Use of Frequent Itemset Mining Techniques to Analyze Business Processes

Vladimír Bartík and Milan Pospíšil

Faculty of Information Technology, Brno University of Technology, Božetěchova 2, Brno, Czech Republic

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Abstract: Analysis of business process data can be used to discover reasons of delays and other problems in a business process. This paper presents an approach, which uses a simulator of production history. This simulator allows detecting problems at various production machines, e.g. extremely long queues of products waiting before a machine. After detection, data about products processed before the queue increased are collected. Frequent itemsets obtained from this dataset can be used to describe the problem and reasons of it. The whole process of frequent itemset mining will be described in this paper. It is also focused on description of several necessary modifications of basic methods usually used to discover frequent itemsets.

1 INTRODUCTION

The paper is focused on application of data mining techniques to the data, which describe business processes. This task is called process mining and it is focused on analysis of information from event logs that were produced by business processes. In process mining, the typical result is the process description, which is previously unknown.

In this paper we assume that we know description of a process but the process log often needs further analysis. Our objective is to obtain other knowledge, which should be a previously unknown, potentially useful and valid knowledge, which leads to an improvement of a business process.

There are two basic kinds of data mining techniques: descriptive and predictive. In process mining, predictive techniques, such as classification, can be, for example, used to predict events leading to delay in a process. Based on the learning dataset collected in past, a user can be warned during the process about high probability of a problem appearing in the process.

On the other hand, descriptive techniques, such as association rules or frequent itemsets can help an analyst to a better understanding of the reasons of various problems in a business process. The main objective of this contribution is the use of frequent itemsets to describe some critical moments in the business process. Its application will be presented using a business process in a manufacturing company.

The event log used for our analysis is usually in a form of a relational table. Therefore, it is possible to use an arbitrary data mining method to analyze it. Frequent itemsets and association rules have been originally designed for transactional databases usually used in the market domain. But it is not a problem to adapt it for analysis of relational data. Some issues regarding this adaptation will be described in the following sections in detail.

Data used for analysis are collected by the simulator of producing history, which takes the event log as an input. The simulator takes the input and scans the queues before each production machine. If the queue before a production machine in a relatively short time interval is rapidly increased, the products and their properties are stored into a database table, which is used subsequently for analysis.

The organization of our paper will be following. After a summarization of works related to our problem, the simulator of producing history and the way how the data for analysis are obtained is described in Section 3. Then, in Section 4, the process of frequent itemset mining and several related problems needed to be solved are described. Then, results of our process mining method are shortly summarized in Section 5 and an outline of various extensions of our approach for the future and conclusion are contained in the last sections.

Bartík, V. and Pospíšil, M.

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2 RELATED WORK

Data mining techniques are often used in business process management. The research area is referred to as Process Mining (Van der Aalst et al., 2007, 2011). It is focused on analysis of information from event logs that were produced by business processes. The results can be used to build a model from an unknown process (this is called process discovery (Rozinat et al., 2009)) or to make the model more precise.

Data mining techniques can also be used to analyse the behaviour of a process, for example decision tree based classification can be used to predict the process performance (Wetzstein et al., 2009). Another prediction based method was proposed in (Polato et al., 2014). In (Grigori et al., 2001) it was shown that classifiers can also be used to predict the execution time of the process, based on case attributes and time information of preceding tasks.

Descriptive data mining techniques have not been used frequently in the process mining yet. This paper shows that one of them (frequent itemsets mining) can be used to describe the problems which occur during the process and understand them.

Association rules and frequent itemsets were first introduced in (Agrawal et al., 1993). Mining association rules was primarily designed for usage in transactional data, typically used in the market domain. Here, the goal is to find sets of items, which occur frequently together in the same transaction. A lot of algorithms for mining frequent itemsets in transactional data have been developed. The Apriori algorithm (Agrawal and Srikant, 1994) is probably the most famous of them because of its simplicity. On the other hand, the FP-Growth algorithm (Han et al., 2000) proved to be much more efficient than the Apriori algorithm. A lot of modifications and improvements have been proposed for both algorithms.

