

Project-oriented Education as a Platform for Transfer of Math KSA

Borislav Lazarov

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria

Keywords: KSA Transfer, Project-oriented Education, Decontextualization.

Abstract: Under consideration is the transfer of math knowledge, skills and attitude (KSA) built in project-oriented education to curriculum problems using mathematical modeling. Two teaching experiments were conducted to study the existence or lack of such kind of transfer. There were two experimental groups of regular 7th grade students. The first experiment (held in December 2013) aimed to clarify how students apply their KSA developed through project-oriented activities in solving standard curriculum problems. The outcomes from this experiment were far from satisfactory – we registered just partial and incomplete (or none) transfer of KSA for the largest proportion of the population. The second experiment (in November 2014) was done in more complex educational environment which included dynamic geometry software. This small change caused an unexpectedly larger positive effect in students' activeness and acquisition of the topic. The statistics showed that the largest part of the students is trying to apply (more or less successfully) the knowledge in another context. This gives us reason to believe that the project-oriented education is an appropriate platform for decontextualization of math KSA.

1 INTRODUCTION

Different viewpoints regarding the transfer of knowledge, skills and attitude (KSA) are presented in a number of publications, e.g. a comprehensive study on the transfer of math KSA to solving physics problems (Roorda et al., 2014) is done from actor-oriented perspective. In fact, any application of math KSA is a kind of transfer. For instance, the use of computers in solving math problems requires *conceptual understanding of mathematics and understanding how to apply mathematics* (Gravemeijer, 2014) which is, par excellence, transfer of KSA.

Our standing point is slightly different: we consider the **transferability** and **multifunctionality** of KSA as an indicator for student's competence of synthetic type (synthetic competence). This is especially important for the middle school math KSA because of the higher level of abstraction, on one hand, and on the other hand, the fact that *"mathematical behavior" is ... describing reality through constructs and processes which have universal application* (European Commission, 2004). However, the mathematics KSA are usually developed in some clinical conditions and our observations are that, when built in such way, the KSA are rarely applicable outside the particular educational context (Lazarov, 2014). So, every time we speak about synthetic competence, the

next question about **decontextualization** stands on agenda: *are math KSA transferable and multifunctional*. Recently our efforts to find a platform enabling the transfer of math KSA are connected mainly with the integrated approach and project-oriented education (POE). Below we share our findings in two teaching experiments with 7th grade students (middle of the secondary school). This population has been chosen in compliance with the expectations of the European Commission: *the key competences to be built at the end of the compulsory education* (European Commission, 2004) which is the 8th grade in Bulgaria.

2 MODIFICATION OF THE EDUCATIONAL PARADIGM WITH RESPECT TO POE

The educational paradigm in Bulgarian traditional school could be stated as *math KSA to be formed in classroom environment focused on reaching the official educational standards* (Bulgarian Ministry of Education, 2006). A nontraditional form that serves this paradigm is the project-based education (or project-based learning, also project-based problem solving): it aims at forming KSA in a particular subject like

physics, biology, etc., in the frame of the classroom activities; the indicators of progress are related to the educational standards (Dimitrova, 2008). In one sentence: the project-based education operates in the same educational context as the traditional classroom education, upgrading it with new types of activities. The domain of the project-oriented education is extracurricular and facultative forms or interdisciplinary activities. The educational goals of POE are connected with the applications of KSA which are (expected to be) built during regular school practice in a different educational context. The role of POE is auxiliary and aims to support the transition of analytical subject knowledge into synthetic one. From this perspective, the following educational paradigm could be determined:

- basic KSA to be formed in traditional classroom practice;
- the transferability and multifunctionality of KSA to be checked in different context;
- KSA to be turned into competence of synthetic type by providing appropriate platform (like POE).

However, sometimes new KSA are required for building bridges between school subjects that are included in a particular project initiative and these KSA become a product of POE. Then a reverse question appears: *is such new knowledge applicable to some curriculum topics and how does it correspond to the educational standards*. In other words *are the KSA, that are formed in auxiliary educational activity, transferable to the traditional classroom context*.

