Norm Selection for Evaluation Criterion for Placement Planning of Active Damping Devices in Structure

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Abstract: Active vibration control has been widely investigated in civil engineering. This study considers the problem of selecting a norm for an evaluation criterion for the planning of the placement of active damping devices (ADDs) in a structure in active vibration control. Using a 4-degree-of-freedom system as an example, we compare the commonly used 2-norm and ∞-norm, and show that the 2-norm is a suitable choice for the performance index of the placement planning of ADDs.

1 INTRODUCTION

Since active structural control exhibits good control performance, it has been attracting great attention. An active structural control system has been designed using many control methods, for example, classical control (Gucu, 2006), modern control (She et al., 2010), advanced control (Zhang et al., 2014), and predictive control (Tsuji et al., 2012).

Due to the constraints on the cost and structure, active damping devices (ADDs) have usually been placed only on the top floor or at the base of the structure. However, the situation has been changed in the last decade. Along with the progress of the technologies in mechatronics in recent years, both of the cost and the size of ADDs have been reduced greatly. This provides flexibility in the selection and placement of an active structural control system (For example, (Tokkyokiki Corporation, 2015)).

Take a four-story building as an example. An ADD was placed on the first floor in (Yoshida et al., 1995), on the second floor in (Gucu, 2006), and on all floors in (She et al., 2010). However, no explanations for the placement were given. And it is questionable why they placed the ADDs in those way and whether or not it is really necessary to place ADDs on all floors. To solve these problems, we introduced the 2-norm of a control system to evaluate the control performance (Miyamoto and She, 2015). In this study, we extend the result in (Miyamoto and She, 2015) and explore the commonly used norms for the purpose of evaluating the placement planning of ADDs.

2 STRUCTURAL MODEL OF 4-STORY AND CONTROL SYSTEM

To make the discussion simple, we use a four-story structure (Figure 1) in this paper. The achieved results can easily be extended to an *n*-story structure. The motion of the system is described by

$$M_s\ddot{x}(t) + C_s\dot{x}(t) + K_s x(t) = E_u u(t) + E_g \ddot{x}_g(t), \quad (1)$$

where

$$\begin{cases} x(t) = [x_1(t),\ x_2(t),\ x_3(t),\ x_4(t)]^T, \\ u(t) = [f_{u1}(t),\ f_{u2}(t),\ f_{u3}(t),\ f_{u4}(t)]^T, \end{cases}$$

$$M_s = \mathrm{diag}\{m_1,\ m_2,\ m_3,\ m_4\},$$

$$C_s = \begin{bmatrix} c_1+c_2 & -c_2 & 0 & 0 \\ -c_2 & c_2+c_3 & -c_3 & 0 \\ 0 & -c_2 & c_3+c_4 & -c_4 \\ 0 & 0 & -c_4 & c_4 \end{bmatrix},$$

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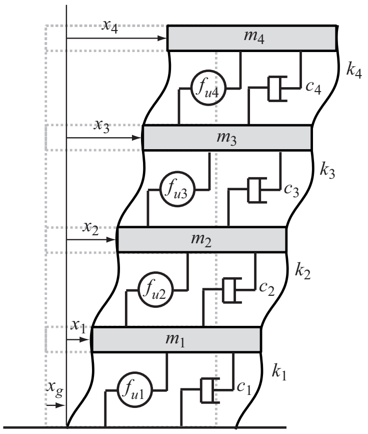

Figure 1: Dynamic model of four-story structure.

$$K_s = \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix},$$

$$E_u = \begin{bmatrix} b_1 & -b_2 & 0 & 0 \\ 0 & b_2 & -b_3 & 0 \\ 0 & 0 & b_3 & -b_4 \\ 0 & 0 & 0 & b_4 \end{bmatrix},$$

$$E_g = [m_1,\ m_2,\ m_3,\ m_4]^T,$$

where the meanings of the parameters and variables are given in Table 1. And E_u indicates the placement of ADDs, and b_i $(i = 1,2,3,4)$ in E_u is given by

$$b_i = \begin{cases} 0, & \text{the } i\text{th floor does not have an ADD,} \\ 1, & \text{the } i\text{th floor has an ADD.} \end{cases} \tag{2}$$

