Periodic Takagi-Sugeno Observers for Individual Cylinder Spark Imbalance in Idle Speed Control Context

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Abstract: This paper aims to present a systematic methodology for designing periodic observers for cyclic nonlinear systems represented by Takagi-Sugeno models. An application to idle speed control of a spark-ignition engine will be proposed. Thanks to the estimated individual cylinder values, we can detect an imbalance of each cylinder (unbalanced cylinder). Based on a dynamic hybrid model, some simulation results will prove the efficiency of our method.

1 INTRODUCTION

In natural as in artificial systems, cyclic behaviors can be observed (walk action, flying wings, spark-ignition (SI) engine). The precursor study of (Bolzern et al., 1986) presents a periodic representation of such a system and the work of (Bittanti and Colaneri, 2000) defines the concept of periodic transfer function.

Regarding the engine application, (Chauvin et al., 2007) consider the system as continuous-time linear periodic and build an input estimation for torque combustion of an engine, which is a critical value in control problem. Moreover, the system can be written using a particular discrete-time domain, the crank-angle domain whose efficiency for this kind of study has been demonstrated in (Yurkovich and Simpson, 1997). This work also introduces what they call “fuzzy control” for automotive applications, which is close to the Takagi-Sugeno (TS) (Takagi and Sugeno, 1985) representation used in the present paper. Discrete-time nonlinear periodic systems represented by TS models have been analyzed by plenty of papers, such as (Lendek et al., 2012) and (Lendek et al., 2013a) for new Lyapunov functions construction or (Kerkeni et al., 2009) and (Lendek et al., 2013b) for stabilization. An application to an automotive problem is presented in (Kerkeni et al., 2010) to estimate the air flow inside each cylinder.

The context for this study is idle speed control (ISC). The regulated value is the engine speed, which has to be as low as possible to reduce fuel consumption and pollution during idle phases, but not too low to avoid stalling. Moreover, disturbances may occur in this idle phase because of the starter asking for torque when an electronic device is turned on (lights, radio, GPS, air conditioner and so on…). Because it deals with fuel and pollution, this control problem becomes an environmental one.

The control inputs used for such a control are the throttle angle that pilots the air entering into the engine and the spark advance angle. This angle represents the difference between the moment the spark appears and the Top Dead Center (TDC).

Concerning our application, it is well known from (Grizzle et al., 1991) that all the cylinders of an engine are not working exactly the same way. This leads to a need of individual spark advance control.

Using spark advance for idle speed control has been done in many studies from the literature, with different ways: A mathematical approach taking into account delay and uncertainties (Bengea et al., 2004), an electricity one with current ion that circulates in the spark plug circuit (Shamekhi and Ghaffari, 2005), a torque-based model and control with values for simulation (Bohn et al., 2006), a mathematical set analysis for the problem of safety in digital control of a linear model (De Santis et al., 2006), an electronic-based work with knock sensors

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to tune spark advance (Di et al., 2010) or three control papers that use Adaptive Posicast Control (APC) and Model Predictive Control (MPC) on nonlinear models that are linearized by the authors (Yildiz et al., 2007), (Di Cairano et al., 2008) and (Di Cairano et al., 2011).

The originality of this paper is to distinguish the behavior of the different cylinders using individual cylinder observers in the engine considered as a discrete-time (crank-angle domain) nonlinear periodic system. Comparing to the presented studies, this paper takes into account the nonlinearities instead of linearizing them.

The paper is organized as follows: first, the engine model used to develop the observers is presented, then the observer design methodology is described and finally some simulations results, based on different scenarios, are provided.

2 ENGINE MODEL

2.1 Temporal Domain Model

The model used for developing the observers and the controllers in this paper is mainly based on the work of (Balluchi et al., 2010) that proposes an hybrid model (i.e., a continuous-time model triggered with discrete-time events). The physical phenomena occurring in the SI engine are modelled with polynomial equations whose coefficient values are given for a Magneti Marelli engine. The model is structured as in Figure 1.

