Robust Affine Projection Algorithm using Selectively Shrunk Error Component

Seung Hun Kim[1], Jae Jin Jeong[1], Gyogwon Koo[1] and Sang Woo Kim[2]

[1]*Department of Electrical Engineering, POSTECH, 77 Cheongam-Ro, Nam-Gu, 790-784, Pohang, Gyeongbuk, Republic of Korea*

[2]*Department of Electrical Engineering and the Department of Creative IT Engineering and Future IT Innovation Laboratory, POSTECH, 77 Cheongam-Ro, Nam-Gu, 790-784, Pohang, Gyeongbuk, Republic of Korea*

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Abstract: A novel robust affine projection algorithm (APA) is proposed, which selectively shrinks error components in an error vector according to their individual possibilities of being interrupted by the impulsive noise. In existing robust APAs, if there exists only one error component interrupted by the impulsive noise, all error components of an error vector are shrunk using common step sizes which are inversely proportional to the norm of the error vector. This improper scaling results in performance degradation with a high impulsive noise probability and projection order. In this paper, we derive a modified minimization criterion considering the individual possibilities of error components from a geometric interpretation. For a wide range of impulsive noise probability and a high projection order, the performance of the proposed algorithm is verified in various system identification events including an abrupt system change. The proposed algorithm showed the fastest convergence rate and the lowest steady-state mean square deviation compared to the previous robust APAs and a recent variable step-size affine projection sign algorithm.

1 INTRODUCTION

Adaptive filters are applicable to various fields including echo cancellation, system identification, active noise control, and they are designed according to system environments and a designer's purpose (Sayed, 2003). Normalized least-mean-square (NLMS) algorithm is the most popular adaptive filter algorithm due to its simple implementation, but it shows a degraded convergence behavior with a colored input signal or an impulsive interference.

To overcome its weakness to colored input signal, affine projection algorithm (APA) (Ozeki and Umeda, 1984) and modified versions of APA (Kong et al., 2007; Kim et al., 2009; Shin et al., 2004; Paleologu et al., 2008) were suggested, but there still remained the convergence problem with impulsive noise. To improve robustness against impulsive noise, affine projection sign algorithm (APSA) (Shao et al., 2010) was introduced, which combined the APA and the $\mathcal{L}_1$-norm minimization concept from the normalized sign algorithm (Arikan et al., 1994). After that, there have been several researches on variable step-size algorithms for the APSA (Shin et al., 2012; Yoo et al., 2014; Zhang and Zhang, 2013). However, the APSA has a slow convergence rate compared to APA because it was derived from the minimization criterion on the $\mathcal{L}_1$-norm of the error vector.

To give robustness to APA without losing its fast convergence rate, several approaches using modified step-size, which were designed to be robust against impulsive noise, were introduced (Vega et al., 2010; Song and Park, 2014). These algorithms have not only faster convergence rate compared to APSA but also robustness to impulsive noise. Nonetheless, when the impulsive noise arises frequently or a high projection order is needed, they show degraded performance. This is because they adopt common step size to all error components of the error vector. Even though there exists only one interrupted component within the error vector, the other uninterrupted components would become extremely small values by the step size, and this inappropriate shrinkage undermines the filter performance. Therefore, the error components should be selectively shrunk according to their individual possibilities of being interrupted.

This paper proposes a novel strategy for selective shrinking of error components in APA, and aims

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to design a robust APA which shows invariant performance even with a high impulsive noise probability and a high projection order. Being motivated by (Rey Vega et al., 2008), the strategy is derived from a geometric interpretation on location relationships between a hypersphere and hyperplanes. The hypersphere has its center at the current weight vector, and its radius is the expected norm of the difference vector between current and previous weight. In addition, the hyperplanes are sets of weight vectors satisfying *a posteriori* error vector equals 0. Each hyperplane for each error component is judged with no exceptions if the impulsive noise disturbs. That is, if a hyperplane is out of the hypersphere, the hyperplane would be parallel translated to meet the hypersphere. The minimization criterion is obtained from this geometric interpretation, and the corresponding weight update equation is derived.

