Filling Accuracy Analysis of the Rocket Propellant based on the Flowmeter Measuring Model

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Keywords: Propellant Filling System, Flowmeter Measuring Model, Filling Accuracy, Valve-closed Delay.

Abstract: The high filling accuracy of rocket propellant is an important guarantee for the success of the rocket launch. In view of the factors that affect filling accuracy of the rocket propellant in the filling system of the spaceflight launch site, the algorithm of propellant filling accuracy calculation based on the flowmeter measuring model is proposed in this paper. It respectively carries through mathematical analyses for the different factors affecting the filling accuracy. Through the proposed algorithm, numerical calculation has been carried on the comprehensive filling accuracy of rocket propellant under the existing filling process. It can provide theoretical basis and data support for optimizing filling control process and improving filling accuracy in the launch site, so as to further improving the success rate of rocket launch.

1 INTRODUCTION

The rockets propellant filling system is an important part of the spaceflight launch site. It mainly fulfills the task of the rocket propellant filling. High filling accuracy of rocket propellant is an important guarantee for the success of the rocket launch, so the precision is a basic requirement for the filling system (Deng, 2012). Therefore researching filling accuracy of the rocket propellant is of great significance to ensure complete success of the rocket launch.

The basic filling quantity of rocket propellant is measured by the level gauge of rocket tank, and the quantitative-filling quantity is measured by the flowmeter in the filling storeroom. It starts quantitative-filling filling when reaching the specified level. The valve automatically closes when the flowmeter measures to the quantitative-filling quantity. The existing filling system adopts the filling model of volume-level, which measures the filling quantity by flowmeter, to meet the needs of filling quantity (Zhuang, 2005).

There are some factors that affect the filling accuracy of rocket propellant in the propellant filling system of the spaceflight launch site. It could increase the risk of rocket filling and launch. Therefore, filling accuracy of the existing filling system in the launch site needs to be analyzed. The factors that influence the filling accuracy need to be improved, to improve the accuracy of the rocket filling quantity, so as to improve the safety and reliability of rocket filling and launch.

The rest section of the paper is organized as follows: Section 2 introduces the related work on filling accuracy of rocket propellant. Section 3 introduces the proposed algorithm, which is the algorithm of filling accuracy based on flowmeter measuring model. Section 4 analyzes the actual filling accuracy of rocket propellant. Finally, section 5 makes conclusion.

2 RELATED WORK

At present, there are few special researches on actual filling accuracy of propellant filling system. However, the researches on the some factors that affect the accuracy of propellant filling have been carried out.

The filling measuring model based on weight measurement is proposed in literature (Xiang, 2014). It designs and analyzes the filling measuring model, and analyzes the filling accuracy based on the proposed model. It also compares the filling accuracy with the existing volume-level measuring model, and improves the filling accuracy. However, there are some problems in the proposed model as follows: First, it does not consider the system error that caused by other equipment when calculating the

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filling accuracy. For the literature, the filling accuracy affected by the equipment can be ignored when carries through comparison between the two models, because the measuring error caused by these equipments are consistent. Actually, equipment performance can cause certain error value of filling quantity. Second, when calculating the filling accuracy, whether or not taking the volumetric measurement error (Ma, 2013) into account. For the literature, it needs to compare the filling accuracy based on the existing model with the filling accuracy based on the proposed model, and the weight measurement model eliminates the effect of the volumetric measurement error. So it must take the volumetric measurement error into account when calculating the filling accuracy. When analyzing the filling accuracy of propellant filling system, it does not need to consider the effect affected by volumetric measurement error, if we take the filling quantity given by the rocket department as reference.

In order to exactly calculate actual filling accuracy of the rocket propellant in the spaceflight launch site, the algorithm of propellant filling accuracy calculation based on the flowmeter measuring model is proposed in this paper. It takes the factors that affect filling accuracy of the rocket propellant into account. The factors include flowmeter measuring, valve-closed delay, maintenance of flowmeter set-zero and leakage of pipeline. It respectively carries through numerical analyses and mathematical calculation for the different factors affecting the filling accuracy. Through the proposed algorithm, the actual filling accuracy of the existing rocket propellant filling system has been figured out. It can provide theoretical basis and data support for optimizing propellant filling control process and improving filling accuracy in the launch site. The following analyses the factors that affect the filling accuracy.