The state-space equation is

$$\begin{cases} \dot{\xi}(t) = A\xi(t) + Bu(t) + B_d\ddot{x}_g(t), \\ y(t) = C\xi(t), \end{cases} \tag{3}$$

where the state vector is $\xi(t) = \left[x^T(t),\ \dot{x}^T(t)\right]^T$, the output is $y(t) = x(t)$, and

$$A = \begin{bmatrix} 0 & I_4 \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix},\ B = \begin{bmatrix} 0 \\ -M_s^{-1}E_u \end{bmatrix},$$

$$B_d = \begin{bmatrix} 0 \\ -E \end{bmatrix},\ C = \begin{bmatrix} I_4 & 0 \end{bmatrix},\ E = [1,\ 1,\ 1,\ 1]^T.$$

In order to verify the relationship between the placement of ADDs and control effect, it is necessary to construct a control system. In this study, we construct a simple optimal control system by minimizing the following performance index

$$J = \int_0^\infty \left\{\xi^T(t)Q\xi(t) + u^T(t)Ru(t)\right\}dt, \tag{4}$$

where Q (≥ 0) and R (> 0) are weighting matrices such that $(Q^{1/2}, A)$ is observable. A feedback control

Table 1: Meanings of parameters and variables ($i = 1,2,3,4$).

Symbol	Meaning
m_i [kg]	Mass of the ith story
c_i [Ns/m]	Damping of the ith story
k_i [N/m]	Stiffness of the ith story
x_i	Displacement of the ith story
f_{ui}	Output of the ith ADD
x_g	Displacement of the ground

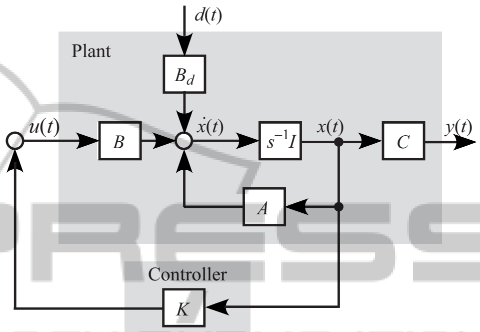

Figure 2: Configuration of optimal active-structural-control system.

law is given by

$$u(t) = K\xi(t), \tag{5}$$

$$K = -R^{-1}B^TP, \tag{6}$$

$$A^TP + PA + Q - PBR^{-1}B^TP = 0. \tag{7}$$

As a result, the transfer function of the system from the input to the output is

$$G(s) = C[sI - (A+BK)]^{-1}B, \tag{8}$$

where s is the operator of the Laplace transform. An optimal state-feedback active-structural-control system is shown in Figure 2.

3 SELECTION OF NORM FOR PLANNING INDEX

First, we consider the norms for a signal. A norm of a signal $u(t) = [u_1(t),\ u_2(t),\ \cdots,\ u_n(t)]^T$, $\|u\|$, has the following properties:

1. $\|u\| \geq 0$;
2. $\|u\| = 0 \Leftrightarrow u(t) = 0,\ \forall t$;
3. $\|au\| = |a|\|u\|,\ \forall a \in \mathbb{R}$; and
4. $\|u+v\| \leq \|u\| + \|v\|$.

Table 2: Relationships of the norms between input, output, and system.

	$\|u\|_2$	$\|u\|_\infty$	$pow(u)$
$\|y\|_2$	$\|G\|_\infty$	∞	∞
$\|y\|_\infty$	$\|G\|_2$	$\|G\|_1$	∞
$pow(y)$	0	$\leq \|G\|_\infty$	$\|G\|_\infty$

1-norm, 2-norm, and ∞-norm of the signal are defined as follows.

$$\|u\|_1 = \sum_{i=1}^{n} \int_{-\infty}^{\infty} |u_i(t)|dt, \quad (9)$$

$$\|u\|_2 = \left\{ \int_{-\infty}^{\infty} u^T(t)u(t)dt \right\}^{1/2}, \quad (10)$$

$$\|u\|_\infty = \sup_t |u_i(t)|. \quad (11)$$

On the other hand, if

$$pow(u) = \left\{ \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} u^T(t)u(t)dt \right\}^{1/2} \quad (12)$$

exist, we call the signal a power signal. Note that a nonzero signal can have zero average power. So, Property 2 does not hold for (12) and $pow(\cdot)$ is not a norm.