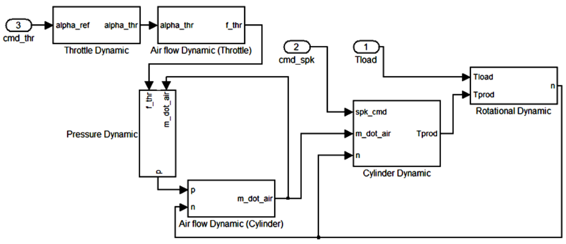

Figure 1: Structure of the engine model.

As detailed in the introduction, the two controlled inputs of the engine are the throttle angle in degrees (°) and the spark advance per each cylinder in degrees before TDC (°). The hybrid model from (Balluchi et al., 2010) can be translated into a continuous-time state-space representation:

$$\dot{x}(t) = f^{p}\left(x(t), u(t), T_{load}(t)\right) \qquad (1)$$

with 10 states and 2 control inputs: $x_1 = n$ (speed of the crankshaft in rpm), $x_2 = T_{air}$ (torque produced by the air path in Nm), $x_3 = p$ (pressure in the intake manifold in mbar), $x_4 = \alpha$ (throttle angle in degrees), $x_5 = \eta$ (efficiency due to spark advance), $x_6 = \varphi$ (spark advance angle before top-dead center, in degrees), $x_7^i = \dot{m}_{air}^i$ (mass air flow in the i-th cylinder) for $i \in \{1, \ldots, 4\}$, $u_1 = cmd_{thr}$ (command of the throttle angle in degrees) and $u_2 = cmd_{spk}$ (command of the spark advance angle, same for all spark plug).

Where $f^{p}\left(x, u, T_{load}\right)$ is a nonlinear function which depends on continuous event "$\left(x, u, T_{load}\right)$" and discrete one "$p$" which corresponds to different functioning phases of the engine, see table I.

Table 1: Engine phases (in function of crankshaft degrees).

Phase / Cylinders	C1	C2	C3	C4
P1 (0° to 180°)	I	C	H	E
P2 (180° to 360°)	C	E	I	H
P3 (360° to 540°)	E	H	C	I
P4 (540° to 720°)	H	I	E	C

Where "I" stands for "Intake", "C" for "Compression", "E" for "Expansion" and "H" for "Exhaust". Such a dynamic temporal system can be written in another base, the crank-angle domain as it has been presented in the past (Yurkovich and Simpson, 1997). Moreover, this domain, based on crankshaft angle, is very close to reality, where the Eelectronic Controller Unit (ECU) of an engine is triggered with a sample time related to crankshaft degrees (for our application, this time is considered as a signal every six crankshaft degrees to fit our engine test bench). This signal (i.e., every six degrees) is doubled with another one when a dead centre is reached, i.e. every 180 crankshaft degrees.

2.2 Crank-angle Domain Model

The crank-angle domain can be considered as a special discrete-time domain that can take into account the "discrete events" described in the hybrid model from (Balluchi et al., 2010). These events allow the model simulating the multiple phases of an engine. As presented in the introduction, such a change can be represented via a periodic discrete-time nonlinear system. For a 4-strokes engine, the phase cycle is completed in 2 rounds, i.e. 720 crankshaft degrees (change of phase every 180 crankshaft degrees). The conversion starts with moving from time to crank-angle domain:

$$\frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt} = \frac{dy}{d\theta}\omega \qquad (2)$$

with ω speed in degrees per second;

$$\frac{dy}{dt} = \frac{dy}{d\theta}\frac{60n}{360} = \frac{dy}{d\theta}6n \Rightarrow \frac{dy}{d\theta} = \frac{1}{6n}\frac{dy}{dt} \qquad (3)$$

In order to convert nonlinear continuous model into a discrete one, the Euler transformation is used:

$$\frac{dy}{dt} \approx \frac{y(k+1)-y(k)}{T_s} \qquad (4)$$

Using (4), the following recursive law can be defined:

$$y(k+1) = y(k) + \frac{T_S}{6n(k)}\frac{dy}{d\theta} \qquad (5)$$

where $y(k+1)$ denotes the signal taken at the (k+1) instant and T_S the sampling time, chosen appropriately not to lose information.