Our dataset obtained by the simulator is in a form of a relational table. Mining frequent itemsets in relational tables is quite different because some of the attributes are continuous and they need to be discretized. Discretization can be very simple, if we use equi-depth or equi-width discretization, but there have also been several advanced methods proposed, such as distance-based methods described in (Miller and Yang, 1997) and (Li et al, 1999), where discretization is based on clustering. After discretization, we can consider one record in a table as a transaction and use a slight modification of basic methods of frequent itemset mining.

A set of frequent itemsets can also be further used as a classifier, as it was presented in (Liu et. al, 1999) and (Bartik, 2007). Therefore, the results of our paper can be used also to a predictive analysis of event logs afterwards.

3 ACQUIREMENT OF THE DATA FROM AN EVENT LOG

Our approach is used in the manufacturing company, which is producing doors. The event log describes the whole production process. The process is implemented by a set of various production machines, each of which is responsible for one aspect of a door production. One record in the event log represents a production task of one product at one machine.

We also have a set of attributes for each product. The manufacturing company needs to know how the attributes of various products to be produced affect the queues before some of the machines. Some of them lie on the critical path, which consists of a set of machines, which work in serial. These machines are most probable to have a long queue before it and that is why we have to focus our attention on these machines.

To analyze the queues before the machines and to obtain the data for analysis, we use the simulator of production history.

3.1 Data Acquirement

The task, for which we use the simulator of production history, is to analyze the queues before the machines on the critical path and if the queue becomes very long, we have to store the information about last products produced by that machine immediately before the increase of the queue into a special relational table.

We have to mention that the length of a queue is not the most important criterion to store that information because a long queue can be caused by higher number of products at the input of the production line and this does not necessary mean a problem in the business process.

Data about the products, which possibly cause some problem, are collected in the moment when the queue is increased rapidly in a short time interval. The queue before each machine in the critical path is monitored continuously and the actual length of the queue is compared with its length recorded, for example, one hour before. If the difference is greater than a constant specified by the user, then information about the last batch of doors is stored into the dataset for analysis. In our business process of a



Figure 1: Histogram of queue lengths at a production machine.

manufacturing company, this constant has been set to a value of 100. But this number can be different for other processes

There is an example of a histogram of queue lengths at one production machine depicted in Figure 1. The points in time, in which the queue increased rapidly and therefore the data about products are collected into the dataset, are labelled by the arrows. Data are stored into a relational table and the task will be to obtain properties of products, which occur frequently together in records collected in moments when the queue is rapidly increased.

3.2 Description of the Data

Each record in the input relational table represents one piece of product. In our case, the table consists of 17 attributes, both continuous and categorical. Continuous attributes include primarily proportions of each product, which should be discretized in some suitable way. The categorical attributes are stored in a form of strings and they represent some visual properties and material of which the product is made.

The information about production machine, on which the increased queue was identified, is also assigned to each record. This allows manager to specify if he needs to mine frequent itemsets representing "problematic" products for all machines together or for a specific machine that he currently needs to be analyzed.

We also collect the information about time, when the product has been processed by the machine but this information is not needed for frequent itemset mining but it can be used for further analysis of data.

The algorithm for collecting the data was executed for an event log, which describes the production process during the interval of two years. The relational table consists of approximately 15000 records for three production machines at the critical path.

There is no need for complex pre-processing of this data, except discretization of continuous attributes, which will be discussed later.

3.3 Problems of Data Quality and Their Solutions

There are several types of problems that could be resolved to improve frequent itemset mining accuracy. In this section, they will be reviewed and summarized. Here is the list of common problems:

- Incomplete Measurement. Some workplaces are measured only partially. Only start or finish time information is available. The worst case is when no measurement is available – the information can be derived from context tasks around. But in this case, the precision of data can be significantly lower.
- Cluster Measurement. This happens when setup time and errors are measured together with work time.
- Hidden Subprocess. If the execution time of task is measured, task could contain a subprocess with unknown execution time – we know only execution time of the whole process but sometimes we

need to know the times of its subtasks to better predict real execution time or analyze the event log. This problem is similar to the process discovery problem.

- Changes in Time. Real processes are not static; their execution times change in time. There are two possible solutions – adjust method to changes or ignore changes and work only with new relevant data. We have to choose between these two solutions. It depends on the concrete situation. When changes of processes are small and slow, methods could be easily adjusted, when changes are larger, using only the newer data may be a better solution.
- Other Reasons of Wrong Execution Time. If the queue before a production machine increases, it is not caused by a problem of the process in all cases. There are also other reasons of it, such as the worker had a break or the machine had some failure etc. These cases should not be collected into the dataset for analysis. The problem should be solved in the pre-processing phase, if we are able to detect these situations.