To verify the performance of the proposed algorithm, the simulations for system identification with randomly generated system coefficient vector are performed. The proposed algorithm shows the fastest convergence rate and the lowest steady-state mean square deviation (MSD) compared to the previous robust APAs (Vega et al., 2010; Song and Park, 2014), and the recent variable step-size APSA (Yoo et al., 2014) for a wide range of the impulsive noise probability and various projection orders.

Section 2 introduces the conventional affine projection algorithm and its geometric meaning. Section 3 handles the robust projection concept introduced in (Vega et al., 2010; Rey Vega et al., 2008) and the proposed algorithm. Section 4 shows the performance of the proposed algorithm, and Section 5 is a conclusion.

2 CONVENTIONAL AFFINE PROJECTION ALGORITHM

In conventional APA, the weight vector $\mathbf{w}_i$ is recursively updated from the previous weight vector $\mathbf{w}_{i-1}$, the input matrix $\mathbf{U}_i$ and the error vector $\mathbf{e}_i$ as follows:

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu \mathbf{U}_i \left(\mathbf{U}_i^T \mathbf{U}_i\right)^{-1} \mathbf{e}_i, \qquad (1)$$

where μ is a step-size, and

$$\begin{aligned} \mathbf{U}_i &= [\mathbf{u}_i \ \mathbf{u}_{i-1} \ \cdots \ \mathbf{u}_{i-K+1}] \\ \mathbf{u}_i &= [u(i) \ u(i-1) \ \cdots \ u(i-M+1)]^T \\ \mathbf{e}_i &= \mathbf{d}_i - \mathbf{U}_i^T \mathbf{w}_i = [e_1(i) \ e_2(i) \ \cdots \ e_K(i)]^T . \end{aligned}$$

Here, M is the length of the input vector which is equal to the length of an unknown system coefficient, $\mathbf{w}^o$, and K is the number of input vectors used, which is often called the projection order. Also, the error vector is obtained from the desired system output vector $\mathbf{d}_i = [d(i) \ d(i-1) \ \cdots \ d(i-K+1)]^T$. Each component of $\mathbf{d}_i$ is calculated from $d(i) = \mathbf{u}_i^T \mathbf{w}^o + v(i)$ where $v(i)$ is a measurement noise.

For the specific case of $\mu_i = 1$, the weight update equation for APA can be regarded as the following optimization problem

$$\min_{\mathbf{w}_i} \|\mathbf{w}_i - \mathbf{w}_{i-1}\|^2 \quad \text{subject to } \mathbf{d}_i = \mathbf{U}_i^T \mathbf{w}_i. \qquad (2)$$

If we define the hyperplane as

$$\begin{aligned} \mathcal{H}_j(i) \overset{\triangle}{=} \{ & \text{set of all vectors } \mathbf{w} \text{ satisfying} \\ & d(i-j+1) - \mathbf{u}_{i-j+1}^T \mathbf{w} = 0\}, \end{aligned} \qquad (3)$$

then $\mathbf{w}_i$ becomes the projection from $\mathbf{w}_{i-1}$ onto the following intersection

$$\bigcap_{j=1}^{K} \mathcal{H}_j(i). \qquad (4)$$

3 ROBUST PROJECTION ALGORITHM

In (Rey Vega et al., 2008), the robust NLMS was introduced. The algorithm was derived from the location relationship between a hypersphere and a hyperplane. The hypersphere has the squared radius of $\delta_{i-1} = E\left[\|\mathbf{w}_i - \mathbf{w}_{i-1}\|^2\right]$ with center at $\mathbf{w}_{i-1}$, and the hyperplane $\mathcal{H}_i$ is the set of all $\mathbf{w}$ satisfying $d(i) - \mathbf{u}_i^T \mathbf{w} = 0$. When the impulsive noise interrupts the desired system output, $\mathcal{H}_i$ would be out of the hypersphere, and $\mathbf{w}_i$ is obtained from the constricted projection onto $\mathcal{H}_i$ to satisfy $\|\mathbf{w}_i - \mathbf{w}_{i-1}\| = \sqrt{\delta_{i-1}}$.