The first is flowmeter measuring. The filling quantity is measured by vortex-flowmeter in the filling system. (Yang, 2004) The vortex-flowmeter is a kind of new type speed instrument on the basis of the principle of fluid oscillation. Its output signal is pulse frequency signal or standards current signal that is proportional to the flow, and can be long-distance transmission. The output signal is only related with the flux, not affected by temperature, pressure, composition, viscosity and density of the liquid. The measuring accuracy of vortex-flowmeter is only 1%, the measuring accuracy is not high, and can lead to higher error.

The second is valve-closed delay. The valves used in the filling system are high pressure pneumatic ball valve. Its working principle is that opens or closes the flow path of the propellant under the pneumatic pressure. When the rocket propellant filling finished, in view of the time when the valve closed, the filling automatic control process is designed as follow. When the filling finished and the PLC received the end signal, the filling-valve and overflow-valve are closed at the same time. Meanwhile, the relevant valves on the filling pipeline are closed. When flowmeter measures to the filling quantity, PLC sends out the instruction of close-valve. It has a certain time delay from PLC instruction issued to the valve fully closed, the valve-closed delay could cause error of propellant filling.

The third is maintenance of flowmeter set-zero. Filling control system adopts PLC control model. Take the second-level quantitative-filling (Yan, 2004) for example, when PLC receives the second-level signal, firstly the secondary instrument of the flowmeter is set zero, and the set-zero operation cannot be instantly restore, which need to keep 0.5 seconds, to ensure that the secondary instrument performs normal set-zero action. The maintenance of flowmeter set-zero could cause error of the propellant filling.

The fourth is leakage of pipeline. The pipeline of filling system in the spaceflight launch site is longer. In the process of propellant filling, it’s hard to avoid leakage of pipeline, including the outer leakage and the inner leakage. The leakage of pipeline could cause certain error of the propellant filling.

3 ALGORITHM OF FILLING ACCURACY BASED ON FLOWMETER MEASURING MODEL

Through analysis on the factors that affects the filling accuracy, the specific error value of filling quantity caused by each factor has been calculated, including the error value caused by flowmeter measuring, the error value caused by valve-closed delay, the error value caused by maintenance of flowmeter set-zero and the error valve caused by leakage of pipeline, etc. Then the actual filling accuracy of rocket propellant can be calculated. When carrying through numerical calculation, the filling quantity given by the rocket department is taken as the reference, taking no account of the influence of volumetric measurement error. The specific analysis is as follow.

3.1 Numerical Analysis of Error Caused by Flowmeter Measuring

The existing filling system adopts the filling model of volume-level. Namely, it measures filling quantity by flowmeter, meanwhile adopts the liquid-level quantitative-filling way. Flowmeter start measure from zero when it receives the liquid-level signal, until flowmeter counts to the certain value. General the second liquid-level is taken as the liquid-level of quantitative-filling. Therefore, the measuring value of the flowmeter before the quantitative-filling liquid-level does not affect the actual filling accuracy, only affects actual effect of display. Actually, the error value of flowmeter measuring equals the error value of quantitative-filling measuring. The value could be calculated according to the metering accuracy of flowmeter, namely 1%.

Define: the error value caused by flowmeter measuring is E_1 (L), the quantitative-filling quantity is b (L), the measuring accuracy of flowmeter is m. The calculation formula of error value caused by flowmeter measuring is as follow:

$$E_1 = b \times m \tag{1}$$

Specific calculation data are shown in table 1.

Table 1: Error value caused by flowmeter measuring.