3 OBSERVER DESIGN

Acting as "virtual sensors", observers are a good alternative for automotive problems where adding a sensor is neither an economic valuable solution nor a production commodity one. These observers are able to provide unmeasured information to the controller in order to ensure the optimization of the engine functioning.

3.1 Periodic Takagi-Sugeno Observers

In this section, the theory of these particular observers (i.e., periodic nonlinear ones) will be presented. Let us consider a reduction of the previously-detailed model (1) translated into crank-angle domain with the transformation presented in (5). For commodity of writing and reading, the term k will be omitted and the term $k+1$ will be represented using a "$+$" in index:

$$\begin{cases} x_r^+ = A_d^p(x)x_r + B_d(x)u_r + E_d^p \\ y = C_d x_r \end{cases} \qquad (6)$$

With p denoting the period, i.e. $p \in \{1,\ldots,4\}$, $A_d^p(x) = I + (T_S/6n)A_r^p(x)$ where $A_r^p(x)$ stands for the matrix of the reduced periodic temporal model and $A_d^p(x)$ stands for the discrete one, by the same way, $B_d(x) = \frac{T_S}{6n}B_r$, $C_d = C_r$ and $E_d^p = \frac{T_S}{6n}E_r^p$.

Contrary to what can be read into the literature, this paper presents an alternative of the classical linearization: Dealing directly with the nonlinearities of the system using a particular representation of these nonlinear systems, the so-called Takagi-Sugeno (TS) models (Takagi and Sugeno, 1985). These TS models have the advantage of being an exact representation of the nonlinear model they represent. Let us consider a model with m nonlinearities and their bounds, $m_{\max}$ and $m_{\min}$. A TS model can be written with 2^m subsystems.

The membership functions must be constructed with measurable values in order to verify a premise vector fully measured (i.e., $\hat{z}(k) = z(k)$). The scalar nonlinear functions $h_i(z)$ must verify the property of convex sum $\sum_{i=1}^{m} h_i(z) = 1$. As presented in plenty of papers on Takagi-Sugeno models such as (Takagi and Sugeno, 1985) (Tanaka and Wang, 2001) (Lendek et al., 2010), the scalar functions are related to the nonlinearities bounds: The $h_i(z)$ can be obtained using, for instance,

$$h_1(z) = \frac{nl_{1_{\max}} - nl_1}{nl_{1_{\max}} - nl_{1_{\min}}} \times \ldots \times \frac{nl_{m_{\max}} - nl_m}{nl_{m_{\max}} - nl_{m_{\min}}} \qquad (7)$$

So, a periodic discrete TS model is obtained from (6):

$$x_r^+ = \sum_{i=1}^{m} h_i(z)\left(A_{d\_i}^p x_r + B_{d\_i}u + E_{d\_i}^p\right) \qquad (8)$$

From (8), a periodic Takagi-Sugeno (i.e., nonlinear) observer can be written based on (Guerra et al., 2012):

$$\begin{aligned} \hat{x}_r^+ &= \sum_{i=1}^{m} h_i(z)\left(A_{d\_i}^p \hat{x}_r + B_{d\_i}u + E_{d\_i}^p\right) \\ &+ \left(\sum_{i=1}^{m} h_i(z)S_i^p\right)^{-1}\left(\sum_{i=1}^{m} h_i(z)K_i^p\right)(y - \hat{y}) \end{aligned} \qquad (9)$$

For commodity, the sum of $h_i(z)$ is omitted in the following sections of this paper, and the term $\sum_{i=1}^{m} h_i(z)A_{d\_i}^p$ is replaced by $A_{d\_z}^p$. The product $\left(\sum_{i=1}^{m} h_i(z)S_i^p\right)^{-1}\left(\sum_{i=1}^{m} h_i(z)K_i^p\right)$ is written as

$\left(S_z^p\right)^{-1}\left(K_z^p\right)$ keeping in mind that the z does not represent the same $h_i(z)$.