From the similar aspect, in (Vega et al., 2010), the robut APA was introduced. When an intersection $\bigcap_{j=1}^{K} \mathcal{H}_j(i)$ is out of the hypersphere, $\mathbf{w}_i$ is obtained from the constricted projection onto the intersection to satisfy $\|\mathbf{w}_i - \mathbf{w}_{i-1}\| = \sqrt{\delta_{i-1}}$. The derived update algorithm depends on the variable step size which is inversely proportional to the norm of the error vector, and it is an approximate solution obtained from the assumption that the points on hypersphere are close to each other when $\sqrt{\delta}$ is small.

However, such existing robust APAs including also (Song and Park, 2014) multiplied the same step size for all components of $\mathbf{e}_i$. That is, if there is only one $e_j(i)$ interrupted by the impulsive noise, the other components in $\mathbf{e}_i$ are also shrunk even though they are not interrupted. More reasonable solution is to apply selective step-size to each error component according to their individual possibilities of being interrupted by the impulsive noise.

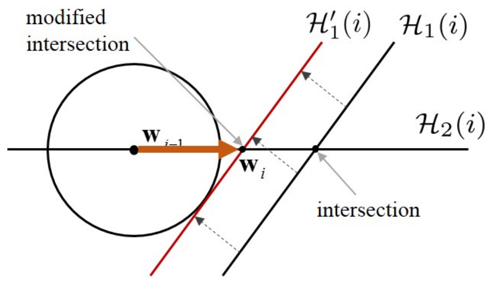


Figure 1: Geometric interpretation of the proposed algorithm for the simplest case ($M = 3, K = 2$).

To obtain selective shrinking strategy, the proposed algorithm finds a new intersection by moving the hyperplanes which are out of the hypersphere to the surface of the hypersphere. The distance between $\mathbf{w}_{i-1}$ and j-th hyperplane is calculated as $h_j(i) = |e_j(i)|/\|\mathbf{u}_{i-j+1}\|$. That is, if $h_j(i)$ exceeds $\sqrt{\delta_{i-1}}$, then the corresponding $\mathcal{H}_j(i)$ should be revised to meet the distance $\sqrt{\delta_{i-1}}$. From this, the modified minimization criterion with fixed step-size $\mu = 1$ is obtained as follow

$$\min_{\mathbf{w}_i} \|\mathbf{w}_i - \mathbf{w}_{i-1}\|^2 \quad \text{subject to } \mathbf{f}_i = \mathbf{U}_i^T \mathbf{w}_i, \tag{5}$$

where $\mathbf{f}_i$ is defined as, for $1 \le j \le K$,

$$f(i-j+1) = \mathbf{u}_{i-j+1}^T \mathbf{w}_{i-1} + \min\left(|e_j(i)|,\ \|\mathbf{u}_{i-j+1}\|\sqrt{\delta_{i-1}}\right)\operatorname{sign}(e_j(i)).$$

Note that when $h_j(i)$ does not exceed $\sqrt{\delta_{i-1}}$, $f(i-j+1)$ is just same as $d(i-j+1)$.

After solving (5), a new weight update equation for the proposed algorithm is obtained such as

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mathbf{U}_i\left(\mathbf{U}_i^T\mathbf{U}_i\right)^{-1}\Sigma_i\mathbf{e}_i, \tag{6}$$

where

$$\Sigma_i(m,n) = \begin{cases} \min\left(1, \sqrt{\delta_{i-1}}\frac{\|\mathbf{u}_{i-m+1}\|}{|\mathbf{e}_m(i)|}\right), & \text{if } m = n \\ 0, & \text{otherwise} \end{cases}$$

and $\delta_{i-1} = \alpha\delta_{i-1} + (1-\alpha)\min\left(\delta_{i-1},\ \left(\frac{e_1(i)}{\|\mathbf{u}_i\|}\right)^2\right)$.