Rocket level	Quantitative-filling quantity Theoretical value (L)	Flowmeter accuracy	Error value (L)
R1	2560	1%	25.6
R2	1190	1%	11.9
R3	1130	1%	11.3
Y1	1390	1%	13.9
Y2	1160	1%	11.9
Y3	1230	1%	12.3

3.2 Numerical Analysis of Error Caused by Valve-Closed Delay

3.2.1 Mathematical Analysis

The high pressure pneumatic valve of filling system conforms to the quick-opening flow characteristic when valve closes. Valves provided with the flow characteristic have larger flow when the opening is smaller. With the increase of the opening, the flow increases rapidly and is close to the largest soon. Keep on adding the opening, the change of flow is small. Therefore it is called quick-opening flow characteristic (Pan, 2011). The function relationship between the relative flow and f relative excursion is: $dq = Kq^{-1}dl$. Generating into the boundary conditions, we can obtain the function relationship of quick-opening flow characteristics, the formula is as follow:

$$q = \frac{Q}{Q_{\max}} = \frac{1}{R}\sqrt{1 + \left(R^2 - 1\right) \cdot \frac{L}{L_{\max}}} = \frac{1}{R}\sqrt{1 + \left(R^2 - 1\right) \cdot l} \tag{2}$$

In the above formula, R is the ratio that valve can control between maximum flow and minimum flow, namely the adjustable ratio. Q is the flow that passes through the valve. Qmax is the maximum value of flow that passes through the valve. $Q/Q_{\max}$ is the relative flow. $L/L_{\max}$ is the relative excursion.

For the valve of quick-opening flow characteristic, the gain K is proportional to the reciprocal of flow Q, or $K \propto 1/Q$. With the increase of flow, the gain decreases.

The flow characteristic curve when valve closes is as follow:

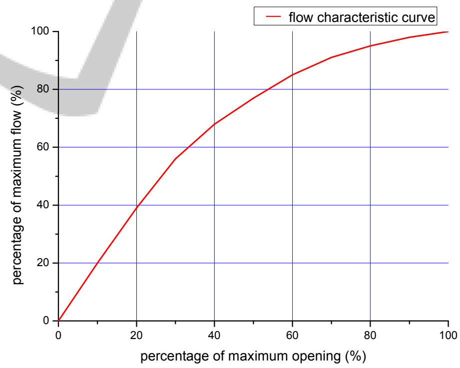

Figure 1: Flow characteristic curve when valve closes.

3.2.2 Numerical Calculation of the Error

From experiment we can get the time delay is 1.5s, the transmission speed of electrical signal is quite fast, and it can be ignored.

Table 2: Error value caused by valve-closed delay.

Rocket level	Flow velocity when filling finished (L/min)	Time (s)	Error value (L)
R1	300	1.5	5.6
R2	300	1.5	5.6
R3	150	1.5	2.8
Y1	300	1.5	5.6
Y2	300	1.5	5.6
Y3	150	1.5	2.8

Define: the error value caused by valve-closed delay is E_2 (L). Then the error value can be calculated through the above formula. The specific calculation data are shown in table 2.

3.3 Numerical Analysis of Error Caused by Maintenance of Flowmeter Set-zero

Filling control system adopts PLC to control. Take the second-level quantitative-filling for example, when PLC receives the second-level signal, the secondary instrument of flowmeter is set zero. The set-zero operation cannot be instantly restore, it need to keep 0.5 seconds, to ensure that the secondary instrument perform the set-zero action. The maintenance of flowmeter set-zero could cause the error of propellant filling quantity.

Define: The error value caused by maintenance of flowmeter set-zero is E_3 (L). The flow velocity when receives the second liquid-level signal is v_3 (L/min). The time of valve-closed delay is t_3 (s). Then the error calculation formula caused by maintenance of flowmeter set-zero is as follow:

$$E_3 = v_3 \times t_3 \tag{3}$$

Specific calculation data are shown in table 3.

Table 3: Error value caused by maintenance of flowmeter set-zero.

Rocket level	Flow velocity (L/min)	Time (s)	Error value (L)
R1	300	0.5	2.5
R2	300	0.5	2.5
R3	150	0.5	1.25
Y1	300	0.5	2.5
Y2	300	0.5	2.5
Y3	150	0.5	1.25

3.4 Numerical Analysis of Error Caused by Leakage of Pipeline

3.4.1 Mathematical Analysis

In order to calculate the error value of propellant filling quantity caused by leakage of pipeline, the pipeline leakage model needs to be established, as shown in Fig.2. When the fissure of equipment is regular, and the fissure size, physical and chemical properties and parameters of the leakage material are known, the leakage quantity can be calculated according to related equations of the hydrodynamics. When the fissure of equipment is irregular, the fissure size can be instead of equivalent size. (Ma, 2008)

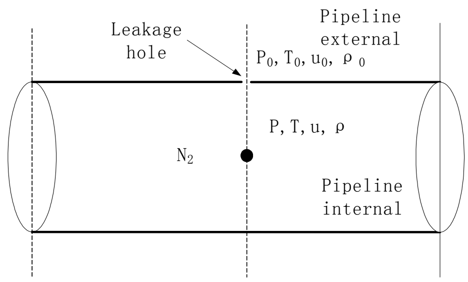

Figure 2: Leakage model of the filling pipeline.