3 INCORPORATING POE IN SCHOOL STRATEGY

The project-oriented education could be successfully implemented as an upgrade of the traditional classroom practice but there should be enough space for it. The general mathematics schedule is overloaded with topics closely related to the educational standards. This is why some project-oriented initiatives should be accommodated as facultative ones. In the years 2013/2014 and 2014/2015, we orchestrated two teaching experiments to clarify the degree of transferability of math KSA from POE context into regular classroom problem-solving, and as a result to evaluate the achievement of the compulsory math standards (Bulgarian Ministry of Education, 2006). Both experiments took place in the frame of an elective math course with 7th grade students.

The integrated approach was adopted as a school strategy in a Bulgarian private secondary school (Lazarov and Severinova, 2014). The mathematics curriculum was split into two independent parts: the compulsory content for the Bulgarian schools and a complementary mathematics according to the Level Up Maths course (Pledger, 2009). There is a poor connection between these two parts which causes serious problems for students while building even some basic KSA in many math topics. We saw a possible cure in organizing POE and we applied it. However, whether POE could be such cure at all, we tried to understand by examining the transfer of some math KSA.

4 PARAMETERS OF THE STUDY

The experimental groups were of 16 students for the first experiment and 10 students for the second, all of them 7th graders (13-14 years old). Students were divided in teams (up to 3 members) and each team was given a project assignment. The preparation and project activities lasted 4 weeks.

4.1 Project Assignments

The Level Up Maths course (ibid.) contains three lessons on solving simultaneous equations: one dedicated to the graphical method, another one about some analytical methods and a special lesson called Top Profit where two optimization problems of linear programming were given to illustrate "the big idea" of implementation of simultaneous equations. We prepared project assignment for the teams which included:

- three given problems to be solved in paper-and-pencil (P&P) mode,
- another problem of the same type to be posed and solved,
- dynamic-geometry-software (DGS) applets to be composed for illustration and experimental work,
- computer presentation to be designed and performed.

4.2 The Assignment Problems

The first two problems for the project assignments were modifications of the ones from the textbook (Pledger, 2009); the third problem was taken (directly or slightly modified for the different teams) from a university math course (Jagdish and Lardner, 1989).

The fourth problem was to be stated and solved by the teams. Here is one set of assignment problems.

Problem 1. A company manufactures two phone models: X and Y. It takes 4 hours to produce each item of type X and the company makes a profit of 5 levs¹ from such an item; it takes 5 hours to produce each item of type Y and the company makes a profit of 11 levs from it. The company can produce at most 300 phones of each type and has 1600 hours to complete the order. Find the number of phones of each type that should be produced to make the largest profit.

Problem 2. A company manufactures two bicycle gears: X and Y. It takes 4 hours to produce a gear X and the company makes a profit of 7 levs from it; it takes 2 hours to produce a gear Y which makes a profit of 3 levs. The company can produce at most 400 gears of type X and 300 Y gears; it has 2000 hours to complete the order. Form four equations as follows: about the maximum number of hours; about the maximum number of each gear type; about the profit. Plot the graph of the first three equations (on a sheet of paper and by GeoGebra). Find the point that gives the largest profit.

Problem 3. A company manufactures two products: X and Y. Each item of type X needs 5 units of raw materials and 2 units of energy; each item of type Y needs 3 units of raw materials and 4 units of energy. The company has available 105 units of raw materials and 70 units of energy. It can make a profit of 200 levs from each item X and 160 levs from each Y. Find the number of items of each type that should be produced to get the largest possible profit.

Problem 4. State and solve a problem about top profit following the model of Problem 3. (Figure 1)

Our own problem:

A company produces two products: K and T. For producing one K product are used 10 ones of raw materials and 6 ones of energy; to produce on product T are used 4 ones of raw materials and 8 ones of energy. The profit of one product K is 100 lv and for one T is 80 lv. The company has 410 ones of raw materials and 140 ones of energy. What amount of each product could be produced to find the largest profit? What is that profit?

Figure 1: One of the 4th (students') problems.

4.3 Data Collection and Indicator

The data to be analyzed were collected from the students' P& P work, from the project presentations, and from the results of a control test. The indicator we observed was

¹Lev is the Bulgarian currency

Transfer of knowledge and skills about simultaneous equations built in a POE context to standard problem-solving skills, including:

- I1 – introduction of relevant notations;
- I2 – composition of proper equations (modeling a situation);
- I3 – skills in solving linear simultaneous equations;
- I4 – qualitative interpretation of the quantitative data.

