Mobile Sensor Path Planning for Iceberg Monitoring using a MILP Framework

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Abstract: We look at the task of iceberg monitoring using a single mobile sensor, and we suggest a modular framework for this. The focus is on path planning for which we come up with a novel strategy, which includes solving a static optimization problem often to account for changes. We formulate the optimization problem in a MILP framework, and we illustrate how this yields acceptable computational time for problem size of about 15 icebergs. We also suggest a tuning rule for weighting between different objectives in the optimization formulation, which we demonstrate in simulations. Initializing the optimization with the previous solution can improve computational time dramatically. Finally, we discuss how we easily can add extra features to our framework.

1 INTRODUCTION

Unmanned Aerial Vehicles (UAVs), or more general Unmannd Aerial Systems (UAS), have been studied for a long time. The military has recognized the utility of UAVs as early as WWI. During the Cold War the efforts of developing UAVs for surveillance and reconnaissance missions increased dramatically, and the Vietnam War was the first war where UAVs got put into substantial use (Cook, 2007).

In modern times, UAVs performing surveillance and reconnaissance missions see applications in civilian life as well as the military. Examples of applications are environmental monitoring - which include weather, wildfire and polar monitoring (Chmaj and Selvaraj, 2015), traffic surveillance (Peng et al., 2012), agriculture (Watts et al., 2012) and much more.

In this paper we will study tracking of icebergs. Radar, satellite imagery, shipboard sensors, drift buoys and visual observation have traditionally been used for the tracking and forecasting of icebergs (Timco et al., 2005). However, we envision UAVs to be important in the future of tracking icebergs (Eik, 2008). UAVs has the advantage over satellites when comparing price and maneuverability, in addition to spatial and temporal resolution. Satellites can only follow predefined trajectories. In the Northern hemisphere the satellite coverage is pore, and you can expect only coverage within hours interval. This calls for a real-time solution for monitoring icebergs, where we propose UAVs as mobile sensors to be a cost-effective solution.

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1.1 Contribution

Our contribution in this paper is a framework for monitoring of moving targets with a single UAV. We introduce a novel strategy for doing path planning. The assumptions we make in Section 3 enable us to reduce the path planning problem to the targets visitation problem (TVP) (Grundel and Jeffcoat, 2004). Furthermore, we propose a formulation for this problem using Mixed Integer Linear Programming in Section 3.1.

1.2 Previous Work

Path planning for UAVs is currently a popular research topic, and it has been for the last 10 years. There are many different approaches to path planning for UAVs visiting different objects or targets. The most basic approach to this problem is the Traveling Salesman Problem (TSP). TSP is one of the most studied computational problems in the last 60 years(Applegate et al., 2011) The TSP formulation for this problem is simply to find the shortest path that contains all targets. Reference (Applegate et al., 2011) may serve as a starting point for TSP.

An extension of the TSP problem is to add profits to each target. TSP with profits adds the objective of collecting maximum of profits without exceeding a

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travel cost. A starting point for this research may be (Feillet et al., 2005).

In this paper we use another approach with basis in TSP, the targets visitation problem (TVP) (Grundel and Jeffcoat, 2004). The difference between TVP and TSP is that TVP also prioritize targets of high value early in the visitation sequence. We will come back to how we assign different values to the targets in our case.

Besides formulations that take basis in TSP there are multiple different approaches for a framework for continuous trajectory planning. In (Haugen and Imsland, 2013), the path planning for mobile sensors is formulated as a dynamic optimization problem. This problem is discretized into a large-scale nonlinear programming problem and solved. In another approach, (Walton et al., 2014) formulate more complex dynamics both for the mobile sensors and the targets. Then, they present different complex problem formulations and solve the optimization numerically.

In this paper we divide the path planning into different modules. In (Skoglar et al., 2012), they use a similar approach and divide the sensor management into different subtask. Then, they solve the path planning by using Bayesian estimation and search methods. Another approach is to include all objectives into a single objective function like (Pitre et al., 2009).

In (Alidaee et al., 2009), the authors formulate a target search for m UAVs in a Mixed Integer Linear Programming. This is similar to the formulation in this paper.