For a stable system, $G(s)$, its H_2 norm is

$$\begin{aligned} \|G\|_2 &= \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{Trace}\left\{ G^T(-j\omega)G(j\omega) \right\} d\omega \right\}^{1/2} \\ &= \left\{ \frac{1}{2\pi j} \oint \mathrm{Trace}\{G^T(\bar{s})G(s)\} ds \right\}^{1/2}, \end{aligned} \quad (13)$$

and its H_∞ norm is

$$\|G\|_\infty = \sup_{0\leq\omega\leq\infty} \sigma_{\max}\left\{G(j\omega)\right\}. \quad (14)$$

Note that, for a square matrix $\Phi = [\phi_{ij}] \in \mathbb{C}^{n\times n}$, Trace in (13) is $\mathrm{Trace}\{\Phi\} = \sum_{i=1}^{n} \phi_{ii}$, and $\sigma_{\max}(\Phi)$ in (14) is the maximum singular value of Φ.

The definitions of induced norms give the relationships of the norms between the input, $u(t)$, the output, $y(t)$, and the system $G(s)$ (Table 2) (Doyle et al., 2009):

$$\|G\|_1 = \sup_{\|u\|_\infty \neq 0} \frac{\|y\|_\infty}{\|u\|_\infty}, \quad (15)$$

$$\|G\|_2 = \sup_{\|u\|_2 \neq 0} \frac{\|y\|_\infty}{\|u\|_2}, \quad (16)$$

$$\|G\|_\infty = \sup_{\|u\|_2 \neq 0} \frac{\|y\|_2}{\|u\|_2} = \sup_{pow(u)\neq 0} \frac{pow(y)}{pow(u)}. \quad (17)$$

It is clear from the above relationships that, to suppress the vibration caused by an earthquake and/or a typhoon, it is suitable to use the 1-norm or the ∞-norm of the system from the disturbance to the output to evaluate the effect caused by a disturbance, because the worst output caused by the disturbances can be evaluate by the 1- or ∞-norms of the system.

However, those norms may not be suitable for the evaluation of the placement of active damping devices. The reason is as follows. To suppress vibration quickly and efficiently, for a given control input with limited power, the bigger the output produced by the control input is, the more desirable the system is. From this viewpoint, the use of the 2-norm of the system from the control input to the output is suitable for the planning of the placement of ADDs. We use a numerical example to examine and this and compare the use of norms in the next section.

Let k be the number of ADDs. The placement problem is divided into two cases:

(1) k is not fixed: The optimal problem is

$$\max_{b_1,b_2,b_3,b_4\in\{0,1\}} \|G\|_2. \quad (18)$$

(2) k is fixed: The optimal problem is

$$\max_{b_1,b_2,b_3,b_4\in\{0,1\},\ \sum_{i=1}^{4} b_i = k} \|G\|_2. \quad (19)$$

Note that these combinatorial optimization problems are an integer programming problem, and can easily be solved using well-known solvers.

4 NUMERICAL EXAMPLE

For simplicity, we only verify and compare the 2-norm and ∞-norm of the system in this section.

The parameters of a four-story structure are (Yoshida et al., 1995)

$$\left\{ \begin{array}{l} m_1 = 0.828,\ m_2 = m_3 = 0.842,\ m_4 = 0.640, \\ k_1 = 400,\ k_2 = 1600,\ k_3 = 1302,\ k_4 = 160, \\ c_1 = 7.5,\ c_2 = c_3 = c_4 = 0.02. \end{array} \right. \quad (20)$$

For the design of a control system, the weighting matrices in (4) were chosen to be

$$Q = 300 \times \mathrm{diag}\{100,\ 100,\ 100,\ 100,\ 1,\ 1,\ 1,\ 1\},\ R = I_k, \quad (21)$$

where k is the number of ADDs.