In order to study the convergence of such an observer, the estimation error $\tilde{x}_r = x_r - \hat{x}_r$ and its dynamic has to be considered:

$$\tilde{x}_r^{\,+} = \left(A_{d\_z}^p - \left(S_z^p\right)^{-1}\left(K_z^p\right)C_d\right)\tilde{x}_r \quad \ldots\ldots(10)$$

Let us define the quantities:

$$\Gamma_{ij}^p = \begin{pmatrix} -\tilde{P}^p & (*) \\ S_j^p A_{d\_i}^p - K_j^p C_d & -S_j^p - S_j^{p^T} + \tilde{P}^{p+1} \end{pmatrix} \quad (11)$$

Theorem 1: According to what has been demonstrated in the work of (Kerkeni et al., 2009), the prediction error is globally asymptotically 4-periodically stable if there exists symmetric matrices $\tilde{P}^p > 0$, matrices S_i^p, K_i^p such that the following LMI conditions from Tuan (Tuan et al., 2001) hold for all engine phases and Γ_{ij}^p defined in (11):

$$\Gamma_{ii}^p < 0,\ i, p \in \{1,\ldots,4\} \quad (12)$$

$$\frac{2}{3}\Gamma_{ii}^p + \Gamma_{ij}^p + \Gamma_{ji}^p < 0,\ i, j, p \in \{1,\ldots,4\}, i \neq j \quad (13)$$

3.2 Application to Torque Observers

Three cascade periodic TS observers can be designed to rebuild the torque produced by spark advance (see Figure 2).

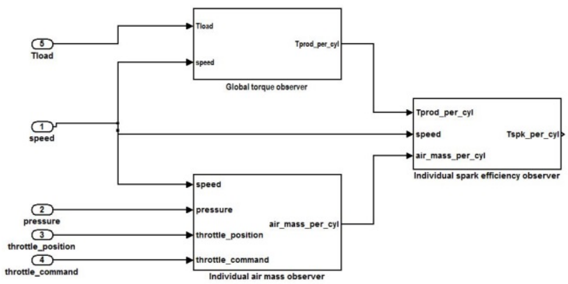

Figure 2: Cascaded observers.

The first observer will provide the air mass inside each cylinder using the measures of pressure, speed, throttle angle and command, thanks to the following reduced model:

$$x_r^{\,T} = \begin{pmatrix} x_3 & x_4 & x_7^1 & x_7^2 & x_7^3 & x_7^4 \end{pmatrix}^T \quad (14)$$

$$A_r^1(x) = \left[\begin{array}{cc|c} -RT_V(c_1 + c_3 x_1) & RT_V(s_1 + s_2 x_4) & \\ 0 & -1/\tau_{thr} & O_{3\times4} \\ c_1 + c_3 x_1 & 0 & \\ \hline O_{3\times2} & & O_{3\times4} \end{array}\right],$$

$$B_r = \begin{pmatrix} 0 \\ 1/\tau_{thr} \\ \hline O_{4\times1} \end{pmatrix}, C_r = \begin{bmatrix} I_2 \\ \hline O_{4\times2} \end{bmatrix}^T$$

$$E_r^1(x_1) = \begin{pmatrix} RT_V\left(s_0 - c_0 - c_2 x_1\right) \\ 0 \\ c_0 + c_2 x_1 \\ \hline O_{3\times1} \end{pmatrix} \quad (15)$$

With $RT_V = RT/V$, $O_{n\times m}$ a n-by-m zero matrix, I_n the identity n-by-n matrix. The data is given in the following table using the convention $x \times 10^y = xe^y$:

Table 2: Engine parameters.

RT_V	c_0	c_1	c_2	τ_{thr}
$2.152e^5$	$8.279e^{-4}$	$3.041e^{-6}$	$8.5e^{-8}$	$8.35e^{-2}$
c_3	s_0	s_1	s_2	a_0
$2.245e^{-9}$	$7e^{-4}$	$3.9e^{-4}$	$5.78e^{-5}$	-0.625
b_0	e_0	h_0	h_1	h_2
59.68	-1074	0	$1.265e^5$	$2.145e^9$

The matrix $A_r^1(x)$ denotes the A matrix in temporal domain during the first phase (i.e., crankshaft between 0° and 180°). The third row expresses, thanks to the coefficient $c_1 + c_3 x_1$, the phase of intake for the first cylinder. Using the same logic, it is possible to build three matrices respectively called $A_r^2(t)$, $A_r^3(t)$ and $A_r^4(t)$ to complete the cycle, see Table 1.