This paper is organized as follows. We present the problem formulation in Section 2. In Section 3, we get into the task of path planning. First, we present the strategy we apply to the path planning in this paper with the appurtenant assumptions. Then, in Section 3.1 we formulate the optimization formulation and the constraints for the path planning. In Section 3.2 we come up with a rule to assist tuning between the objectives presented in Section 3.1. We handle the implementation and complexity of the path planning problem in Section 4. We study a single case of monitoring 12 icebergs and show how weighting between the objectives influences the solution in Section 5. Finally, we discuss the result, conclude and discuss future work in Section 6 and 7.

2 PROBLEM FORMULATION

In our case, icebergs are the desired target for monitoring and we use an UAV as the mobile sensor. Additionally, we have some a priori estimate of the location of the icebergs to which we assign some uncertainty. Ultimately, we desire an actuator input for the UAV, which exploits our a priori information, iceberg and UAV models, sensors, and possibly other information, to keep track of the icebergs.

We choose to approach this problem in a modular fashion. An advantage of dividing the problem into different subtasks, is that it gets easier and safer to implement. Two subtasks are, for example, path planning and autopilot. If the path planning fails, the autopilot will still keep the UAV in the air. The different tasks can have different sampling time. While the UAV will need to change its actuator input multiple times a second, the path planning might not be necessary to execute more than every other minute or even rarer.

Figure 1 illustrates the division and the dependencies between the different subtasks. A more detailed description of each task follows:

- UAV: We view the UAV as a unit consisting of both the physical structure of the airplane together with an autopilot and measurement instruments. We assume that the autopilot is able to set actuator inputs for the plane based on waypoints. The UAV must also have a sensors for discovering icebergs, for instance thermal or optical cameras, and/or radars.
- Observer/Measurement Processor. The measurements from the sensors must be processed. Furthermore, a path planner will need a continuous position estimate of all the icebergs.
- Path Planner. A path planner will use the estimated positions of icebergs to come up with a set of waypoints for the UAV that will minimize the position uncertainty the icebergs. In addition, a path planner must manage the set of icebergs of interest. If a new icebergs appear or icebergs leave the area of interest, the path planner must update the set of icebergs. Another task for the path planner is to plan a path for searching for an icebergs not located at the estimated position.

In this paper, we will not focus on using image processing to obtain position and velocity measurements of objects in water from a camera. An excellent reference for doing this is (Leira et al., 2015). The focus in the paper will be the task of the path planning. We will not go into the task of adding and removing icebergs from the set of icebergs under observation, as this is simple. Searching for icebergs that are not at their estimated position might not be trivial, but it differs from the path planning part and for that reason it will not be considered here.

3 PATH PLANNING

The main task for the path planner in our framework will be to take a set of iceberg position points and that

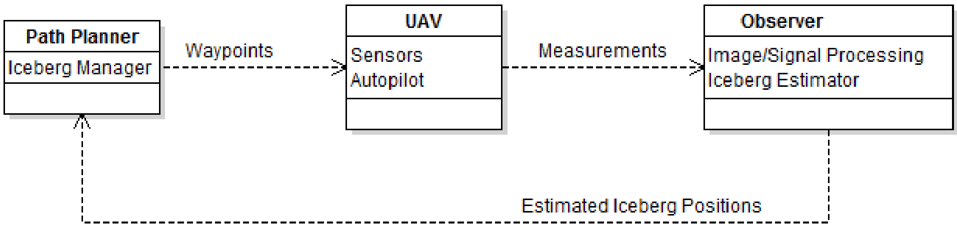

Figure 1: Framework for iceberg monitoring.

of the UAV and decide on a sequence for the UAV to visit the icebergs.

Strategy: We plan to solve the path planning problem often. Then, we apply only the first iceberg of the sequence to the UAV. When the UAV reach the iceberg, we solve the path planning problem again and apply the first iceberg of the new sequence to the UAV, and so on.