Here, same as in (Vega et al., 2010), δ is updated using $e_1^2(i)/\|\mathbf{u}_i\|^2$ instead of $\|\mathbf{U}_i\left(\mathbf{U}_i^T\mathbf{U}_i\right)^{-1}\mathbf{e}_i\|^2$, because it has lower computational complexity and gives better results especially when the impulsive noise environment. Note that the proposed algorithm selectively shrinks the error components by using the scale factor matrix Σ_i in (6). Therefore, we named the proposed algorithm as selectively shrunk error APA (SSE-APA). The summary of the SSE-APA is given in Table. 1.

The SSE-APA needs additional computational cost for calculating the scale factor matrix. The input vector norm can be obtained from the diagonal component of $\mathbf{U}_i^T\mathbf{U}_i$, so $2K$ multiplications and K comparisons are needed. Also, 2 multiplications, 1 addition and 1 comparison are needed to calculate the moving average of δ_i.

Table 1: Proposed Algorithm Summary.

<u>Initialization</u> : $\delta_0 = 0.001,\ \kappa = 0.5,\ \Sigma_0 = I_{K\times K}$

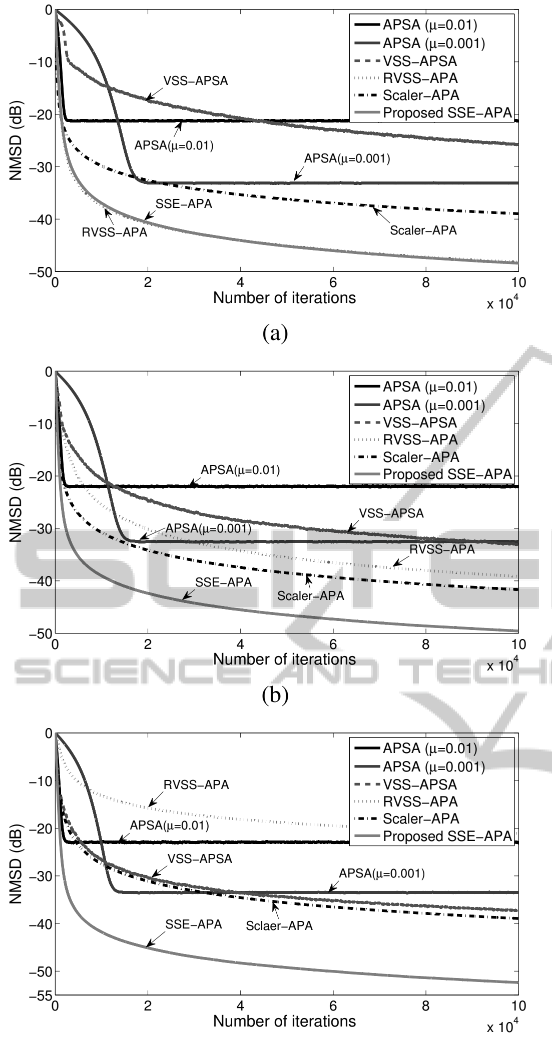
<u>Loop</u> : **for** $j = 1 : K$
  if $\frac{|e_j(i)|}{\|\mathbf{u}_{i-j+1}\|} > \sqrt{\delta_{i-1}}$
    $\Sigma_i(j,j) = \sqrt{\delta_{i-1}}\frac{\|\mathbf{u}_{i-m+1}\|}{|\mathbf{e}_m(i)|}$
  else
    $\Sigma_i(j,j) = 1$
  end
end

<u>Weight update equation:</u>
$\mathbf{w}_i = \mathbf{w}_{i-1} + \mathbf{U}_i\left(\mathbf{U}_i^T\mathbf{U}_i\right)^{-1}\Sigma_i\mathbf{e}_i$

In Fig. 1, a geometric representation for (6) is drawn. The plane $\mathcal{H}_1(i)$ is out of the sphere, so it is moved to $\mathcal{H}_1'(i)$, and $\mathbf{w}_i$ is obtained from the projection onto the modified intersection. The simplest case with $M = 3$, $K = 2$ is considered, because it is not possible to visualize the hypersphere and the hyperplanes.