This study used the ground acceleration data of Noto Peninsula earthquake [Figure 3 (a) and (b)], which has a low frequency, and Kobe earthquake [Figure 3 (c) and (d)], which features randomness, as disturbances in simulations (Japan Meteorological Agency, 2015).

The peak-to-peak values (PPVs) of the maximum displacements for Noto Peninsula earthquake for different placement plans of ADDs are shown in Table

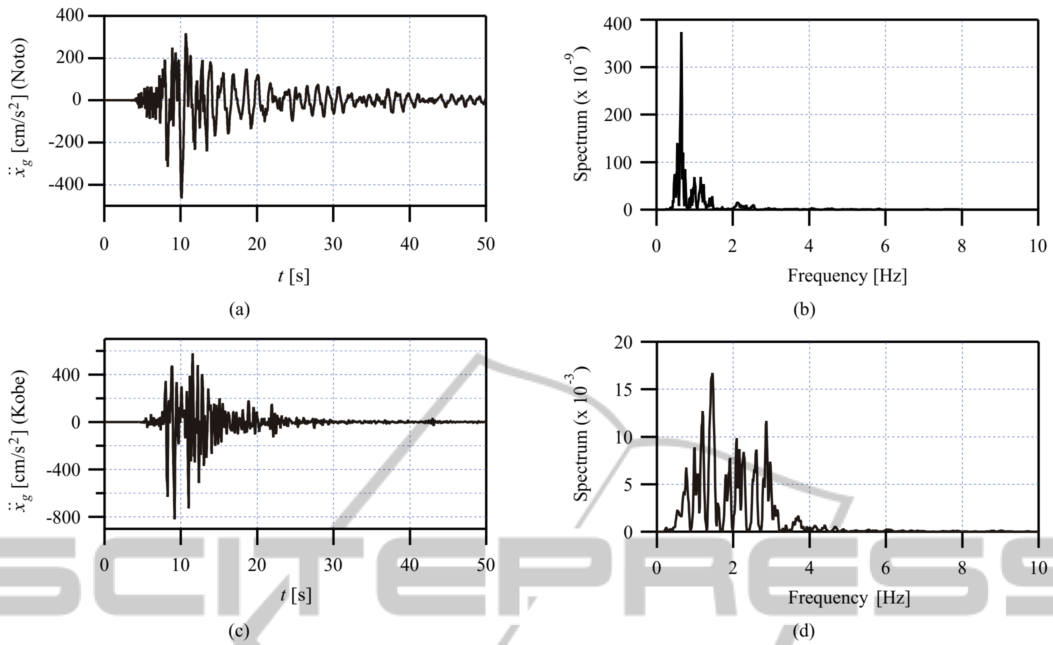

Figure 3: Input earthquake waves: (a) Noto Peninsula earthquake, (b) Power spectrum of Noto Peninsula earthquake; (c) Kobe earthquake; and (d) Power spectrum of Kobe earthquake.

Table 3: Relationship between the placement of ADDs (upper row) and the PPV of the maximum displacement for Noto Peninsula earthquake [cm] (lower row).

Sys1234	Sys123	Sys124	Sys12
7.27 (4F)	7.39 (4F)	7.66 (4F)	7.85 (4F)
Sys134	Sys13	Sys14	Sys1
7.98 (4F)	8.15 (4F)	8.50 (4F)	8.84 (4F)
Sys234	Sys23	Sys24	Sys2
11.36 (4F)	11.54 (4F)	11.95 (4F)	12.18 (4F)
Sys34	Sys3	Sys4	
13.33 (4F)	13.65 (4F)	15.68 (4F)	

Table 4: Relationship between the placement of ADDs (upper row) and the PPV of the maximum displacement for Kobe Peninsula earthquake [cm] (lower row).