This strategy is inspired by model predictive controll (MPC). MPC exploits knowledge of a process model and constraints, and minimize some optimization criteria to calculate a sequence of actuator inputs over a control horizon. Then, MPC applies only the first actuator input of the sequence before repeating the calculations using updated information about the process. The controller continues to only apply the first actuator input of each solution sequence before redoing the calculations.

To be able to apply this strategy, we need a fast and efficient path planning algorithm. The strategy also enables us to make some assumptions and simplifications:

- We assume <u>no</u> UAV dynamics in the path planning, and merely decides the order of which the UAV should visit the icebergs. This enables us to simplify the formulation for the path planning problem. This is a natural assumption if the field of view (FOV) of the UAV is larger than its turning radius. If two or more icebergs are close, we can consider them one point if the FOV will be able to cover them both.
- We assume that we have an initial estimate of the position of the set of icebergs, for example from satellite imagery. In addition, we assume that we have an initial value for the position uncertainty of each iceberg. We can calculate an uncertainty of the position estimate based on the time since the observation.
- The icebergs move much slower than the UAV. This enables us to consider the iceberg stationary when formulating the path planning problem. A typical iceberg velocity is of order 0.1 m/s (Eik, 2009), while we expect an UAV to move 22-25 m/s (UAV Factory, 2012).

Figure 2 illustrates the problem. In the figure we

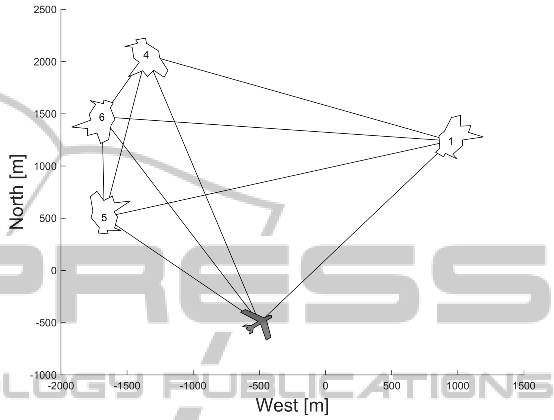

Figure 2: Path Planning Problem with UAV and 4 icebergs. The numbers inside the icebergs are the position uncertainty value for each iceberg.

have drawn the problem as a graph to make the similarities to the traveling salesman problem (TSP) clear. The UAV and each iceberg is a node in the graph. If we do not consider the uncertainty of each iceberg and with the stated assumptions we want a shortest path that starts from the UAV and connects all the icebergs. This is the traveling salesman problem.

The traveling salesman problem is a specific problem of the general class Mixed Integer Linear Programming (MILP) (Bektas, 2006). MILP problems are optimization problems containing integer variables either in the objective function, in the constraints or in both.

This motivates us to solve the path planning problem with an optimization approach. The first objective of the optimization approach will be, as with a TSP problem, to find the shortest distance between each node in the graph. Second, we desire to reduce the position uncertainty of each iceberg. We expect the position uncertainty of each iceberg to vary with the time since the UAV observed it. This objective reduces to sort the iceberg according to their position uncertainty. The optimization must weigh between these two objectives.

An advantage with the strategy we choose is that we do not need an accurate model of the icebergs. Modeling icebergs is difficult, especially since getting other measurements than position and velocity from the air is hard. By having a problem that we solve often we can take new measurements into account and thus compensate for model inaccuracies that will accumulate over time.

3.1 Optimization Formulation for Path Planner

We use a similar approach as (Bektas, 2006) to formulate the optimization problem in a MILP framework. We consider N nodes in the optimization problem, which is the number of icebergs in addition to the UAV. Furthermore, we have two sets of optimization variables. The first is a binary matrix, y_{path}, of dimension $N \times N$. Each element, $y_{path}(i,j)$, represent the path from node i to j. The element is 1 if the path included and 0 if not. The second optimization variable is an integer vector, t, of length N. This contains the sequence of each node in the visiting order.

We can now formulate the optimization problem as

$$\min F(y_{path}, t(i)) = -\sum_{i=1}^{N} \sigma_{nodes}(i)(N - t(i)) + \mu D \tag{1}$$

The position uncertainty of each iceberg is represented by a number in the vector σ_{nodes}, where the first element is the position uncertainty of the UAV, which is 0. A higher number represent a higher uncertainty, and thus an increased desire to visit the iceberg. D is the total distance traveled by the UAV and μ is a tuning variable we use to weight between the two objectives.