As detailed in the previous section, the next step is to convert this temporal domain into crank-angle domain (6). Two nonlinearities appear in the system, $nl_1 = T_s/6n$ and $nl_2 = RT_V(s_1 + s_2 x4)$. A TS model based on four rules can be written thanks to the classical nonlinear sector approach. The two nonlinearities lead to 2^2 subsystems and are bounded:

$$\begin{cases} n_{min} = 600rpm \\ n_{max} = 1000rpm \\ \alpha_{min} = 0.05° \\ \alpha_{max} = 10° \end{cases} \rightarrow \begin{cases} nl_{1min} = T_S/6n_{max} \\ nl_{1max} = T_S/6n_{min} \\ nl_{2min} = RT_V(s_1 + s_2\alpha_{min}) \\ nl_{2max} = RT_V(s_1 + s_2\alpha_{max}) \end{cases} \quad (16)$$

As explained before, the premise vector is fully measured because composed of speed and throttle angle values. The sample time for the observer can be considered as $T_s = 180°$. Applying Theorem 1 and using the LMI Toolbox of MATLAB, the observer gains are obtained from the previous conditions. This observer can estimate the air mass inside each cylinder without adding any new sensor.

Two other observers can be built by the same way: One for the contribution of each cylinder in the global produced torque, using the following reduced model:

$$x_r^T = \begin{pmatrix} n & T_{prod}^1 & T_{prod}^2 & T_{prod}^3 & T_{prod}^4 \end{pmatrix}^T \tag{17}$$

$$A_r^1(x) = \begin{pmatrix} a_0 & 0 & 0 & 0 & b_0 \\ \hline & & O_{4\times5} & & \end{pmatrix}, B_r = \begin{pmatrix} -b_0 \\ \hline O_{4\times1} \end{pmatrix}$$

$$C_r = \begin{pmatrix} 1 \\ \hline O_{4\times1} \end{pmatrix}^T, E_r = \begin{pmatrix} e_0 \\ \hline O_{4\times1} \end{pmatrix} \tag{18}$$

The last nonlinear periodic observer can be used to get the spark advance efficiency. Based on the two previous presented one, this observer is constructed with the exactly same methodology as the other ones. The output of the observer (i.e., $\hat{y}$) is the total torque produced by the entire engine (i.e., T_{prod}). The reference torque comes from the second observer while the first one is used in the C matrix. The reduced model used to reconstruct the state vector is the following:

$$x_r^T = \begin{pmatrix} T_{spk}^1 & T_{spk}^2 & T_{spk}^3 & T_{spk}^4 \end{pmatrix}^T \tag{19}$$

$$A = O_{4\times4}, B = O_{4\times1}, E = O_{4\times1} \tag{20}$$

The C matrix is periodic and based on the equation from (Balluchi et al., 2010): $T_{prod} = T_{air} \cdot T_{spk}$:

$$C^1 = \begin{pmatrix} 0 & 0 & 0 & T_{air} \end{pmatrix} \tag{21}$$

4 SIMULATION RESULTS

This subsection proves the interest of the developed methodology and the presented periodic nonlinear observers for detecting imbalance.

Let us start with a nominal scenario (i.e., a classic stabilization around the speed reference value, 800 rpm, with neither any disturbance nor any imbalance, and the spark command set to its optimal value). The following figures details the speed (figure 3), the air command (figure 4), the observers results and their estimation errors (figures 5 to 9) that converge to 0%.

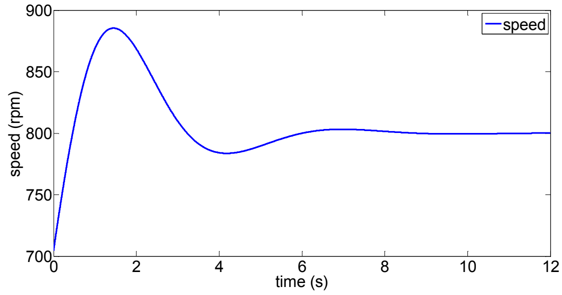

Figure 3: Stabilized speed around speed reference.