4 FREQUENT ITEMSET MINING

In this section, the problem of mining of frequent itemsets in relational tables used in our process mining task will be defined formally. Then, our modification of the basic Apriori algorithm will be described, together with other various solutions of related problems.

4.1 Formal Definition

Assume that we have a relational table *R*, which is defined on domains D_1 , D_2 , ..., D_n . It is defined as a an ordered pair $R = (H, R^*)$, where H is the heading of a relational table and R* is its body, which contains records. The heading of a table is defined as a set $H = \{(A_1:D_1), (A_2:D_2), ..., (A_n:D_n)\}$, where $A_i \neq A_j$, for each $i \neq j$. A_i , for each i=1,2, ..., n are the attributes of a table and D_i are their corresponding domains (sets of possible scalar values of an attribute, which must be of the same type). The body R* of a relational table is defined as a relation $R^* \subseteq D_1 \times D_2 \times ... \times D_n$.

If the domain D_i of an attribute A_i is finite, then the attribute is categorical. On the other hand, if the domain is infinite and there is an ordering defined for that attribute then the attribute is continuous (or quantitative). We have to find a set of frequent itemsets in a relational table. A frequent itemset *FI* in a relational table *R* is defined as a set of predicates *p* of a form $\{a_1, a_2, ..., a_n\}$. This set of predicates with their values must correspond with a given count of rows from the body R*. The count of rows must be higher than a minimum specified by means of minimum support threshold. The set FI of all frequent itemsets can be then specified as:

$$FI = \{ fi \mid support(fi) \ge minsup \}$$
(1)

The value of support s for a set of predicates S is defined as a ratio of row count in the table, in which their values correspond to the values contained in the set of predicates S to the overall count of rows in the relational table. It can be expressed as follows:

$$s(fi) = \frac{n}{N}$$
(2)

where N is the count of all rows in the table R and n is the count of rows, which correspond to the set of items S.

If a set of predicates $p = \{a_1, a_2, ..., a_n\}$ has a support value higher than a minimum support threshold, then there must exist a set *Rows*, which contains records from the table R, for which:

$$\forall a_k \in p: a_k \approx r_i: \frac{|Rows|}{N} \ge minsup \tag{3}$$

The expression $a_k \approx r_i$ denotes that values contained in a given predicate a_k are contained in the record r_i of the relational table R.

If A is a categorical, Boolean attribute or a numerical attributes with a small domain of values, the expression $a_k \approx r_i$ is defined as:

$$(A = v) \approx r_i \Leftrightarrow \exists j : A_i = A \land v_{ij} = v \quad (4)$$

On the other hand, if A is a continuous attribute with a large domain, it is defined as:

$$(A = [l, h]) \approx r_i \Leftrightarrow \exists j: A_j = A \land l \le v_{ij} \le h$$
(5)

The equation (5) leads to the problem of continuous attribute discretization, where the values of the l and h values must be set.

Except support and confidence, there have several alternative measures of frequent itemset and association rule frequency proposed. This includes the *crossSupportRatio* measure (Xiong et al., 2003), which means the ratio of support value of the least frequent item to support value of the most frequent item. The next possible measure is the *allConfidence* (Omiecinski, 2003), which is defined as the minimum confidence value among all possible association rules generated from that itemset.

In our work, the support and confidence measures proved to be sufficient to represent the frequency of itemsets and association rules.

4.2 Description of the Method

In general, there are two basic kinds of methods available to discover frequent itemsets: Apriori based algorithms and methods based on the FP Growth algorithm. Since the efficiency of the algorithm is not the critical problem in our application, we have decided to use the Apriori based algorithm in the first phase. This algorithm works in two following iterative phases.

The first of them is the generation of k-itemsets – itemsets with k items (candidates): In the first iteration, the frequent 1-itemsets are generated from the database. In all consecutive iterations, a set of frequent k-itemsets is generated from a set of frequent (k-1)-itemsets obtained in the previous iteration.