The progress of this indicator we examined by a control test of 4 standard problems, taken from a training book (Savova et al., 2010) for the exam at the end of the 7th grade (see the Appendix). The first two problems are routine ones and allow reproductive approach; the third one requires transfer of some algebraic knowledge and skills to geometrical context. The fourth problem is also standard but at that moment the students have not been introduced to this particular problem-solving technique yet, so this problem seems to be of productive (creative) type. However, there is some common sense reasoning that allows to solve the fourth problem without calculations.

5 KSA BUILT DURING THE POE

The outcomes of the project work in both experiments were satisfactory in general.

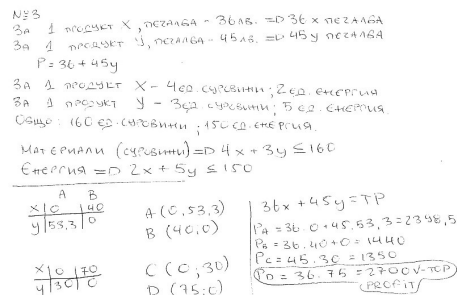


Figure 2: P& P fragments from a team project.

In figures: 5 of 6 teams during the first experiment and 3 of 4 teams during the second one presented their projects. Most of the teams demonstrated reasonable KSA in both graphical and analytical method (Figure 2), they prepared accurate diagrams and drawings. Some students used shorthand as thick and cross to avoid long explanations in their P& P part of the project.

The presentation files were properly designed. Most of the students demonstrated considerable level of reasoning and interpretation of the solutions during the performance (Figure 3). There were several marginal cases in the 4th problem because the prob-

lem data were not adjusted (the students chose figures from their common sense perspective) and even then the teams gave sensible explanations. All these observations gave us the reason to claim that math KSA concerning the modeling of real-life situations by simultaneous equations and solving them are formed in these particular project-oriented initiatives.

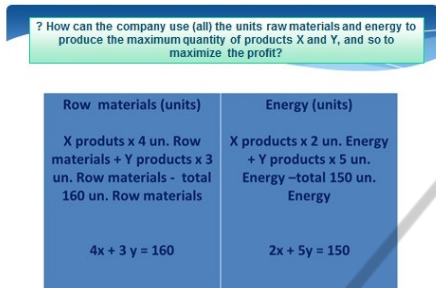


Figure 3: Analyzing data to form equations (screen from a team presentation).

We did not perform enough preparation for the dynamic-geometry-software activities during the first experiment mainly because of our improper estimation of the time necessary for students to learn computer applications. After one introductory lesson about GeoGebra we made just a few demonstrations how to apply the software in solving simultaneous equation graphically. This gap was filled up during the second experiment by two more lessons dedicated to DGS applets. The DGS skills built during these two lessons were enough for the half of the students to make appropriate applets (Figure 4).

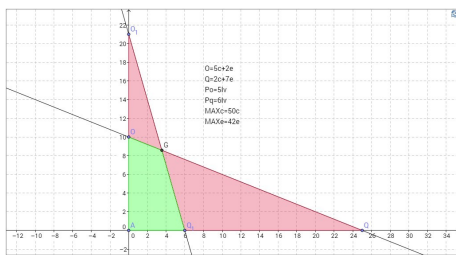


Figure 4: Students' graphical illustration for the 4th project problem.

6 TEST RESULTS

We find the students' achievements in both experiments close to the stated educational goal. But the test results of the first experiment are significantly lower than the ones of the second experiment.

6.1 The First Experiment

The test was done by 6 students. Of them: 1 student managed to apply the method to solve 2 of the 4 problems by simultaneous equations and the rest by common sense and basic skills; 1 student applied the method to 1 problem and solved two more problems; 3 students tried to apply a kind of a mix of 'ideas', but far from satisfactory; 1 student did nothing. The rest of the group did not do the test but showed poor knowledge and skills in solving problems (very similar to the test ones) during the regular lessons after the experiment.

6.2 The Second Experiment

The test was done by 9 students. One of them returned blank sheet. The first problem was solved by the other 8 and they all applied the simultaneous equations. The second problem was solved by 4 students completely and 1 succeeded to compose the system of equations. The third problem was solved by 3 of the students and 1 succeeded to compose the system of equations. Four students composed a relevant system for the last problem but none managed to solve it; one of them made qualitative justification that there is no solution of the modeled situation.

