

# Many-Valued Logic through Its History

Angel Garrido

*Department of Fundamental Mathematics, Faculty of Sciences UNED, Paseo del Rey, 9, 28040, Madrid, Spain*

Keywords: Mathematical Logic, Vagueness, Uncertainty, Many-Valued Logics, Fuzzy Logic.

Abstract: Our purpose is to contribute here to the searching for the origins of many-valued logics and, within them, as a special case that of “Fuzzy Logic”, also called by different manners, as Diffuse Logic, either Heuristic Logic, or ‘logique floue’ (in French), etc. It is also our goal to relate how was welcome to many-valued logics in our Iberian Peninsula, which is just another province of the world philosophical universe.

## 1 INTRODUCTION

Searching for the origins could lead too far and eventually disperse, which, as we know is not very convenient for a job pretending to be research. So we will refer to these first signs that appear in the East (China, India...), and then we may analyze the problem of “future contingents”, treated by Aristotle in *Peri hermeneias*, or *De Interpretatione*.

That issue would then central in medieval times, as during the Scholasticism, with William of Ockham, and Duns Scotus, or Richard of Levenham, among others, looked at from different point of views, for his relationship with determinism and ‘Divine Foreknowledge’. Then, this issue is taken up by Spanish Jesuits Luis de Molina or Francisco Suarez, and even the great polymath G. W. Leibniz dedicated his time.

Even then there is a dark time for the logic, and reappearing in the nineteenth century, philosophers and mathematicians such as George Cantor, Augustus De Morgan and George Boole, Gottlob Frege, ... There was born the new set theory, now called “classic”, but then also had terrible enemies, as the then almighty Leopold Kronecker, who from his professorship in Berlin did everything possible to hinder the work of Cantor, and the rise of those new ideas.

## 2 MANY-VALUED LOGICS AND LWOW-WARSAW SCHOOL

Parallel to this, there arises a new kind of thought

and way of seeing must be the act of philosophizing: the Polish Lwow-Warsaw School (LWS, by acronym). This is happening like tributaries of a great river and sub-tributaries, departing from Leibniz, from masters to disciples. Start with Bernard Bolzano, which influence-much about his disciple, Franz Brentano. This, in turn, greatly influence on all his subsequent students.

Among these disciples of Franz Brentano will be one that particularly interested us. This was the Polish philosopher Kazimierz Twardowski, who shared many characteristics with his teacher: love for precision and clarity of ideas, charisma among those who treated him, preference for the spoken to the written word, etc... From his chair in the city of Lvov spread many of the ideas of Franz Brentano, adding their own. Led to a circle of people, all they very interested in a compulsory renewal of philosophical studies, especially from the point of view of logic. In a certain sense, served a function similar (but independent) to the Vienna Circle (Wiener Kreis), or later the Berlin Circle, because very different and singular characteristics. It's called Lvov-Warsaw School. Its members took the logical-philosophical and mathematical studies at Poland to the forefront of global world research. It was during the “interbellum”, or period between the two World Wars, i.e. ranging from 1918-1939. Then, rouse the Diaspora, after the war and by the strong communist dictatorship.

Many notable names among the members of this school of logic, but could cite to Jan Lukasiewicz, Stanislaw Lesniewski, Alexius Meinong, Kazimierz Ajdukiewicz, Tadeusz Kotarbinski, Mordechaj Wajsberg, Alfred Tarski, Jerzy Slupeecki, ..., or the

justly remembered, Andrzej Mostowski. Also must be cited Jan Wolenski (as vindicator of the LWS' memory), or Helena Rasiowa, Roman Sikorski, etc. Among them, one of the most interesting must be Jan Lukasiewicz, the father of many-valued logics (MVLs, by acronym). Jan Lukasiewicz began teaching at the University of Lwow, and then at Warsaw, but after World War II must to continue in Dublin and then at Manchester.

At first, Jan Lukasiewicz introduced the three-valued logic and then generalized to the infinite-valued. That possibility modulation can be expressed by a membership function, which is to come all the unit interval  $[0,1]$ , instead of being reduced to the dichotomy of classical logic: True vs. False, 0 vs 1, White vs Black, etc., allowing the treatment of uncertainty and vagueness, important not only from the theoretical point of view, but from the applications.

Lukasiewicz was the mentor of Alfred Tarski, whereas officially it was Stanislaw Lesniewski. His biographers, Anita and Solomon Feferman, state that, "along with his contemporary, Kurt Gödel, he changed the face of logic in the twentieth century, especially through his work on the concept of truth and the theory of models." Tarski had gone to the USA to participate in a conference when Nazi troops invaded his native Poland and could not return to it. Over time, he was in California most powerful logical school of his time; in fact, you can consider continuing the tradition inaugurated by the Lwow-Warsaw School, outside the continent in ruins (Europe). Its 'Semantic Theory of Truth' is one of the greatest achievements of the human thinking of all time.