Fig.2 shows the gas leakage process of filling pipeline. The gas inner pipeline is nitrogen, and there is a small leakage hole somewhere on the pipeline. As is shown in Fig.2, the parameter (P, T, u, ρ) respectively express the pressure, temperature, leakage velocity and gas density nearby the leakage hole which on the pipeline internal. The parameter (P_0, T_0, u_0, ρ_0) respectively express the pressure, temperature, leakage velocity and gas density nearby the leakage hole which on the pipeline external.

In the process of the filling pipeline gas tightness check, gas flow process can be taken as reversible and adiabatic process of ideal gas. It follows the state equation and Poisson equation of ideal gas. The following equation can be obtained through the energy conservation equation and momentum conservation equation

$$\frac{k+1}{k}\ln\left(\frac{P_1T_2}{P_2T_1}\right)+\frac{M}{RG^2}\left(\frac{P_2^2}{T_2}-\frac{P_1^2}{T_1}\right)+\frac{4fL_e}{D}=0 \tag{4}$$

In the above formula, D is diameter of the pipeline (mm). K is the specific heat capacity. f is the friction coefficient. u is the gas leakage rate (m/s). G is the gas flow (m^3/s). R is the gas constant.

In the pore model, in view of the aperture is smaller, pressure is not affected by gas leakage, and the gas expansion process is isentropic. Therefore gas leakage rate is constant, and is equal to the initial maximum leakage rate (Dong, 2002).

a) The gas leakage calculation

The leakage rate that gas leak from the fissure is related to its flow state (Zou, 2010). Therefore, it needs to determine the gas flow belongs to sonic flow or subsonic flow when calculating the leakage. The former is called the critical flow, the latter is called the subcritical flow (Beirami, 2006), (Boonen, 2009).

In allusion to the filling pipeline gas tightness check, from numerical calculation we can get: $\frac{p_0}{p} < \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$. Therefore, the gas leakage of filling pipeline belongs to the sonic flow. In the formula, p is the medium pressure within the pipeline (Pa). p_0 is the environmental pressure (Pa). k is the gas adiabatic index, that is, the ratio between Cp and Cv.

When the gas flow is the sonic flow, the leakage is:

$$Q_g = C_d Ap\sqrt{\frac{Mk}{RT}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \tag{5}$$

In the above formula, C_d is the gas leakage coefficient, if the fissure shape was round, $C_d = 1.00$, if the fissure shape is triangle, $C_d = 0.95$, if the fissure shape is rectangle, $C_d = 0.90$. M is the molecular weight. p is the medium pressure (Pa). R is the gas constant (J/(mod • K)). T is the gas temperature (K).

If considering the leakage rate that affected by the gas decrease or pressure reduce inside the pipeline, the calculation of leakage rate is too complex (Cazauran, 2009), (Zhang, 2010). In process of the filling pipeline gas tightness check, in view of pressure of pipeline internal is higher, and the leakage is very small, so assume that the gas pressure inner the pipeline is invariable when carries through calculation.

b) Liquid leakage calculation

The liquid leakage rate can be calculated by Bernoulli equation of hydromechanics (Ben-Mansour, 2010), the leakage rate is as follow:

$$Q_0 = C_d A\rho\sqrt{\frac{2(p-p_0)}{\rho} + 2gh} \tag{6}$$

In the above formula, Q_0 is the liquid leakage rate (kg/s). C_d is the liquid leakage coefficient. A is the area of fissure (m^2). ρ is the density of the liquid leakage (kg/m^3). p is the gas pressure inner the pipeline (Pa). p_0 is the environmental pressure (Pa). g is the acceleration of gravity (9.8 m/s^2). h is the liquid-level height above the fissure (m).