The total distance of the traveled path is

$$D = \sum_{j}^{N} \sum_{i}^{N} y_{path}(i,j) d(i,j). \tag{2}$$

The matrix d contains the distances from node i to j in element $d(i,j)$. Notice that this enables us to include weather effects like wind by having a longer distance to a point than from depending on flying with or against the wind.

Second, we must make sure that each node is not visited and left more than once

$$\sum_{i}^{N} y_{path}(i,j) \leq 1 \quad \forall j \quad \text{and} \tag{3}$$

$$\sum_{j}^{N} y_{path}(i,j) \leq 1 \quad \forall i. \tag{4}$$

If we desire a circular path, these constraints should be equality constraints. A linear path containing N nodes needs $N-1$ paths

$$N - 1 = \sum_{i=1}^{N} \sum_{j=1}^{N} y_{path}(i,j). \tag{5}$$

The UAV must be the first point in the path. To ensure this we must set the first element of the t vector equal to 1:

$$t(1) = 1. \tag{6}$$

We do not allow the UAV to visit an iceberg more than once. To ensure this each element of the t vector must be unique:

$$t_i \neq t_j \qquad \forall i \neq j. \tag{7}$$

The values of t must be be within the number of nodes in the problem:

$$1 \leq t \leq N. \tag{8}$$

Finally, it is important that the optimal path is connected. If we demand that each consecutive node that are connected through a path is later in the visiting sequence we avoid subcycles. The visiting sequence is controlled by the t vector, making the constraint:

$$t_j - t_{i+1} \geq -N(1 - y_{path}(i,j)) \quad \forall i \neq j. \tag{9}$$

3.2 Tuning

In the optimization formulation μ weights between the shortest distance and reducing the position uncertainty. If the constant is set too high, the solution will be equal to the TSP solution. Opposite, if the constant is set too low the solution will be equal to a sorting of the icebergs with the highest uncertainty first. It is difficult to avoid having to tune this trade-off. However, it is possible to deduce a tuning rule to help select the value of the tuning constant.

The traveling distance for the UAV and the uncertainty value are of different magnitude. To compare them we need to scale them accordingly. First, we suggest to calculate the maximum obtainable uncertainty value. We can calculate this value by sorting the uncertainty values of the iceberg in ascending order, multiply element wise with a vector from 0 to $N-1$, and sum the vector:

$$F_{1,max} = \sum_{i=0}^{N-1} \sigma_{nodes,sorted}(i) \cdot i \tag{10}$$

Second, the matrix, $d(i,j)$, contains all the distances from node i to j. We can calculate the average distance for the entries in this matrix through the following equation:

$$d_{avg} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} d(i,j)}{N(N-1)} \tag{11}$$

Finally, we calculate a rough estimate of the distance traveled by multiply the average distance in the distance matrix by the number of paths we need in our optimization problem:

$$D_{est} = d_{avg}(N-1) \tag{12}$$

Now, the objective function, equation (1), using these calculated constants is

$$F = -F_{1,max} + \mu D_{est} \tag{13}$$

We want to have the two terms in the objective to be of comparable size, and a natural choice for μ will therefore be $\mu = \frac{F_{1,max}}{D_{est}}$. At this point we introduce an additional tuning constant τ making the choice for μ:

$$\mu = \tau \frac{F_{1,max}}{D_{est}} \tag{14}$$

This new tuning constant will be more intuitive to set. A value of zero for τ puts all the weight on minimizing the position uncertainty of the iceberg, while a value of one gives approximately equal weight to the two objectives. However, as τ is set to a large value (higher than one), the weight is put on minimizing the traveling distance of the UAV. In the simulations in Section 5 we will compare using different values for τ.