But the writings of Jan Lukasiewicz suffered after a long slumber, which took care to leave an engineer Azeri, Lotfi Asker Zadeh, who had studied in Tehran, he prosecuted studies at MIT and eventually made landfall as a professor at the University of California, Berkeley. The one who would see their potential utility in 1965, firstly obtaining a generalized version of the classical theory of sets, now denoted by FST, acronym of the so-called "Fuzzy Set Theory", and later, its application to logic, creating the "Fuzzy Logic", particularly with the "fuzzy" proposition's modifiers and fuzzy rule-based systems, very useful for instance, on expert systems, such as the Mamdani, either the Takagi-Sugeno-Kang, or Yatsumoto's method.

Another interesting aspect that we must note is that even in his own age was not the only Zadeh proposing similar methods for the treatment of

uncertainty. So, we have the case of Max Black or Dieter Klaua that without getting resonance therefore proposed similar ideas. Also the same Bertrand Russell had treated such issue. We must not forget that Zadeh, an engineer, knew Jan Lukasiewicz investigations so explained his colleague and good friend, the brilliant American logician Stephen Cole Kleene.

But over time, was in Eastern countries where these ideas came to fruition, creating a powerful technological "boom", with new techniques based on "fuzzy" concepts. This trend was particularly strong in Japan, and then it spread to other countries close to the Japanese country, such as South Korea, China or India.

Because it is surely more in Japan where are developed to study the applications of Fuzzy Logic. Thus, in the case of Prof. Michio Sugeno, already in his Ph. D. thesis proposed the use of fuzzy integrals and fuzzy measures. Then there have been many followers of this movement to seek profit fuzzy logic and fuzzy measures. Such is also the case of the known Japanese Prof. Kaoru Hirota, among others.

And much later these ideas, even more application came to Western countries, both European and American, and today it will admit, with brilliant studies both from a mathematical point of view and its philosophical implications, as always connected therewith. Some emerging countries, such as Brazil or Turkey, are currently dumped in the investigation of all these theories and associated methods.

One of the most unique cases in the history of Artificial Intelligence is to Romania, a country which many consider poor and backward. It is not at all in regard to science. We have the most landmark in the mathematician Grigore C. Moisil, who introduced the computer at their country, and after he left a very brilliant school of researchers from Romania devoted to Computer Science.

After World War II, Grigore Moisil started teaching mathematical logic as he understood that the new emerging field of computers would have enormous repercussions for the social fabric of society. He continued on the ideas of Claude E. Shannon on Circuits, and fundamentally the Jan Lukasiewicz advances on Many-Valued Logics, where eventually derive the Fuzzy Logic.

The so-called Algebras Lukasiewicz-Moisil are an attempt to semantic consistency Logical n-valued Lukasiewicz. His study was followed by Gheorghe Georgescu and Aphrodite Iorgulescu, from Bucharest; Cristian Calude, Gheorghe Paun

(membrane computing), and many others, in different areas.

Also worthy of mention and full admiration the figure of Prof. Solomon Marcus, whose mentor was once said Prof. Moisil, and has made great contributions to various fields of mathematics, such as Mathematical Analysis, or Computational Linguistics, of which is one of the founders and principal contributors.

Of course there and increasingly in-out most of the new publications. But many of the best papers on Many-Valued Logic proceeds currently of good European Universities and very active research groups; for instance, in Warsaw, Prague, Ostrava, Vienna, Lisbon, Opole, Barcelona, Madrid, Toulouse, Pamplona, Granada, etc.

Other remarkable researchers multivalued logics (and in particular the Fuzzy Logic) have created a solid and consistent basis for these theories. Such has been the case for the Czech teachers Petr Hájek, Charles University, Prague, or Vilem Novak, University of Ostrava. Both have a powerful research groups and publications are among the most valued in this field.

And they are not alone, as for example, in France we have the important task of disclosure and investigation of Didier Dubois and Henri Prade.

Or in Germany, the cases of Hans-Jürgen Zimmerman or Siegfried Gottwald, disciple and follower of the work of Dieter Klaua.

In Poland they follow the great tradition of the Lwow-Warsaw School of logic, and this is complemented with contributions to research the uncertainty topic through the Rough Sets, introduced by Zdislaw Pawlak, and continued by Andrzej Skowron, among others.

### 3 RECEPTION OF MVLS (MANY-VALUED LOGICS) IN THE IBERIAN PENINSULA

There are certain groups, mostly centered around a “hub”, core or accumulation point, from which new ideas and impulses radiate growing, in the nucleus – or kernel- of each of these “core engine” is usually a -more or less-veteran researcher, well connected and with prestige.