3.4.2 Numerical Calculation of the Error

From the above analysis we can know the specific calculation process, it is as follow. First of all, we calculate the equivalent fissure size according to the pressure drop values and gas leakage formula.5 in the process of gas tightness check. The actual pressure drop value is within 1%, so the value of 1% is used in the calculation. Then we calculate the liquid leakage rate according to the liquid leakage formula.6. Finally we calculate the error value of propellant filling according to the actual quantitative-filling time.

Define: the error value of filling quantity caused by leakage of pipeline is E_4 (L). The liquid leakage rate is Q_0. The quantitative-filling time is t_4 (s). The quantitative-filling velocity is v_4. The quantitative-filling quantity is b. Assuming that the liquid leakage rate is constant in the process of quantitative-filling. According to the real experimental data, gas pressure drop within 1% in half an hour in the process of gas tightness check. The quantitative-filling time can be calculated through the quantitative-filling quantity and the quantitative-filling time velocity. The error value caused by leakage of pipeline before the quantitative-filling liquid-level does not affect the actual filling accuracy, only affects the actual display effect. Calculation formula is as follows:

$$E_4 = Q_0 \times t_4 = Q_0 \times \frac{b}{v_4} \tag{7}$$

Specific calculation data are shown in table 4.

Table 4: Error value caused by leakage of pipeline.

Rocket level	Quantitative-filling quantity Theoretical value (L)	Time (s)	Error value (L)
R1	2560	512	3.20
R2	1190	238	1.49
R3	1130	452	1.41
Y1	1390	278	1.73
Y2	1160	232	1.45
Y3	1230	492	1.54

4 NUMERICAL ANALYSIS OF THE ACTUAL FILLING ACCURACY

From the above numerical calculation and analysis we can know, the infection of different factors to the actual filling accuracy is different, and it is positive or negative that the infection effect of different factors to the error value of filling quantity.

Define: Fac1 expresses the error caused by flowmeter measuring. Fac2 expresses the error caused by valve-closed delay. Fac3 expresses the error caused by maintenance of flowmeter set-zero. Fac4 expresses the error caused by leakage of pipeline. Then, the influence factor of Fac1 is "±", the influence factor of Fac2 is "+", the influence factor of Fac3 is "+", the influence factor of Fac4 is "±".

Define: the actual filling error of rocket propellant is E (L). The formula can be got as follow:

$$E = \pm E_1 + E_2 + E_3 \pm E_4 = \pm(b \times m) + E_2 + (v_3 \times t_3) \pm (Q_0 \times \frac{b}{v_4}) \quad (8)$$

The error calculation results of rocket propellant filling under the existing filling process are shown in table 5.

Table 5: Actual filling accuracy of rocket propellant.

Rocket level	E1 (L)	E2 (L)	E3 (L)	E4 (L)	Total error (L)	Filling accuracy
R1	±25.6	+5.6	+2.5	±3.20	-20.7～33.7	-0.81%～1.32%
R2	±11.9	+5.6	+2.5	±1.49	-5.29～20.0	-0.44%～1.68%
R3	±11.3	+2.8	+1.25	±1.41	-8.66～15.35	-0.77%～1.36%
Y1	±13.9	+5.6	+2.5	±1.73	-7.53～22.0	-0.54%～1.58%
Y2	±11.9	+5.6	+2.5	±1.45	-5.25～20.0	-0.45%～1.72%
Y3	±12.3	+2.8	+1.25	±1.54	-9.79～16.35	-0.79%～1.33%

In order to more intuitive reveal the influence that the actual filling accuracy affected by different factors, we convert the data in the above table into graph form, as shown in Fig.3.

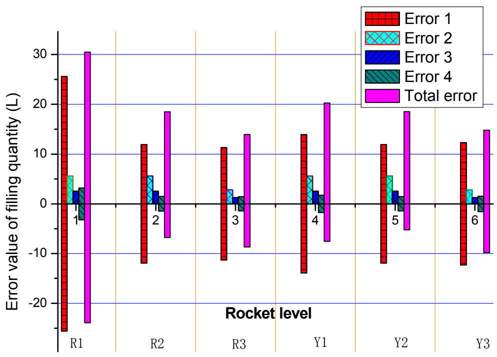

Figure 3: Filling accuracy contrast fig of the rocket propellant.

