new coordinates. Since, by definition, such coordinate are a diffeomorphism (i.e. infinitely differentiable), registration in the unitary domain guarantees an implicit registration in the image domain through the inverse change. This allows the morphing between textures from different expressions and subjects.

Figure 3 shows the texture deformation of two different expressions (sadness and happiness) of the same person, figure 4 the deformation of the same expression (anger) for two different persons and figure 5 the deformation between different identities and expressions. For all cases, deformations in the cartesian image domain are shown in first rows, while the corresponding deformations in the unitary Laplacian domain are shown in second rows. Each column corresponds to the deformation at different times, $t = 0, 0.1, 0.3, 0.5, 0.7, 1$, with the original textures at $t = 0$ and $t = 1$. The implicit registration of texture coordinates achieved in the unitary Laplacian domain allows smooth interpolation of texture color values, regardless of the expression and subject identity. Original expressions and identities are recovered by the inverse mapping from the unitary to the cartesian image domain. Since Laplacian coordinate changes are infinitely differentiable, texture deformation is smooth in both, unitary and cartesian image domains.

4 CONCLUSIONS

Solutions to the Laplacian constitute a unique tool for defining smooth coordinate changes that could put into correspondence different meshes. Such an elastic registration, could be the final stage in the synthesis of facial expressions including their texture. In order to get realistic expressions, the Laplacian system weights should be tuned according to the geometric deformation that the surface undergoes, so that coordinate changes solve the Laplacian in a manifold.

In this paper, we have presented a first study on the properties of the Laplacian in manifolds for understanding the deformation that the coordinate change induces. We have shown the potential of such solutions for expression synthesis by presenting a user-friendly texture morphing and a texture expression synthesis from a sketch. The promising results encourage further research on the use of Laplacian in manifolds for affective avatars synthesis. First, the role of the Dirichlet conditions will be investigated to actually register anatomies without mesh folding. Such registration will be used to get a model of face identity and expression for the synthesis of realistic avatars. Simultaneously, temporal models for and 4D data (3D geometry+time) will be used to build a model of facial expression deformations to animate static face meshes or to synthesize realistic avatars.

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