4 IMPLEMENTATION AND COMPLEXITY OF PATH PLANNING

MILP is in general an NP-complete problem with computing time growing exponentially with problem size (Mahajan and Ralphs, 2010). Therefore, since our problem is a MILP problem, we will get an exponential growth in computational time with problem size. However, with a modern computer and state-of-the-art solver we will have an acceptable solution time for a problem size of sufficiently small size.

To demonstrate the solution time of our problem we solve the target visitation problem from equation (1) with an increasing problem size. To implement the optimization formulation, we used the YALMIP language developed by (Lofberg, 2004). To solve the problem we use the solver CPLEX from IBM (IBM, 2011).

To setup up the target visitation problem we chose to randomly set positions of the icebergs within a given area. In addition, we randomly assign position uncertainty values between 1 and 10 for each iceberg. Table 1 contains the parameters for setting up the problem.

Figure 3 illustrate the worst case, average and best solution time in seconds in a logarithmic plot. The graph clearly illustrates the exponential rise in solution time with the number of icebergs to track. However, if the number of icebergs is kept at about 15 icebergs the solution time is about 1-2 minutes. Table 2 lists some of the solutions time in seconds.

There are two advantages with the strategy we have chosen in regard to computational time. First, after solving the optimization problem one time we can use the first solution to initialize the next problem and so on like an MPC. This is called a warm start can greatly reduce the computational time. Second, if the solver is not able to obtain a solution within a given time, the UAV can use the next point in the previous solution as the next waypoint. For these reasons the algorithm can in practice handle up to 20 icebergs.

Table 1: Setup Parameters for each Problem of Different Size.

Iceberg Area	[5000m × 5000m]
Iceberg Uncertainty	[1..10]
UAV position	(-500m,-500m)
UAV Uncertainty	0
Number of Optimizations	50
τ	0.5

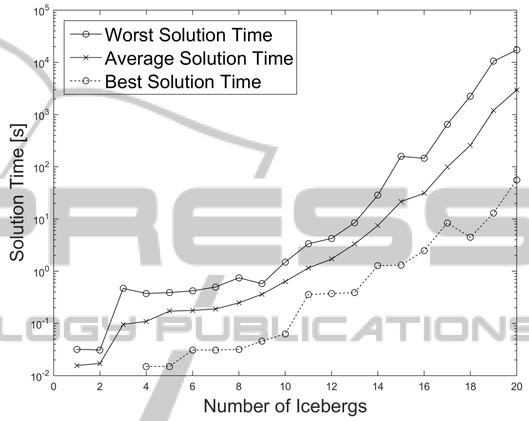

Figure 3: Computational time for different problem size.

Table 2: Average and Worst Case Computational Time.

Icebergs	**2**	**6**	**10**
Avg. sol. time [s]	0.02	0.18	0.64
Worst case sol. time [s]	0.03	0.42	1.50
Icebergs	**12**	**16**	**20**
Avg. sol. time [s]	1.71	31.20	2969.69
Worst case sol. time [s]	4.24	145.16	17269.17

5 SIMULATION

In this section, we simulate iceberg monitoring with 12 icebergs and a single UAV. To demonstrate the path planning algorithm from Section 3.1 with different choices for τ from Section 3.2.

Before we go into the simulation results we present the model we use for the UAV, the icebergs and the position uncertainty of the icebergs.

5.1 Iceberg and UAV Models

For the UAV we use a Dubins Vehicle (Dubins, 1957) as model

$$\begin{bmatrix} \dot{x} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} U\cos(\psi) \\ U\sin(\psi) \\ u \end{bmatrix}, \tag{15}$$

here x is the position, U is the velocity, ψ is the heading of the vehicle and u is the actuator of the UAV. In addition, the actuator input must be within certain limits

$$u \in [u_{min}, u_{max}]. \tag{16}$$

The UAV needs an autopilot to steer it from waypoint to waypoint. We use the line of sight (LOS) algorithm for the UAV autopilot described in section 10.3 of (Fossen, 2011). In addition, we added integral action in the controller with anti-windup from (Caharija et al., 2012).