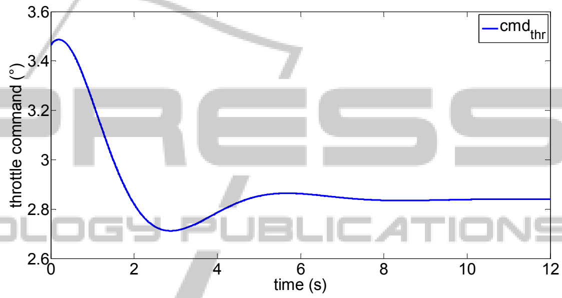

Figure 4: Throttle command in degrees.

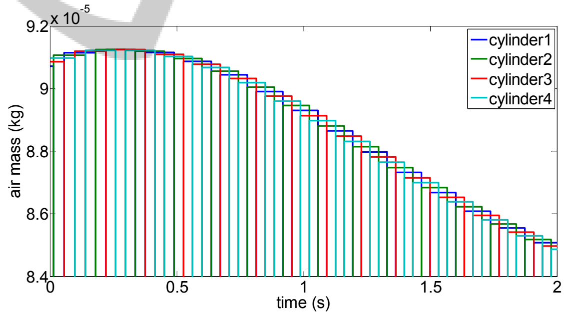

Figure 5: Individual cylinder air mass observer results.

To get the observation error, it is necessary to build to global air mass inside the cylinders, i.e. the air mass used for the torque calculus. Thanks to the engine speed, we can select the air mass inside the cylinder which produces the torque (i.e., the "Expansion" phase in the Table 1).

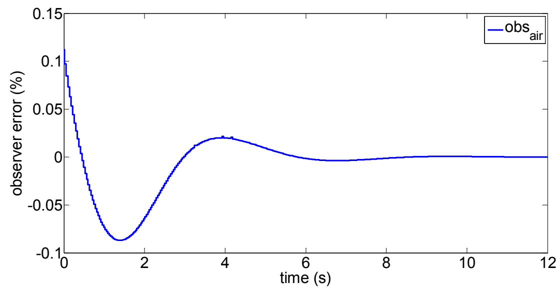

Figure 6: Air mass observer error in percentage.

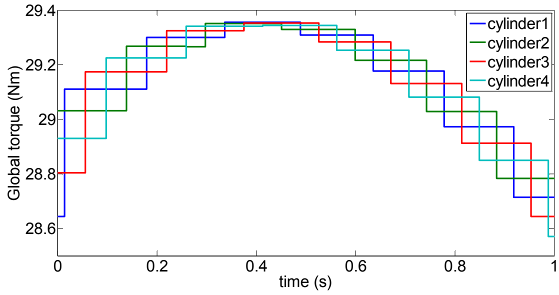

Figure 7: Individual cylinder torque observer results.

To get the torque observer error, we have to realize the same operation as presented before, i.e. using the engine speed to select only the torque produced by the cylinder in the "Expansion" phase according to the Table 1.

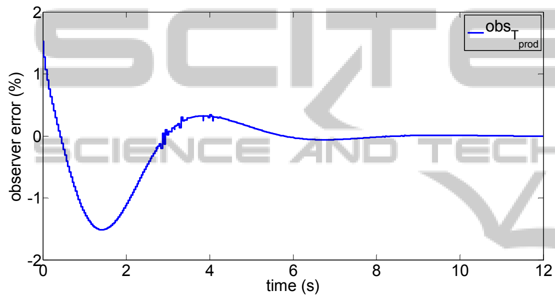

Figure 8: Torque observer error in percentage.

By the same way, we can get the spark advance efficiency thanks to the third observer and build the observation error using the same idea of selecting only the cylinder in the "Expansion" phase. Even if a little static error appears, it stays acceptable (less than 1%)

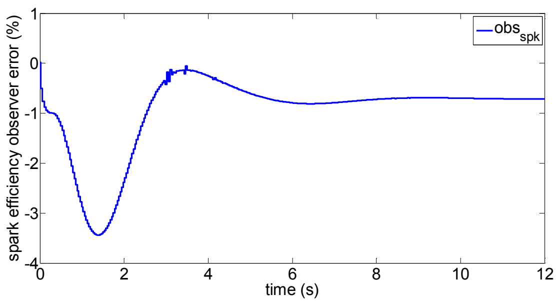

Figure 9: Individual spark efficiency observer error.