7 SOME SPECIAL CASES FROM THE SECOND EXPERIMENT

We are going to give more details about the second experiment because it fits better the educational goal and the outcomes are more representative. Table 1 shows the progress made by students (S1-S9) in solving the test problems (TP1-TP4) according to the indicator items (I1-I4).

Now we are going to analyze the performance of some students whose results could be considered emblematic for the different stages of transfer of KSA.

Table 1: The coverage of the indicator.

	TP1	TP2	TP3	TP4
S1	I1,I2,I3	I1	-	I1, I2
S2	-	-	-	-
S3	I1,I2,I3	I1,I2,I3	I1,I2	I1
S4	I1,I2,I3	I1,I2,I3	I1,I2,I3	I1,I2
S5	I1,I2,I3	-	-	-
S6	I1,I2,I3	-	-	I1,I2
S7	I1,I2,I3	I1,I2,I3	I1,I2,I3	I1,I2,I4
S8	I1,I2,I3	I1,I2,I3	I1,I2,I3	I1
S9	I1,I2,I3	-	-	-

7.1 Excellence

Student S7 gained the full score on the test (the only one). She passed quickly through the first three problems. Then she composed an equation for the fourth problem and observed (comparing coefficients) that it cannot be solved in positive numbers. S7 demonstrated high style of writing, proper introduction and usage of notations, skills in algebraic transformations, qualitative estimation of quantitative values. According to our indicator, we can claim that there is transfer of KSA in full scale.

7.2 Satisfactory

Students S3, S4 and S8 managed to solve the first two problems completely; S4 and S5 solved problem 3 and S3 composed the system of problem 3. Let us stress that these students applied algebraic method to geometrical problem which requires connections between two concepts and corresponding notations (Figure 5). We consider this to be the desirable normal case: transfer of KSA at operational level into new context.

Задача 3. Площади на квадрата и правоъгълника е 85. Общата им страна е 17 cm. На колко е разликата между страните на квадрата?

$$\begin{aligned} S_a - S_b &= 85 \\ a - 5 &= b \\ a - b &= 5 \end{aligned} \quad \begin{aligned} a^2 - b^2 &= 85 \\ (a+b)(a-b) &= 85 \\ a+b &= 17 \\ a+a-5 &= 17 \\ 2a &= 22 \\ a &= 11 \text{ cm} \end{aligned}$$

Figure 5: S4's solution of the 3rd test problem (S_a and S_b stand for the area, P_a stands for the perimeter).

7.3 Below Average

Students S1, S5 and S9 solved completely only problem 1, applying simultaneous equations. S5 and S9 did not present their project and we will skip the discussion about transfer of their KSA. It is more likely they have incomplete knowledge and poor skills, i.e. there was little to be transferred. In contrast to this, S1 presented a very good project. He applied graphical method (by DGS) to get the solutions of the problems but made no analytical calculations. S1 also composed relevant systems to problems 2 and 4 on the test but did not solve them. It remains to clarify whether S1 could solve these two problems if he was allowed to use a computer.

8 CONCLUSIONS, SPECULATIONS AND OPEN QUESTIONS

We found that the transfer of the math KSA from the context of their formation to another context is not to be taken for granted for the 7th grade students. Our conclusion is indirectly confirmed by the representative statistics (CKOKUO, 2013), if we accept the definition of synthetic competence as it is given in (Lazarov, 2013). As a rule, on this stage the formation of the students' expression style and their deductive abilities are still in progress. More able students perform well the reproductive tasks realizing decontextualization of math KSA. But our experiments clearly show that even advanced students (S3, S4 and S8) meet difficulties when they try to apply math KSA in productive mode and in another context. The POE provides one more opportunity for decontextualization of higher order math KSA. Comparing the two experiments, we may explain the better test results in the second experiment with the additional GeoGebra activities. But how this factor influences the transfer of the KSA is still not clear.

The case with S1 shows the need of relevant methodology for computer-supported-problem-solving in teaching-learning traditional math topics. Such methodology could (and should) impact both educational standards and examination assessment policy. The KSA transfer is restricted by the specific subject language. This means that in a POE, students express themselves in metalanguage which includes natural language extended with subject slang, dynamic software and multimedia. But students are allowed to express themselves only in narrow traditional form when doing a regular routine practice in subject education.