Sys1234	Sys123	Sys124	Sys12
10.83 (4F)	11.02 (4F)	11.45 (4F)	11.81 (4F)
Sys134	Sys13	Sys234	Sys14
11.98 (4F)	12.25 (4F)	12.63 (4F)	12.86 (4F)
Sys1	Sys23	Sys24	Sys2
13.41 (4F)	20.34 (4F)	21.41 (4F)	21.90 (4F)
Sys34	Sys3	Sys4	
23.79 (4F)	24.27 (4F)	29.17 (4F)	

3. In the table, the word in the parentheses in the right column shows the place where the maximum displacement occurred; and SysXYZ indicates that the control system has ADDs at the X-th, Y-th, and Z-th floors, for example, Sys1234 means that it has ADDs at the first to the fourth floors.

Tables 3 and 4 show that the ascending order of the maximum PPV for the placement of ADDs is almost the same for those two quite different earthquakes. And Sys1 has the minimum PPV for the use of one ADD, Sys12 is for two and Sys123 is for three ADDs.

Let $PPV_{SysN}^{(\max)}$ be the maximum PPV of SysN. The relative difference of the PPV for the placement of ADDs is defined to be

$$\delta_{SysN} = \frac{PPV_{SysN}^{(\max)} - PPV_{Sys1234}^{(\max)}}{PPV_{Sys1234}^{(\max)}} \times 100\%. \quad (22)$$

It is in the range of 2-56% for Noto earthquake and 2-17% for Kobe earthquake for three ADDs; in the range of 8-83% for Noto earthquake and 9-120% for Kobe earthquake for two ADDs; and in the range of 22-116% for Noto earthquake and 23-169% for Kobe earthquake for one ADD. This clearly shows that, while the increase of the ADDs improves the control performance, suitable placement of ADDs also dramatically changes the control performance. Observing the above results yields the follows. First, since the smallest difference of δ_{SysN} between the use of one and four ADDs for a suitable placement is about 22%, that between two and four is about 8%, and that between three and four is about 2%; there is no need to place ADDs at all floors. Second, the largest difference of δ_{SysN} is as large as 146% for different placement of the same number of ADDs. So, if we choose

Table 5: Norms of active structural control system for (21).

Placement	H_2	H_∞
Sys1234	0.01161	0.004301
Sys123	0.01118	0.004300
Sys134	0.01111	0.004282
Sys124	0.01109	0.004294
Sys13	0.01064	0.004281
Sys12	0.01054	0.004293
Sys14	0.01048	0.004275
Sys1	0.009854	0.004274
Sys234	0.008719	0.003894
Sys23	0.008106	0.003886
Sys24	0.007717	0.003819
Sys34	0.007048	0.003454
Sys2	0.006763	0.003805
Sys3	0.006102	0.003388
Sys4	0.004640	0.001951

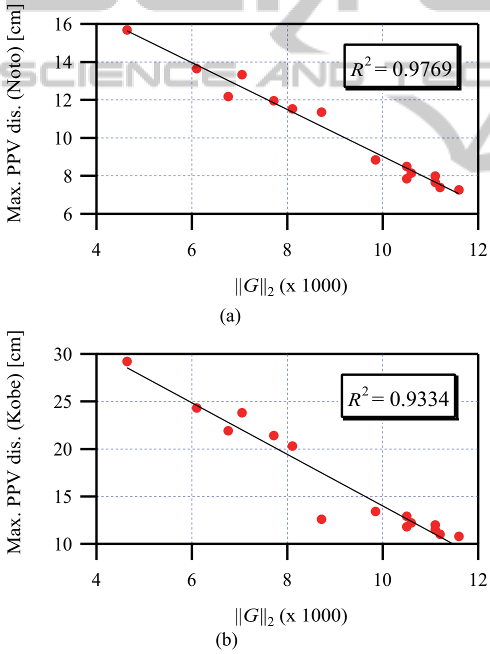

Figure 4: Relationship between the maximum PPV of the output and the H_2 norm of the system for (a) Noto Peninsula earthquake and (b) Kobe earthquake.

suitable floors to place devices, we can use a small number of ADDs to achieve satisfactory aseismic effect.

Table 5 shows the calculated results of the H_2 and H_∞ norms of $G(s)$ for different placement of ADDs. The relationships between the maximum PPV of the output and the H_2 norm is shown in Figure 4, and that between the maximum PPV and the H_∞ norm is shown in Figure 5.