This step consists two phases: concatenation and extraction. The first one generates all possible kitemsets (candidates). The second one extracts the itemsets any subset of which is not contained in frequent itemsets generated in the previous iterations. This results from the fact that the support of a k-itemset cannot be higher than the support of its subset (this property is also called the Apriori property).

The second step is counting and checking the minimum support threshold. All transactions in the database are scanned and if the itemset is found, its support is incremented. Then, the minimum support threshold is checked.

If no new (k+1)-itemsets are generated in some iteration, the algorithm is stopped and the final result is the union of all frequent itemsets generated by previous iterations, which contain I to k items.

The next section contains description of several modifications needed to adapt the Apriori algorithm to use it in our relational dataset generated from the event log for the purpose of process mining.

For the FP-tree method, these modifications can be the same. No other modifications are needed.

4.3 Discretization and Other Necessary Modifications

As it was mentioned above, the first problem needed to be solved is discretization of continuous attributes. This is necessary because of the fact that support of items containing continuous values is much lower than those with categorical values. This leads to a result consisting only from frequent itemsets with categorical values.

In our project, we decided to use the equi-width discretization. The main disadvantage of this approach is the fact that continuous attributes usually do not have uniform distribution and therefore the differences of support values for various intervals of continuous values can differ very much. Influence of this factor will be significantly reduced by arrangements described below.

The next modification is the filtering of frequent itemsets, which is performed after the set of all frequent itemsets is obtained. If we obtain a lot of frequent itemsets with very high values of support, it is necessary to compare the value with its frequency of occurrence in the overall event log.

For example, in the manufacturing company, if we obtain a frequent itemset containing properties of products {height=[x_1 , x_2], edge_surface = 'yyyy'} with the value of support equal to 20%, we have to scan the whole event log and count the support of this itemset within the event log. If the value of support is similar (or higher), the frequent itemset has no significance for the analysis of delays. The support of an itemset in the dataset collected when the queues are rapidly increased should be significantly higher than in the whole event log.

Therefore we have decided to set the minimum support threshold at a quite lower value because a lot of frequent itemsets is filtered. This causes that the time complexity of the algorithm is quite higher but this makes it possible to obtain more interesting frequent itemsets meeting the requirements mentioned above because there is an assumption that problems can be caused by products with some nonstandard properties, which are not very frequently produced and therefore their support in the dataset will probably be lower but its value of interest is higher.

For this purpose, we have defined a new measure called percentage change of support value (*PCS*). It is defined as:

$$PCS(fi) = \frac{s_2(fi) - s_1(fi)}{s_1(fi)}$$
(6)

where s_1 is the support value of frequent itemset fi in the whole event log and s_2 is the support value of the same frequent itemset in the dataset collected when queues are significantly increased. The user has to specify a value of minimum *PCS* value before the process of mining frequent itemsets is started.

Some results of mining frequent itemsets with various values of minimum PCS will be summarized in the next section.

5 EXPERIMENTS AND THEIR RESULTS

As it was mentioned in Section 3, our dataset consists of 15000 records and 17 attributes that describe properties of products present at production machines at the critical path in time when some delay occurred. The value of minimum PCS has been set to 0.1. We recommend setting this value higher than zero to ensure that the frequent itemset is really significant with respect to delays in the business process.

Next, we have to find a suitable minimum support value. The value must ensure that the count of all frequent itemsets is high enough. In our first experiment, we have set it to a value of 30%. The count of frequent itemsets obtained by the Apriori algorithm was 105, but almost all of them have been deleted by our pruning phase because their support in the whole dataset has not been high enough to satisfy the condition of minimum PCS value.

Therefore we recommend setting the minimum support value between 15% and 25%. The Table 1 shows the dependence of the frequent itemsets count on the minimum support.

From Table 1 we can see that the reduction of frequent itemsets with the PCS value is very strong. Due to the very high time complexity of the algorithm for the lowest minimum support value and a small difference between counts of frequent itemsets after pruning, we consider the value 0.2 as the optimal value. But this optimal value can be slightly different with use of other datasets.

Table 1: Counts of frequent itemsets.

Minimum support	After Apriori	After pruning
0,15	1160	110
0,20	593	95
0,25	426	69

Regarding the values in the frequent itemsets obtained by our method, their length does not usually exceed 6 items. The most of them contain attributes describing material, model names or edges of doors produced in the manufacturing company. Only a small number contains information about product's size or other numeric values. This can be caused by the fact that most of products are usually produced in some standard sizes and other sizes appear in data very rarely.