This scenario showed the validity of the designed periodic nonlinear observers. Let us introduce the second scenario: An imbalance changes the behaviour of one or several cylinders. A first simulation can be realized applying such a imbalance on the first cylinder. In the engine model, before the spark efficiency calculus, the spark advance (i.e., the spark advance command) is decreased by 20 degrees (i.e., the spark advance used for the efficiency calculus is $cmd_{spk} - 20$). For this scenario, the spark command is set to the optimal value (i.e., $cmd_{spk} = 26.5°$ and consequently $cmd_{spk} - 20 = 6.5°$).

As presented in Figure 13, the spark advance efficiency observer converges for the first cylinder to the value of 0.81. Then, using the formula for the spark efficiency calculus (22) from (Balluchi et al., 2010), we can return to the spark advance angle.

$$T_{spk} = v_0 + v_1\varphi + v_2\varphi^2 + v_3\varphi^3 \quad (22)$$

Applying the inverse of (22), the spark advance angle φ corresponding to a T_{spk} equal to 0.81 is 6.5°. Comparing to the spark advance command (26.5°), we can, thanks to the periodic nonlinear observer, detect an imbalance on the first cylinder equal to a difference of 20 degrees for the spark advance.

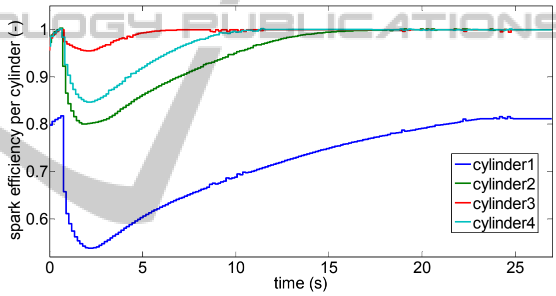

Figure 10: Individual spark advance efficiency observer results with an imbalance on Cylinder 1.

Because all the cylinders can suffer from unbalancing, the next simulation presents the results for two different imbalances applied on the first and third cylinders: Their spark advance will be decreased respectively by 20 and 10 degrees. Figure 14 presents the individual spark advance efficiency observer results in such a scenario.

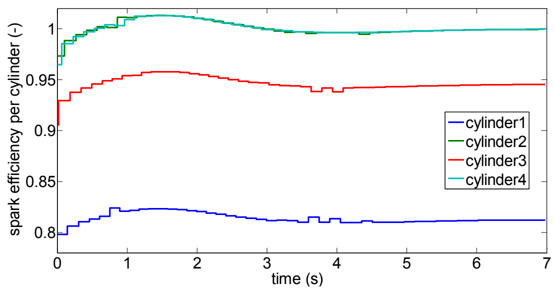

Figure 11: Individual spark advance efficiency observer results with two imbalances.

The spark advance efficiency observer converges to the optimal value, 1, for Cylinders 2 and 4. Cylinder 1 converges to the same value as Figure 12, 0.81, and Cylinder 3 converges to 0.94. Using the inverse of (22), the following spark advance degrees and imbalances can be identified:

$$\begin{cases} \varphi_1 = 6.5° \\ \varphi_2 = 26.5° \\ \varphi_3 = 16.5° \\ \varphi_4 = 26.5° \end{cases} \rightarrow \begin{cases} \text{Imbalance :} -20° \\ -0° \\ -10° \\ -0° \end{cases} \qquad (23)$$

5 CONCLUSIONS

This paper has presented a methodology to manipulate hybrid dynamic systems as periodic models and adapt them to crank-angle domain. Such an adaptation can lead to the construction of a periodic nonlinear Takagi-Sugeno representation, allowing using efficient tools such as LMI. Thanks to this methodology, this paper presents how to build periodic TS observers and demonstrates with examples such as individual cylinder air mass, global produced torque or individual cylinder spark advance efficiency, allowing the detection of an imbalance between the cylinders. This can lead to a tuned spark advance control, including auto-balancing of the cylinders.

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