From the data in Table5 and Fig.3, we can get that the infection of different factors to the actual filling accuracy is different. Fac1 has the greatest influence on the filling accuracy, followed by Fac2, Fac3 and Fac4 has smaller influence on the filling accuracy.

From the above mathematical analysis and numerical calculation, we can know that the actual filling accuracy of rocket propellant is related with the quantitative-filling quantity and quantitative-filling velocity. In order to further analyze the relationship between filling accuracy and quantitative-filling quantity and quantitative-filling velocity, on the one hand, the numerical calculation of the filling accuracy is carried through in the case of 1/2 and 1/4 of the original quantitative-filling quantity, and the calculated results are compared with the filling accuracy under the original quantitative-filling quantity. The results are shown in Fig.4. On the other hand, the numerical calculation of the filling accuracy is carried through in the case of 1/2 and 1/4 of the original quantitative-filling velocity, and the calculated results are compared with the filling accuracy under the original quantitative-filling velocity. The results are shown in Fig.5.

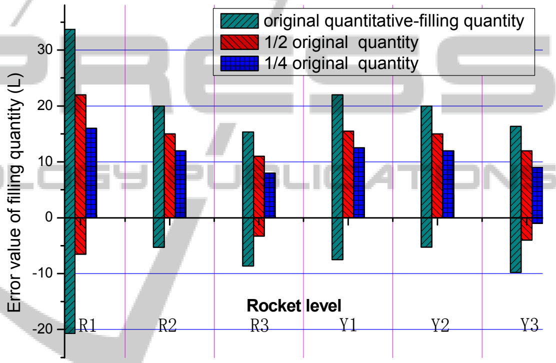

Figure 4: Filling accuracy contrast fig in the case of different quantitative-filling quantity.

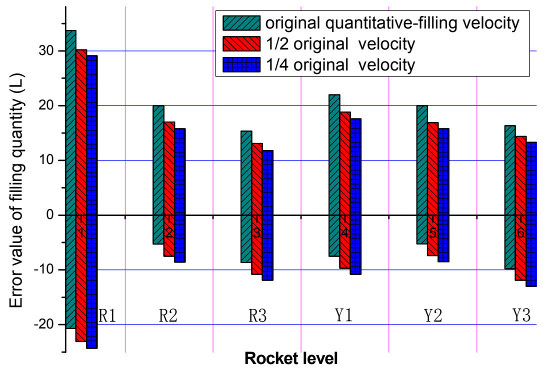

Figure 5: Filling accuracy contrast fig in the case of different quantitative-filling velocity.

Fig.4 shows the filling accuracy contrast in the case of different quantitative-filling quantity, Fig.5 shows the filling accuracy contrast in the case of different quantitative-filling velocity. As is shown in the Fig, the actual filling accuracy of rocket propellant is related with the quantitative-filling quantity and quantitative-filling velocity. The quantitative-filling quantity has much influence on the filling accuracy, and reduce the quantitative-filling quantity can significantly improve the filling accuracy. The quantitative-filling velocity has smaller influence on the filling accuracy, but it can adjust the peak of filling error value. Therefore reduce the quantitative-filling quantity can reduce the maximum of filling error, so as to improve the filling accuracy.

5 CONCLUSIONS

In terms of rocket propellant filling, this paper analyzes the factors that affect the accuracy of the propellant filling in the filling system of the spaceflight launch site. It calculates the error value of filling quantity caused by the different factors, and carries through numerical calculation and analysis for the actual filling accuracy of rocket propellant. It is helpful to optimize filling model and filling process, and provides theoretical basis and data support for the research of improving the filling accuracy. Through numerical analysis we can get that the equipment performance has much influence on the filling accuracy, in the case of definite equipment performance, reduce the quantitative-filling quantity and quantitative-filling velocity can also improve the filling accuracy.

However, it does not consider the influence of the gas-liquid two-phase flow and the propellant temperature rise when calculating the actual filling accuracy. The next research direction is to get the error value of filling quantity caused by the gas-liquid two-phase flow and the propellant temperature rise through simulation calculation, to perfect the factors that affect the accuracy of propellant filling, put forward more effective targeted measures, so as to further improve the filling accuracy of rocket propellant.

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