4 SIMULATION RESULT

To verify the performance of the proposed algorithm, the system identification for the randomly generated $\mathbf{w}^o$ with length $M = 64$ was performed. The colored input sequence was obtained by filtering a zero-mean white Gaussian noise through the first order autoregressive model with its pole at 0.9. During 1000 independent trials, the impulsive noise environment was generated as $v_i = b_i + \eta_i$, where the background noise b_i is a zero-mean white Gaussian noise and the added impulsive noise η_i is the product of a Bernoulli process ω_i and a zero-mean white Gaussian noise A_i, i.e., $\eta_i = \omega_i A_i$. Here, we defined the signal-to-background noise ratio (SBR) and the signal-to-impulsive noise ratio (SIR) as σ_d^2/σ_b^2 and σ_d^2/σ_A^2, respectively, where $\sigma_{(\cdot)}^2$ is the variance of the random sequence $(\cdot)$. The Bernoulli probability $Pr(\omega = 1)$ was randomly selected within (0.01, 0.3) for every independent trial. The parameters for (6) were heuristically decided as $\delta_0 = 0.001$ and $\alpha = 1 - K/(\kappa M)$ with $\kappa = 0.5$. The parameter δ_0 is suggested to be a small positive num-



(a)

(b)

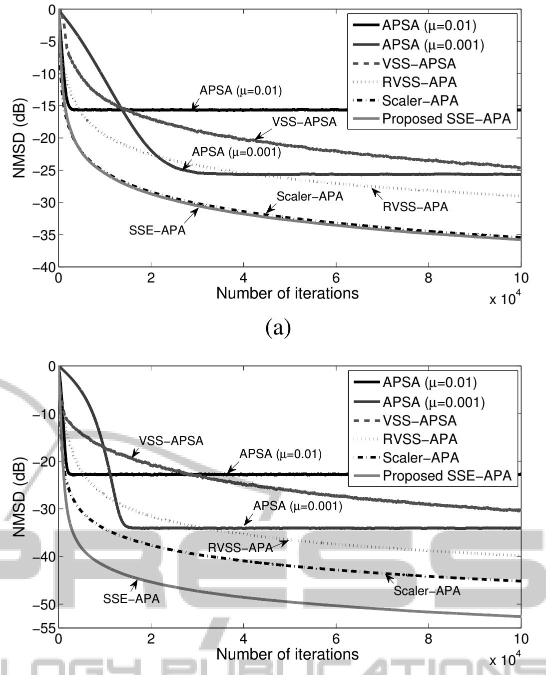
(c)

Figure 2: NMSD learning curves of conventional APSA ($\mu = 0.01, 0.001$), RVSS-APA (Vega et al., 2010), Scaler-APA (Song and Park, 2014), VSS-APSA (Yoo et al., 2014) and the proposed algorithm (a) $K = 2$, (b) $K = 4$, (c) $K = 6$.

ber, i.e., $\delta_0 < 1$, and κ is suggested to be a positive number lower than 10.

To compare the performance with other algorithms, we plotted normalized MSD (NMSD) learning curves in dB scale. Here, the NMSD is defined as $\|\tilde{\mathbf{w}}_i\|^2/\|\mathbf{w}^o\|^2$ where the weight error vector $\tilde{\mathbf{w}}_i = \mathbf{w}^o - \mathbf{w}_i$. The proposed SSE-APA is compared with conventional APSA ($\mu = 0.01, 0.001$), VSS-APSA (Yoo et al., 2014), RVSS-APA (Vega et al., 2010) and Scaler-APA (Song and Park, 2014). For fair comparison, the simulation results were generated using the parameter decision guideline suggested in (Yoo et al., 2014; Vega et al., 2010; Song and Park, 2014).

For the first simulation, the noise ratios were set to SBR = 30 dB and SIR = −30 dB. Figure 2(a) represents the NMSD learning curves when $K = 2$. As can be seen, the proposed SSE-APA and the RVSS-APA had a fast convergence rate and the lowest steady-state MSD compared to other algorithms. When we increased the projection order to $K = 4$ as in Figure 2(b), however, the RVSS-APA and the Scaler-APA showed the severe performance degradation as explained in the text. In contrast, the proposed SSE-APA still had the fastest convergence rate and the lowest steady-state MSD compared to other algorithms. In Figure 2(c), as can be seen, the performances of the RVSS-APA and the Scaler-APA were further worsen when the projection order is increased to $K = 6$. As expected, the proposed SSE-APA had consistently better performance despite of the further increased projection order.



(a)

(b)

Figure 3: NMSD learning curves of conventional APSA ($\mu = 0.01, 0.001$), RVSS-APA (Vega et al., 2010), Scaler-APA (Song and Park, 2014), VSS-APSA (Yoo et al., 2014) and the proposed algorithm (a) SBR$\rightarrow$ 20 dB , (b) SIR$\rightarrow$ -40 dB .

To justify the performance of the proposed algorithm in various environments, the SBR was changed to 20 dB (Figure 3(a)), and the SIR was changed to −40 dB (Figure 3(b)) for $K = 4$. These changes both mean the scaling of the noise, so the performances of the overall algorithms were degraded. However, the proposed SSE-APA still showed the fastest convergence rate and the lowest steady-state MSD compared to other algorithms.

Another important concern about robustness in the

Table 2: Reset Algorithm.

<u>Parameter</u> : $V_T = 3M$, $V_D = \frac{15}{16}V_T$,
$\xi = 10^5$, $\varepsilon = 10^{-6}$,
$\mathbf{M} = \mathrm{diag}\,(1, \cdots, 1, 0, \cdots, 0)$:
$V_T - V_D$ 1's, V_D 0's.

<u>Ctrl update</u> : $\mathbf{c} = \mathrm{sort}\left(\left[\frac{|e_1(i)|}{\|\mathbf{u}_i\|+\varepsilon} \cdots \frac{|e_1(i-V_T+1)|}{\|\mathbf{u}_{i-V_T+1}\|+\varepsilon}\right]\right)$
if $\mathrm{mod}(i, V_T) = 0$
$\mathrm{ctrl}_{\mathrm{new}} = \frac{\mathbf{c}^T\mathbf{M}\mathbf{c}}{V_T - V_D}$
end

<u>Reset decision</u> : $\Delta_i = \left(\mathrm{ctrl}_{\mathrm{new}} - \mathrm{ctrl}_{\mathrm{old}}\right)/\delta_{i-1}$
if $\Delta_i > \xi$
$\delta_i = \delta_0$
else
$\delta_i = \alpha\delta_{i-1} + (1-\alpha)\min\left(\delta_{i-1}, \left(\frac{e_i(1)}{\|\mathbf{u}_i\|}\right)^2\right)$
end
$\mathrm{ctrl}_{\mathrm{old}} = \mathrm{ctrl}_{\mathrm{new}}$

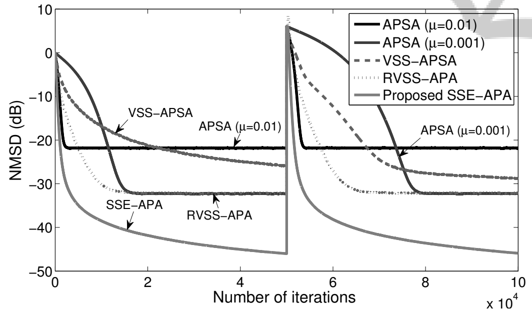


Figure 4: NMSD learning curves of conventional APSA ($\mu = 0.01, 0.001$), RVSS-APA (Vega et al., 2010), VSS-APSA (Yoo et al., 2014) and the proposed algorithm with reset algorithm ($\mathbf{w}^o \to -\mathbf{w}^o$ at iteration$= 5 \times 10^4$).

adaptive filter is an abrupt change in the system coefficient. To track the change in the system coefficient successfully, most variable step-size algorithms using minimum operator need the reset algorithm. Therefore, we adopted the reset algorithm to the proposed algorithm which was introduced in (Rey Vega et al., 2008; Vega et al., 2010) (Table. 2). To verify the tracking performance of the proposed algorithm, the sign of the system coefficient was reversed at half iteration, i.e., $\mathbf{w}^o \to -\mathbf{w}^o$. The simulation was performed with $K = 4$ and the same values for other parameters. The scaler-AP was ruled out because there was no suggested reset algorithm for (Song and Park, 2014). As can be seen in Figure 4, the proposed algorithm showed the fastest convergence rate and lowest steady-state MSD compared to other algorithms even

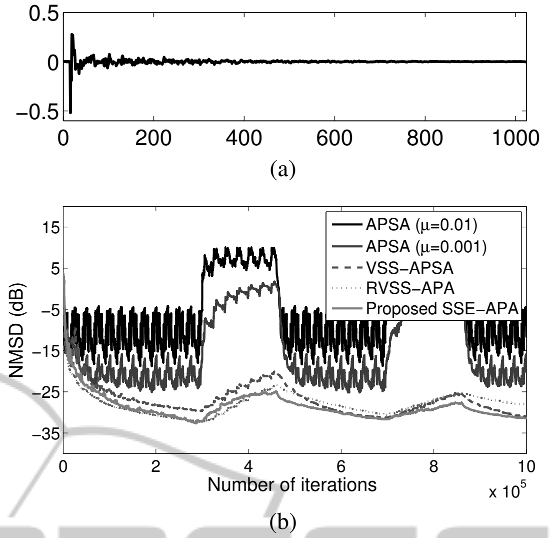


Figure 5: Acoustic echo cancellation with double-talk situation: (a) Room impulse response (b) NMSD learning curves of conventional APSA ($\mu = 0.01, 0.001$), RVSS-APA (Vega et al., 2010), VSS-APSA (Yoo et al., 2014) and the proposed algorithm ($K = 6$).

after the system change.

As the last simulation, to show the performance in a real implementation, the proposed algorithm was applied to the acoustic echo cancellation with double-talk situation. The used room impulse response is plotted in Figure 5(a). A far-end signal and a near-end signal were real speech signals with 8-kHz sampling rate, and two 20-sec near-end signals with 1000 times greater energy than the far-end signal were added before the half and the last iteration, respectively. For two sections which the near-end signals interrupted, i.e, $i \in (300000, 460000)\,\&\,(700000, 860000)$, the proposed algorithm showed a fast convergence rate and a low steady-state MSD compared to other algorithms as in Figure 5(b). The scaler-AP was excluded because it is hard to choose the proper parameter value β (Sayin et al., 2014).

5 CONCLUSIONS

A robust APA using selectively shrunk error component was proposed in this paper. In APA, there both exist interrupted and uninterrupted error components by impulsive noises in an error vector. However, the existing robust APAs applied common step size to all error components in the error vector, so the performance was degraded with a high impulsive noise probability and a high projection order. To overcome this, we proposed the modified minimization criterion to selectively shrinks the error components based on the geometric interpretation. The performance of the proposed SSE-APA was verified with a wide range of impulsive noise probability, various projection orders, lower SBR and SIR, and the system tracking scenario. The simulation results showed that the proposed SSE-APA achieved consistently the fastest convergence rate and the lowest steady-state MSD compared to existing robust APAs and a recent VSS-APSA in various environments with a wide range of the impulsive noise probability.

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