The transfer of math KSA requires appropriate media – the students' natural language is not sufficient to serve this purpose. The common mathematical language is very difficult to be acquired till the end of the 7th grade. The experts in language education introduced the concept of **plurilingualism** when people communicate in a conglomerate of natural languages (Council of Europe, 2011). Spreading this idea on the math education could be very productive in supporting the students' creative work. The natural language and special languages (like mathematical) will be only a part of a language expansion. Synthetic symbols (pictures, drawings, applets, whole screens including music and animation) are created in a linear way but further act as multidimensional semiotics terms and serve as a plurilingualistic tool. Using synthetic symbols, the students get

more channels for communication, but these additional resources are available only in project-based and project-oriented education. We prefer the second one because it provides more opportunities and flexibility in stating goals, indicators for success, etc. From this perspective, it is worth studying the role of DGS in the transfer of math KSA.

ACKNOWLEDGEMENTS

The author thanks to Daniella Severinova for providing the opportunity to put into practice the experimental teaching, to Diana Semkova for the assistance in organizing this particular POE, and to Albena Vasileva for the improvement of the text. The author also appreciates the reviewers' notes which he takes into account in the final editing of the paper.

REFERENCES

Bulgarian Ministry of Education, (2006). Educational standards. Compulsory education, Annex 3, 30-32, <http://www.minedu.government.bg> (In Bulgarian, active in Dec 2014)

Center for Control and Evaluation of the Quality of the School Education (2013). Study of the students literacy in 6th grade. 43-44 (In Bulgarian, active in March 2014)

Council of Europe (2011) Common European Framework of Reference for Languages (Language Policy Unit, Strasbourg, www.coe.int/lang-CEFR) 4-5

Dimitrova, N. (2008). Technological model of project-based education in physics and astronomy. In: Prof. Dimitar Pavlov, Raising the Spirit. Paradigma, Sofia, 143-151 (in Bulgarian)

European Commission (2004). Framework for key competences in a knowledge-based society. <http://ec.europa.eu/education/policies/2010/doc/basicframe.pdf> (active in Nov 2013)

Gravemeijer, K. (2014). Competencies in and for 21st Century. The Korean Society of Mathematical Education. Proceedings of the 2014 Int'l Conference on Mathematical Education, Cheongju National University of Education, October 17-18, 2014

Jagdish, A., Lardner, R. (1989). Mathematical analysis. Prentice-Hall, New Jersey. 401-404.

Lazarov, B. (2013) Developing Synthetic Competence along Individual Educational Trajectory. The Korean Society of Mathematical Education. Proceedings of the International Conf. on Math. Edu. on Creativity and Giftedness (Mokwon Univ., Seo-gu, Daejeon, Korea, August 9-10, 2013) pp 251-262.

Lazarov, B. (2014) Decontextualization. Mathematics and Education in Mathematics. Proceedings of 43th Con-

ference of the UBM, Borovetz, April 2-6 2014, 67-77. (in Bulgarian)

Lazarov, B. & Severinova, D. (2014). Incorporating integrated approach in secondary school. MEST Journal, 15 07, 2(2).

Pledger, K. (editor) (2009). 6-8 Level Up Maths. Heinemann, Essex, 26-27.

Roorda, G., Vos, P. & Goedhart, M. (2014). An actor-oriented transfer perspective on high school students' development of the use of procedures to solve problems on rate of change. International Journal of Science and Mathematics Education (doi:10.1007/s10763-013-9501-1)

Savova, B., Todorova, M. & Zlatilov, V. (2010). Math tests for 7th grade (Testove po matematika za 7. klas). Prosveta, Sofia. (in Bulgarian)

APPENDIX

Control Test Problems

TP1. A student paid 30 levs for two books. If the first book was 25% cheaper and the second one was 50% more expensive, the two books would be of the same price. Find the price of each book.

TP2. Find the larger of two positive numbers if their ratio is 8:5 and their difference equals 27.

TP3. The difference of the areas of two squares is 85 cm². The side of the first square is 5 cm longer than the side of the second one. Find the perimeter of the larger square.

TP4. The copper-zinc ratio in an alloy is 1:4 and in another alloy it is 2:3. How much of each alloy should be taken to produce 10 kg alloy in which the copper-zinc ratio equals 3:2?