One of the first Hispanic scholars giving notice of the new currents was Juan David Garcia Bacca, who in 1936 published his Introduction to modern logic, a work praised by I. M. Bochenski and Heinrich Schölz. Later try so eminent teachers,

between them Alfredo Deaño (editor by Spanish translation of Lukasiewicz’s selected papers), Miguel Sánchez-Mazas (studying and interpreting in deep sense the logico-mathematical works and ideas of Gottfried Wilhelm Leibniz, as –for instance- the known “characteristic universalis”), either Jesus Mosterin (historian and philosopher of science), or Manuel Sacristán (prosecuted in ‘academia’ due to its Marxist point of views), all them very often clashing against a very conservative and nothing good context to innovative ideas.

But one good initiative has been the creation in the old mining town of Mieres, and by the Government of Asturias, named the ‘Research Center for Artificial Intelligence and Soft Computing’, initially around someone well-known as Enric Trillas, which can be considered the father of the introduction of Fuzzy Logic in the Spanish University curricula. This center has attracted many of the most famous international researchers, such as well-known Japanese Professor Michio Sugeno. His topics of research are very broad working, but revolve around fuzzy methods, as well as philosophical implications these carry.

Although I have left for last, a name should not be omitted landmark, from those that appear only from time to time in Spain. We are referring to the Father Pablo Domínguez Prieto (1966-2009), Spanish philosopher and theologian, who wrote the first major book in Spain on the Lvov-Warsaw School, starting for that of his doctoral thesis in Philosophy, who had come to the Complutense University at Madrid (1993). Such work is called *Indeterminación y Verdad. La polivalencia lógica en la Escuela de Lvov-Varsovia (Indeterminacy and Truth)*, and was published in 1995, with a foreword by Arch. J. M. Zyzinski, and showing a very strong influence by Jan Wolenski. Pablo can be considered as one of the Spanish forerunner in the study of MVLS, from the philosophical point of view and in particular of the great Polish contribution (LWS) to logic and mathematical fields. A romantic ‘halo’ comes to close its brief existence, because his passion for the mountain climbing made him want to do in the snow Moncayo mountain, after giving lectures to the nuns of the monastery of Tulebras. And that was his last top headlong he died, leaving orphans these Spanish studies again. Left this short comment must be a tribute to his memory.

Another interesting Spanish author who has been reporting these new streams of logic is Prof. Julián Velarde, with its paper “Polyvalent Logic”, or his book *Formal Logic*, a volume II belonging to its History of Logic, all them around the University of

Oviedo and its service publications, or later, to the Editorial Pentalfa. Also of great interest may be his work *Gnoseology of Fuzzy Systems*, where analyses the deep philosophical connections of these issues.

New research groups have been formed in recent times, as the Spanish institution CSIC (Consejo Superior de Investigaciones Científicas), centered in Barcelona (through the 'Instituto de Inteligencia Artificial'; IIA, by acronym), led by Lluís Godo and Francesc Esteva.

Or the group that belongs to the UPNA (Public University of Navarra), headed by Humberto Bustince.

Either in the University of Granada (lead by Miguel Delgado Calvo-Flores). Also Francisco Herrera, Serafín Moral, Juan José Acero, etc.

Either can be considered the University of Zaragoza, with Prof. T. Calvo.

Even, found some valuable researchers in our own community, Madrid, as the Complutense, Autónoma, Polytechnic, Carlos III, etc., universities. But also in Málaga, Santiago de Compostela, Oviedo, Almería, Albacete, etc.

In Portugal the origins of the study of AI are linked to the names of Luis Moniz Pereira, Helder Coelho and Fernando Pereira, who in 1973 created the LNEC, within which the following year formed a division of Computer Science.

In 1977 the programming language called DEC-10 Prolog is designed, and Helder Coelho who contributes to divulge in Brazil.

In 1984 it is created the Portuguese Association for AI (ARIA, by acronym), which maintains its vitality with many publications and also organizing very interesting conferences, like this FCTA we attended.

## 4 CONCLUSIONS

The prospects offered by AI are immense; in particular the theory and applications of Fuzzy Logic in Computer. For example, research to try to model the human brain using supercomputers, hardware that will become increasingly smaller and cheaper, a mission that walk after Google and other large companies such as IBM. After this "brain mapping" ('cartografía del cerebro', in Spanish) of its operation, how to improve, how to age, etc., are the efforts of scientists and engineer working on programs such as the European Human Brain Project, led by Henry Markram, or American

BRAIN, in which Spanish researchers have participation.

As Ray Kurzweil say (RK, by acronym; the Google engineering director at San Francisco and very clever author of many futuristic well-known works, *"The Singularity is Near"*. Meaning 'