Consensus of Nonlinear Multi-Agent Systems with Exogenous Disturbances

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Abstract: Most existing research concerning the consensus problem of multi-agent systems has been focused on linear first-order or two-order systems without disturbances. However, in practice, most multi-agent systems are complicated nonlinear system subjected to disturbances. In this paper, the coordinated tracking problem for nonlinear undirected multi-agent systems with exogenous disturbances is studied in the framework of consensus theory. The exogenous disturbances generated by both linear exosystems and nonlinear exosystems are considered. Disturbance observers are developed to estimate the disturbances generated by the linear exogenous systems. The Lyapunov stability theorem is used to prove the asymptotical consensus of the systems. The dynamic gain technique is used to construct the disturbance observer for the disturbance generated by a nonlinear exosystem. Based on the adaptive disturbance observer, a consensus protocol is proposed for the nonlinear multi-agent system. Finally, the proposed design approaches are verified though simulation examples.

1 INTRODUCTION

Recently the consensus problem of multi-agent systems has attracted considerable research attention due to the broad applications of consensus algorithms in cooperative control of mobile vehicles flocking (Tanner et al., 2007; Liu Y. et al., 2003), deployment (Corts et al., 2005), formation control (Hu et al., 2008; Konduri et al., 2013), and so on. Multi-agent system is composed of multiple agents. Each agent perceives the surrounding environment, and communicates with other agents.

First-order consensus problems (Bliman, P. A. and Ferrari, T. G., 2008), and second-order consensus problems (Ren, W. and Atkins, E., 2007; Xie, G. M. and Wang, L., 2007) have been studied intensively. However, in reality, mobile agents may be governed by more complicated intrinsic dynamics subjected to disturbances. Examples include tethered satellites, close formation flying and morphing structures with distributed actuators. All these are nonlinear multi-agent systems which are subjected to exogenous disturbances. A few attentions have been paid to the problems of the consensus of nonlinear multi-agent systems (Wu, 2005; Yu et al., 2010). So far, the research of the consensus of nonlinear multi-agent system has been carried out for several specific types of dynamics, such as Euler-Lagrance model (Mei et al., 2011), unicycle model (Sepulchre et al., 2008), rigid body posture model (Ren, W., 2007) and some general nonlinear model satisfying some certain conditions. For general nonlinear systems, it is difficult to handle the consensus control problem under a unified framework.

Few works have considered the consensus problem of nonlinear multi-agent systems with exogenous disturbances. In (MA, G. F. and Mei, J., 2011), the authors have studied the consensus problem of nonlinear multi-agent systems, but they have not considered disturbances. In (Yang et al., 2011), the authors have considered linear multi-agent systems with disturbances generated by linear exogenous systems, and a disturbance observer based protocol has been designed is using linear matrix inequality method. In (Zhang, X. X. and Liu, X. P., 2013), the authors have investigated the consensus problem of linear systems with disturbances generated by nonlinear exosystem by utilizing the dynamic gain technique. In (Das, A. and Lewis, F. L., 2011), Das and Lewis have studied the synchronization of nonlinear systems with disturbances by using a neural network based

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method. However, they have not considered external disturbances generated from nonlinear exogenous systems. In this paper, the consensus problem of nonlinear undirected multi-agent systems with exogenous disturbances is investigated. New disturbance observers for multi-agent systems are derived based on pinning control to estimate external disturbances. The influence of the disturbances is compensated by using proper feedback. Based on the disturbance observers, a consensus protocol is proposed.

This paper is organized as follows. In Section 2, some concepts and useful lemmas are briefly outlined. The consensus of nonlinear multi-agent systems with exogenous disturbances is studied in Section 3. The utility of the new theoretical findings is illustrated by simulation results in Section 4. Finally, some conclusions are drawn in Section 5.

2 PRELIMINARIES

A weighted undirected connected graph G is defined as a triple $G=(V,\varepsilon,B)$, where V denotes the set of nodes, $\varepsilon \subseteq \{(i,j): i,j\in V\}$ denotes the set of edges. Denote $(i,j)\in\varepsilon$ if j is a neighbor of i. The set of neighbors of $i\in V$ is defined as $N_i=\{j\in V:(i,j)\in\varepsilon\}$ .The adjacency matrix is $B=\left[b_{ij}\right]\in R^{n\times n}$ with weighted adjacency elements $b_{ij}\ge 0$. It is clear that if $(i,j)\in\varepsilon$, we have $b_{ij}=b_{ji}>0$, and if $j\ne i$, $\{i,j\}\notin\varepsilon$ we have $b_{ij}=b_{ji}=0$. Assume that there are no self-loops, that is, for any $i\in V$, $(i,i)\notin\varepsilon$. A path between two nodes is a sequence of edges by which it is possible to move along the sequence of the edges from one of the node to the other node. If there exists at least one path from any node to any other node in $G=(V,\varepsilon,B)$, the graph is said to be connected. The matrix D is a diagonal matrix with the elements $\tilde{d}_i$ along the diagonal, where $\tilde{d}_i=\sum_{j\in N_i} b_{ij}, i\in V$. The Laplacian matrix of the weighted graph is defined as $L=D-B$. In an undirected connected graph, the Laplacian matrix is a symmetric matrix.

Consider a nonlinear multi-agent system over an undirected graph $G=(V,\varepsilon,B)$ with n nodes, and the dynamics of agent i is written as:

$$\begin{aligned} \dot{x}_i &= f(t,x_i)+u_i+g(\mathrm{t},\mathrm{x}_i)d_i \\ y_i &= h(\mathrm{t},x_i) \end{aligned} \tag{1}$$

where $x_i\in R^m, u_i\in R^m$, and $d_i\in R^m$ denote the state, the control input, and exogenous disturbance, respectively, $f(t,x_i)$, $g(t,x_i)$,and $h(t,x_i)$ are smooth function in terms of x_i .

Remark 1: Assume the nonlinear dynamic equation $f(t,x)$ satisfies the Lipschitz condition, that is

$$\left\|f(t,x)-f(t,y)\right\|\le l\left\|x-y\right\|, \text{ where } l>0 \tag{2}$$

We say the system described by Eq. (1) asymptotically reaches consensus if $x_i-x_j\to 0$ as $t\to\infty$ for all $i,j\in V$.

3 THE CONSENSUS OF NONLINEAR MULTI-AGENT SYSTEMS WITH EXOGENOUS DISTURBANCES

Assume that the disturbance d_i , $i\in V$ is generated by the following linear exogenous system (Zhang, X. X. and Liu, X. P., 2013)

$$\begin{aligned} \dot{\xi}_i(t) &= A\xi_i(t) \\ d_i(t) &= C\xi_i(t) \end{aligned} \tag{3}$$

where $\xi_i\in R^{m_i}$ is the internal state of the exogenous system, and $A\in R^{m_i\times m_i}$, and $C\in R^{m\times m_i}$ are the coefficient matrices of the disturbance system.

Before proceeding further, we make the following standard assumptions.

Assumption 1: The graph $G=(V,\varepsilon,B)$ describing the interaction topology is connected.

Assumption 2: The matrix pair (A,C) is observable.

The following disturbance observer (Chen, W. H., 2004) is proposed to estimate the unknown disturbance d_i in system (1):

$$\begin{aligned} \dot{\hat{\xi}}_i &= A\hat{\xi}_i+q(\mathrm{x}_i)[\dot{\mathrm{x}}_i-f(t,x_i)-u_i-g(t,x_i)\hat{\mathrm{d}}] \\ \hat{d}_i &= C\hat{\xi}_i \\ \mathrm{q}(\mathrm{x}_i) &= \frac{\partial p(\mathrm{x}_i)}{\partial x_i} \end{aligned} \tag{4}$$

where $z_i \in R^{n_i}$ is the internal state variable of the observer, $\hat{\xi}_i$ and $\hat{d}_i$ are the estimation of ξ_i and d_i. $p(x_i)$ is a function, and the matrix $q(x_i) \in R^{n_i \times n}$ is the observer gain to be designed.

Define the estimate error as

$$e_i = \xi_i - \hat{\xi}_i \quad , \quad i \in V \tag{5}$$

Based on (1), (3) and (4), it is known that

$$\dot{e}_i = [A - q(x_i) g(x_i) C] e_i \tag{6}$$

Then, the following proposition ensures that the disturbance observer (4) can exponentially track the disturbance.

Proposition 1 (Chen, W. H., 2004)**:** The estimation error system (6) is globally and exponentially stable if there exists a gain K such that the following transfer function is asymptotically stable and strictly positive real:

$$H(s) = C\left(sI - \bar{A}\right)^{-1} K \tag{7}$$

where $\bar{A}$ is as follows

$$\bar{A} = \left(A - K\alpha_0 C\right) \tag{8}$$

The Proof is similar to (Chen, W. H., 2004).

Theorem 1: Under the assumptions 1 and 2, with the disturbance observer (4), the pinning control protocol given by

$$\begin{aligned} u_i = &-\alpha[\sum_{j \in N_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - x_0)] \\ &-g(t, x_i)\hat{d}_i \end{aligned} \tag{9}$$

ensures the asymptotical consensus of the system (1).

Proof. Let

$$\tilde{x}_i(t) = x_i(t) - x_0$$
$$\tilde{x}(t) = [\tilde{x}_1^T(t), \cdots, \tilde{x}_n^T(t)]^T$$
$$e(t) = [e^T{}_1(t), \cdots, e^T{}_n(t)]^T$$

where x_0 is the constant of consistency. One can obtain the following system

$$\dot{\tilde{x}}(t) = -\alpha\left(H \otimes I_n\right)\tilde{x} + F(t, \tilde{x}) + \Psi e \tag{10}$$

$$\dot{\tilde{x}}(t) = \alpha\bar{L}\tilde{x} + F(t, \tilde{x}) + \Psi e \tag{11}$$

where $\bar{L}$=-$\left(H \otimes I_p\right)$,

$$F(t, \tilde{x}) = [\left(f(t, x_1) - f(t, x_0)\right)^T, \cdots$$
$$,\left(f(t, x_n) - f(t, x_0)\right)^T]^T$$

$\Psi = G(t, x)C$,

$G(t, x) = [g(t, x_1)^T, g(t, x_2)^T, \cdots, g(t, x_n)^T]^T$,

$H = L + diag\left(a_{10}, a_{20}, \cdots, a_{n0}\right)$, L is the Laplacian matrix of the weighted graph, $\otimes$ is the Kronecker product, I_n is a n-dimensional identity matrix, $\bar{L}$ is Hurwitz.

According to the definition of Euclidean norm and Lipschitz condition, we can know that

$$\left\|F(t, \tilde{x})\right\| \leq l\left\|[\|\tilde{x}_1\|, \cdots, \|\tilde{x}_n\|]^T\right\| = l\|\tilde{x}\| \tag{12}$$

Now, our aim is to show that $\lim_{x \to \infty} \tilde{x}(t) = 0$.

Define the Lyapunov function candidate as following:

$$V = \tilde{x}^T W \tilde{x} \tag{13}$$

where the matrix W is a positive matrix satisfying the following Lyapunov equation (Zhang, X. X. and Liu, X. P., 2013) $W\bar{L} + \bar{L}^T W = -I$.

Taking the derivative of the Lyapunov function yields

$$\begin{aligned} \dot{V} &= \dot{\tilde{x}}^T W \tilde{x} + \tilde{x}^T W \dot{\tilde{x}} \\ &= \left(\alpha\bar{L}\tilde{x} + F + \Psi e\right)^T W\tilde{x} + \tilde{x}^T W\left(\alpha\bar{L}\tilde{x} + F + \Psi e\right) \\ &= -\alpha\|\tilde{x}\|^2 + 2\tilde{x}^T W\Psi e + 2\tilde{x}^T WF \\ &\leq -\alpha\|\tilde{x}\|^2 + 2\|\tilde{x}\|\|W\Psi\|\|e\| + 2\lambda_{\max}(W)\|F\|\|\tilde{x}\| \\ &\leq -\alpha\|\tilde{x}\|^2 + \theta\|\tilde{x}\|^2 + 2\lambda_{\max}(W)l\|\tilde{x}\|^2 \\ &= -\left(\alpha - \theta - 2l\lambda_{\max}(W)\right)\|\tilde{x}\|^2 \end{aligned} \tag{14}$$

for $\forall\|\tilde{x}\| \geq \left(2\|W\Psi\|\|e\|\right)/(\theta)$,where $\frac{1}{2} \leq \theta < 1$.

When $\alpha > \theta + 2l\lambda_{\max}(W)$,the system (11) is globally and exponentially stable.

Hence, by combining (6) and (11), one can obtain the following system

$$\begin{cases} \dot{\tilde{x}}(t) = \alpha\bar{L}\tilde{x} + F(t, \tilde{x}) + \Psi e \\ \dot{e}_i = (A - q(x_i) g(x_i) C)e_i \end{cases}$$

is globally and exponentially stable.

Finally, one obtains $\lim_{t \to \infty} \tilde{x}(t) = 0$, which completes this proof.

Assume that the disturbance d_i , $i \in V$ is generated from following nonlinear exogenous system (Zhang, X. X. and Liu, X. P., 2013)

$$\begin{aligned} \dot{\xi}_i(t) &= A\xi_i(t) + \phi(\xi_i) \\ d_i(t) &= C\xi_i(t) \end{aligned} \tag{15}$$

where $\xi_i \in R^{m_i}$ is the internal state of the exogenous system, $A \in R^{m_i \times m_i}$, and $C \in R^{m \times m_i}$ are the coefficient matrices of the disturbance system.

Remark 2: Assume the nonlinear disturbance $\phi(\xi_i)$ satisfies the Lipschitz condition, that is

$$\left\|\phi(\xi_{i1})-\phi(\xi_{i2})\right\| \le c\left\|\xi_{i1}-\xi_{i2}\right\|, \text{ where } c>0 \quad (16)$$

A disturbance observer (Chen, W. H., 2004) is proposed to estimate the unknown disturbance d_i in system (1)

$$\begin{aligned}\dot{\hat{\xi}}_i &= A\hat{\xi}_i + q(x_i)\left[\dot{x}_i - f(t,x_i) - u_i - g(t,x_i)\hat{d}_i\right] \\ &\quad +\phi(\hat{\xi}_i)+Q^{-1}\zeta_i e_i \\ \hat{d}_i &= C\hat{\xi}_i \\ \dot{\zeta} &= e_i^T e_i\end{aligned} \quad (17)$$

where

$$e_i = \xi_i - \hat{\xi}_i \quad (18)$$

denotes the estimation error between ξ_i and $\hat{\xi}_i$.

$$\begin{aligned}\dot{e}_i &= \left(A - q(x_i)g(t,x_i)C\right)e_i - Q^{-1}\zeta_i e_i \\ &\quad +\phi(\xi_i)-\phi(\hat{\xi}_i)\end{aligned} \quad (19)$$

Proposition 2 (Chen, W. H., 2004)**:** The estimation error system (30) is globally and exponentially stable if there exists a gain K such that the following transfer function is asymptotically stable and strictly positive real:

$$H(s) = C\left(sI-\bar{A}\right)^{-1}K$$

where $\bar{A}$ is as follows

$$\bar{A} = \left(A - K\alpha_0 C\right)$$

The Proof is similar to (Zhang, X. X. and Liu, X. P., 2013).

Theorem 2: Under the assumptions 1 and 2, with the disturbance observer (17), the pinning control protocol given by

$$\begin{aligned}u_i &= -\alpha\Big[\sum_{j\in N_i} a_{ij}(x_i-x_j)+a_{i0}(x_i-x_0)\Big] \\ &\quad -g(t,x_i)\hat{d}_i\end{aligned} \quad (20)$$

ensures the asymptotical consensus of the system (1).

Proof. Now, it is only need to show that the following system

$$\begin{cases}\dot{\tilde{x}}(t) = \alpha\bar{L}\tilde{x} + F(t,\tilde{x}) + \Psi e \\ \dot{e}_i = \left(A - q(x_i)g(x_i)C\right)e_i - Q^{-1}\zeta_i e_i \\ \qquad +\phi(\xi_i)-\phi(\hat{\xi}_i)\end{cases} \quad (21)$$

is globally and asymptotically stable.

It can be known that the matrix K satisfying Proposition 2 can make the matrix $A-q(x_i)g(x_i)C$ be Hurwitz, and $\bar{L}$ is also Hurwitz.

So, we define a Lyapunov function as follows:

$$\bar{V} = V + \sum_{i=1}^{n} V_i$$

Taking the derivative of the Lyapunov function yields

$$\begin{aligned}\dot{\bar{V}} &= \dot{V} + \sum_{i=1}^{n}\dot{V}_i \\ &\le -\alpha\|\tilde{x}\|^2 + 2l\lambda_{\max}(W)\|\tilde{x}\|^2 + 2\tilde{x}^T W\Psi \\ &\quad -\lambda\left(\|e_1\|^2+\cdots+\|e_n\|^2\right) \\ &\le -\alpha\|\tilde{x}\|^2 + 2l\lambda_{\max}(W)\|\tilde{x}\|^2 + \frac{1}{2}\|\tilde{x}\|^2 \\ &\quad +2\|W\Psi\|^2\|e\|^2 - \lambda\|e\|^2 \\ &= -\left(\alpha - 2l\lambda_{\max}(W) - \frac{1}{2}\right)\|\tilde{x}\|^2 + 2\|W\Psi\|^2\|e\|^2 - \lambda\|e\|^2\end{aligned}$$

Let $\lambda \ge 2\|W\Psi\|^2 + 1$, which leads to

$$\dot{\bar{V}} \le -\left(\alpha - 2l\lambda_{\max}(W) - \frac{1}{2}\right)\|\tilde{x}\|^2 - \|e\|^2$$

because of $\alpha > \theta + 2l\lambda_{\max}(W)$ and $\frac{1}{2} \le \theta \le 1$.

Finally, one obtains $\lim_{t\to\infty}\tilde{x}(t)=0$, which completes this proof.

4 SIMULATIONS

An undirected connected network with 6 agents is shown in figure 1.The weighted values of the graph are given randomly in [0 1]. The dynamic of the multi-agents system is specified as follows:

$$\dot{x}_i = \sin(x_i) + u_i + g(x_i)d_i$$

Let the initial states of agents be [2 -2 6 4 -1 1], and the expected consensus state be $x_0 = 3$, and the controller coefficient is $\alpha=5$.

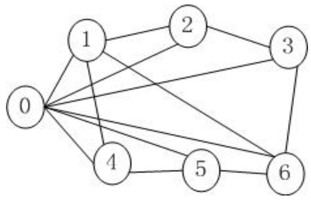

Figure 1: Switching graph of multi-agent systems.

The linear exogenous system is specified as follows:

$$\dot{\xi}_i(t) = 0.2\xi_i(t)$$
$$d_i(t) = 0.1\xi_i(t)$$

The nonlinear exogenous system is

$$\dot{\xi}_i(t) = 0.2\xi_i(t) + \sin(\xi_i)$$
$$d_i(t) = 0.1\xi_i(t)$$

Choose the coefficient matrix of system (1) as $g(x_i) = 1$, and the observer gain $p(\mathrm{x}_i) = 5x_i$. It is easy to verify that the gain matrix of disturbance observer can be chosen as $q(x_i) = 5$. Then the same Hurwitz matrix $A - q(x_i)\mathrm{g}(x_i)\mathrm{C} = -0.3$.
Simulation results are presented as follows:

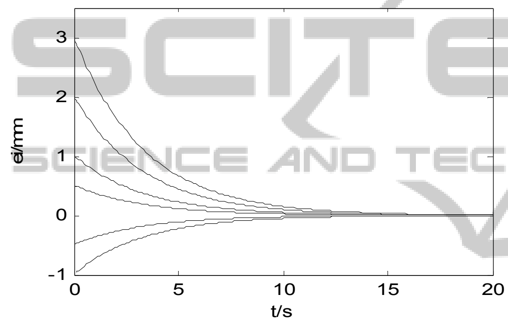

Figure 2: Errors of the disturbances for linear systems.

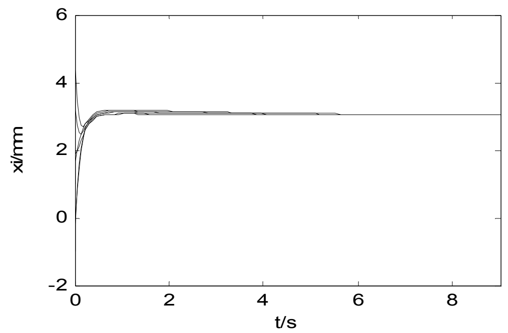

Figure 3: States of multi-agent systems with disturbance observer.

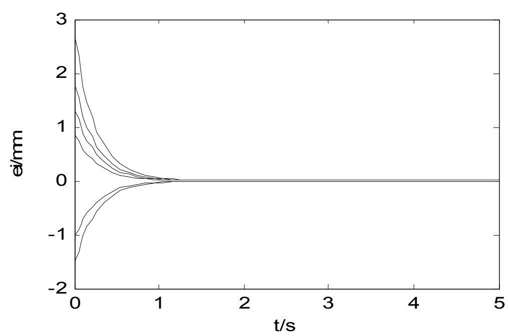

Figure 4: Errors of the disturbances for nonlinear systems.

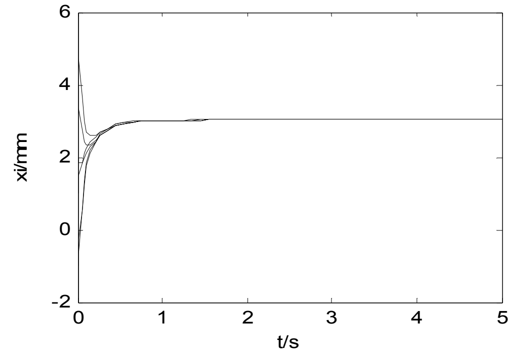

Figure 5: States of multi-agent systems with disturbance observer.

Figure 2 is the error between the disturbances of exogenous linear system and the disturbances estimated by disturbance observer. Figure 3 depicts the final states of the multi-agent systems suffering exogenous disturbances generated by linear exosystems. Figure 4 shows the error between the disturbances of exogenous nonlinear system and the disturbances estimated by disturbance observer. Figure 5 depicts the final states of the multi-agent systems, sustaining exogenous disturbances generated by nonlinear exosystems. From Figures 2-3 and figures 4-5, it can be seen that the consensus algorithm proposed in the paper allows the agents to reach consensus, in the case of exogenous disturbances generated by linear or nonlinear exosystems.

5 CONCLUSIONS

In this paper we have studied the consensus problem of nonlinear multi-agent systems with exogenous disturbances under an undirected topology. For the case when the exogenous disturbance is generated by linear exogenous system, we have shown that, the global exponential convergence of the proposed observer ensures the asymptotical consensus of the nonlinear multi-agent systems with exogenous disturbances. For the case when the exogenous disturbance is generated by nonlinear exogenous system, a disturbance observer with dynamic gain has been designed. The disturbance observers are integrated with the controller by replacing the disturbance in the control law by its estimated value. The numerical simulations on the asymptotical consensus of nonlinear multi-agent systems with disturbances demonstrate the effectiveness of the proposed method.

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