We model the iceberg as moving point with a known velocity and velocity uncertainty

$$\dot{\xi}_i = v_i + w_i(t), \tag{17}$$

here ξ_i is the position, v_i is the known velocity and $w_i(t) \sim (0, Q_i)$ is the uncertainty in the velocity, which have Gaussian distribution with a mean of zero and variance of Q_i. The dimension of v_i, ξ_i and $w_i(t)$ is $\mathbb{R}^2$. The subscript i highlights that all of these values are different for each iceberg. This renders the estimate model for each iceberg to be

$$\dot{\hat{\xi}}_i = v_i, \tag{18}$$

where $\hat{\xi}_i$ is the estimated position of the iceberg.

We need to model the position uncertainty of each iceberg. First, we consider the error in position estimate defined as $\tilde{\xi}_i = \xi_i - \hat{\xi}_i$. We can then calculate:

$$\dot{\tilde{\xi}}_i = w_i(t) \tag{19}$$

A reasonable assumption is that the variance in both direction in the plane will be equal for an iceberg. This means we can set $Q_i = q_i I_{2\times2}$. If we then define the uncertainty, σ_i, as the covariance of the estimation error,

$$\sigma_i = E[\tilde{\xi}(t)\tilde{\xi}^T(t)] \tag{20}$$

and follow the calculations from section 8.1.1. in (Dan, 2006) we get:

$$\sigma_i = q_i t \tag{21}$$

When the icebergs comes within the FOV of the UAV we set the uncertainty to zero. Combining this with derivative of equation (21) we get:

$$\dot{\sigma}_i = q \quad \xi_i \notin \text{FOV} \tag{22}$$

$$\sigma_i = 0 \quad \xi_i \in \text{FOV}. \tag{23}$$

5.2 Simulation Results

We ran simulations with 12 icebergs and a single UAV to do the monitoring. The UAV and the iceberg were spread out over an area of about $[5000m \times 4000m]$. The parameters we used in the simulation are in table 3. Notice that the same value for variance were used for all iceberg. In addition, we used the same values for $w_i(t)$ in all simulations. This is done to be able to compare the effect of only changing the value for τ through the simulations. All simulation run for a time span of $T = 1000$ seconds.

Table 3: Simulation Parameters.

UAV	Icebergs
$x_0 = [-500, 500]^T$ m	$q = 2.5 \cdot 10^{-2}$
$\psi_0 = \frac{\pi}{4}$	$v \in [0.0, 0.4] m/s$
$FOV = 600m \times 600m$	$\xi_{i0} \in [5 \cdot 10^3, 5 \cdot 10^3]$
$u_{max,min} = \pm \frac{g}{U} \tan(5\frac{\pi}{36})$	
$U = 22m/s$, $g = 9.81m/s$	

Our goal is to reduce the overall uncertainty of the icebergs. To compare the simulations with the different value of τ we calculate the performance metric:

$$\varsigma = \int_0^T \sum_{i=1}^{N-1} \sigma_i(t)dt \tag{24}$$

The resulting integral of the total position uncertainty is shown in Figure 4 as a function of τ. With the MPC-like implementation strategy presented in this paper a high value for τ will result in the UAV being stuck in a loop between two points. The value of $\tau = 4.5$ equals the TSP case, which is implemented without the MPC-strategy.

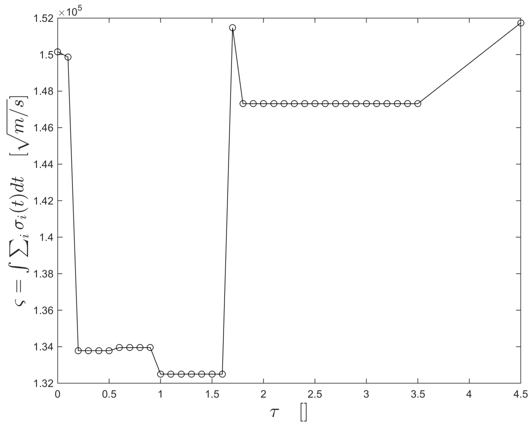

Figure 4: ς in relation to τ for the case of monitoring 12 icebergs with a single UAV. The case of τ = 4.5 is the TSP-solution.