The typical form of a frequent itemset obtained in our dataset, which satisfies the minimum support and minimum PCS is following:

This leads to a conclusion that delays in our business process of door production are mainly affected by some specific values of categorical attributes describing visual properties of doors and the model name of a door.

To prove the correctness of these conclusions, we have used the obtained frequent itemsets for a simple classification. The main idea of this experiment is that we try to predict long queue with use of product parameters. If some count (denoted as c) of products, parameters of which satisfy the frequent itemsets obtained with support 0,2 (for this experiment, we take only frequent itemsets with at least 2 items), these products are classified as "longer queue". The results of classification accuracy and its dependence on the c value, is shown in the Table 2.

Table 2: Results of simple association-based classification.

Value of 'c'	Accuracy of classification
2	67%
4	88%
6	84%

We can see from the results that the longer queues are often caused by more than one product. This is probably caused by the fact that very similar products are usually grouped together into batches before they are produced. We can see that at least 4 products, parameters of which satisfy the frequent itemsets cause delays in most cases.

Of course, not all delays in the process are caused by specific product attributes. There are also reasons like machine failures or big amount of products at the input. Therefore, our approach does not cover all possible problems in the process. On the other hand, our approach solves the problem, which is typical for processes in manufacturing companies and it can substantially help managers in planning of their production.

One of the issues regarding presentation of association rules to managers is their visualization. For this purpose, the simplex representation (Kenett and Salini, 2010) can be used.

6 FUTURE RESEARCH

There are several possibilities to extend the results described in this paper and use the frequent itemsets for other tasks. The main focuses of our future research in this area are described in the next subsections.

6.1 Further Pre-Processing

In the moment, when a problem appears and data are collected, information about a set of products (doors, in our case) is collected. This can include for example 50 products, which are similar very frequently.

Therefore there is a possibility to join these products in one record (or more), which represents the main properties of a set of products. Higher value of support must be assigned to this new record. This step will make the mining simpler and therefore it will probably increase the efficiency of the whole frequent itemset mining process.

6.2 Association-based Classification

Predictions, recommendations, and dynamic optimizations could be realized with use of some predictive data mining technique, such as classification.

As it was proposed in (Liu et. al, 1999), (Bartik, 2007) and Section 5, a set of frequent itemset can be used as a classifier. For each predefined class, a set of frequent itemsets representing the records of that class is discovered. Then, in the classification phase, we are able to compare a new record, class of which is not known, with frequent itemsets for each class and determine the class according to frequent itemsets, which correspond to the record most.

For example, if we separate the processing time attribute into three categories (low, medium, high) and discover frequent itemset for each of them, we are able to predict the delay during the process and warn workers before the problem happens.

6.3 Use of Sequential Patterns

Frequent itemsets can be also extended to take the order of events before the delay into account. Therefore, frequent itemsets could be substituted by sequential patterns. In our event log, the time information is present for each event that is why the order of products that have been processed by the producing machine is easily detectable.

Given a set of sequences (sets of records ordered according to their time) and the support threshold, the task is to find the complete set of frequent subsequences. There are several algorithms proposed for sequential pattern mining, mainly based on the frequent itemset mining algorithms, for example the AprioriAll algorithm or the PrefixScan algorithm based on the FP-Growth method. This can be helpful in the advanced analysis of business processes to find some frequent sequences of events leading to delays or other kinds of knowledge about the manufacturing process.

7 CONCLUSIONS

In this paper, we have proposed the method for analysis of data from event logs based on frequent itemset mining. It can be used to analyze the reasons of problems that can appear during the business process. This can help the analyst to determine products, which usually cause delays at production machines in the manufacturing company.

Our experiments have been executed on the dataset consisting of products, which were processed by the production machine before the problem appeared. All attributes of these products have been collected. Then, our task was to find sets of values, which occur frequently in the processes, where some problem causes the delay. The experiments showed the necessity of a pruning phase, where the support of an itemset in our dataset must be compared to its support measured in the whole event log.

In our future works, except those mentioned in Section 6, we have to find a way to detect records, in which the execution time is measured wrongly. This must be accomplished by a deep analysis of the data and the business process itself.

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