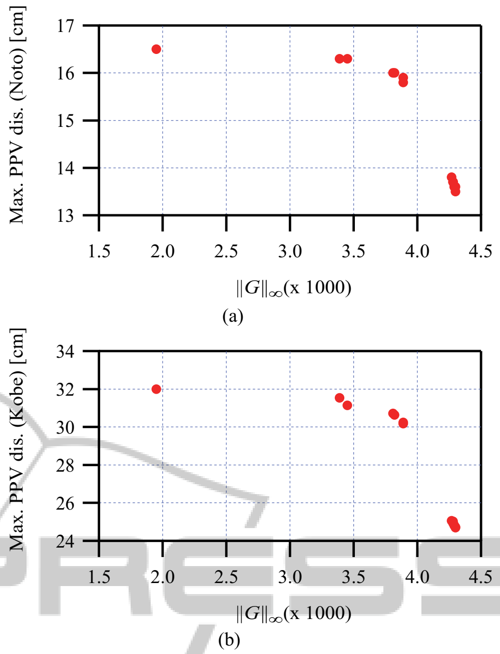

Figure 5: Relationship between the maximum PPVs of the outputs and the H_∞ norm of the system for (a) Noto Peninsula earthquake and (b) Kobe earthquake.

It is clear from Figures 4 and 5 that the maximum PPV of the output is basically in inverse proportion to the H_2 or H_∞ norms of the system. And all of the coefficients of determination, R^2, are larger than 0.9 in Figure 4. However, the H_∞ norms of the system with good control performance all almost have the same value, 0.043, as shown in Table 5. Recall that the H_∞ norm of an SISO system is the peak gain value of its Bode plot. It is easy to understand that the values become almost identical when a vibration mode is suppressed. As a result, if we use the H_∞ norm of the system as a performance index to evaluate the placement of ADDs, it may just show good control performance for those disturbances with the frequency corresponding to the peak gain value of the Bode plot, and does not guarantee the aseismic effect for disturbances with other frequencies. From this viewpoint, we can say that it is not suitable to use the H_∞ norm of a control system to evaluate the placement planning.

On the other hand, as shown in (13), the H_2 norm of a control system is the square root of the area of its frequency-response gain. It is basically different for different placement of ADDs. So, it is suitable to use the H_2 norm of the transfer function from the control input to the output to evaluate the placement planning.

Another question is if the reduction of the number of ADDs results in the increase of the power of the control input of each ADD. To answer this question, we plot the relationship between the maximum PPVs of the outputs and the 2-norms of the control inputs for Kobe earthquake in Figure 6. The figure shows that the power of the control input has no significant correlation. For example, Sys1 uses less input power than Sys23 does, but it yields much better control performance. So, good control performance produced by the control system with a small number of devices, which are given by the optimal placement (18), does not mean the increase of control-input power. This also shows the importance of the optimal placement of ADDs.

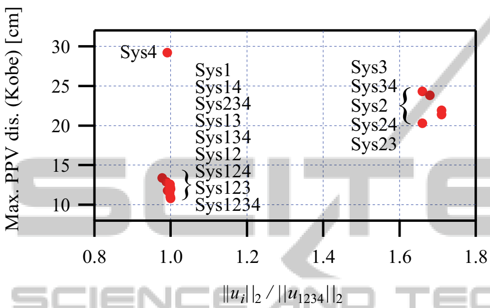

Figure 6: Relationship between the maximum PPVs of the outputs and the 2-norms of the control inputs for Kobe earthquake.

5 CONCLUSION

In this study, we considered the problem of the placement of ADDs to perform active structural control. To suitably evaluate the placement planning, we examined the 1-, 2-, and ∞-norms of a control system from the control input to the output; and employed the H_2 norm for the evaluation. We used a four-story structure as an example to demonstrate the validity of the selection. The following points were clarified.

1. Increasing the number of ADDs does not necessarily lead to the improvement of control performance. Placing a small number of ADDs at suitable floors achieves satisfactory control result.
2. The H_2 norm of the transfer function from the control input to the output is suitable for a performance index to find out an optimal placement of ADDs.

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