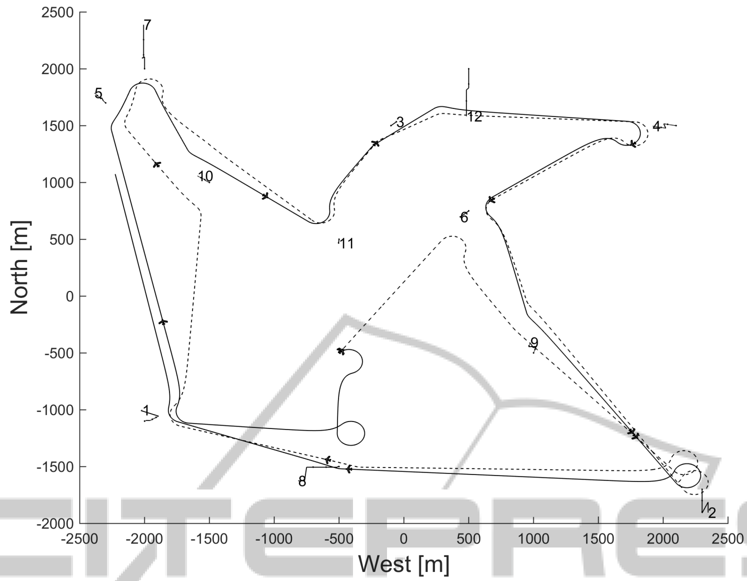

Figure 5: Simulation with 12 icebergs. The dotted path is the optimal path for this case with regard to the metric ς. The whole path is the TSP solution for this case.

Figure 5 shows two simulations of case with 12 icebergs. The UAV illustrated with the blue line is the case $\tau = 1.5$, which is optimal for this case. The other case, where the UAV is plotted with a red line is the TSP-solution for this case.

6 DISCUSSION

The strength of the framework we present in this article is the simplicity. The objective function from equation (1) is intuitive and the tuning rule from Section 3.2 makes it easy to weight between the objectives. Furthermore, the division of the objective of iceberg monitoring into different task makes it easy to separate between tasks like path planning and path following.

A challenge is to weight between the two objectives. The optimal value for τ will vary for each case. However, experience indicates that choosing $\tau = 1$ gives a good trade-off between shortest path and highest uncertainty first.

Another strength of the division of task for the framework is the ability to add extra features. A reason for monitoring iceberg could be to protect an installation or ship. In such a case it will be more important to prioritize icebergs that are close to the object you want to protect. We can easily include this in our objective function by assigning different values for the variance, q_i, for each iceberg based on the distance the iceberg is from our object.

7 CONCLUSIONS AND FUTURE WORK

In this paper we look at the monitoring of a set of icebergs using a single UAV as a mobile sensor. Each iceberg in the set has an estimated position with an appurtenant position uncertainty. We suggest a framework for dividing the task into different subtask separating path planning, autopilot and observer, and we focus on the path planning. For the path planning we come up with a strategy consisting of solving an optimization problem with the icebergs as stationary. The result of the optimization yields a sequence for visiting the icebergs. We apply only the first iceberg in the optimal sequence. When the UAV discovers that iceberg, we solve the optimization again. The advantage of solving the optimization this way is fast computational time and at the same time account for model inaccuracies and changes over time by solving the problem often. In the optimization we use both the objective of achieving shortest distance and visiting the iceberg with highest position uncertainty first. To implement the optimization formulation we use a MILP framework. We also suggest a rule to help tuning between the two objectives in the optimization formulation using a single parameter τ. In simulation we demonstrate how different choices of τ influences the solution for a single case. We use experience with different cases to come with a recommendation for a value of τ. The complexity of our problem is NP-complete. However, with the number of icebergs at about 15, the computational time of our problem is acceptable. Also by initializing our problem with the previous solution, we can yield acceptable computational time for about 20 icebergs. Finally, we discuss how our framework easily can be extended. For example different weighting between icebergs based on their location and not only the time since their last observation.

Future work include:

- Extend the path planning to include management and recovery of lost icebergs.
- Perform experiments with UAV for proof of concept
- Extend the algorithm to allow multiple UAVs

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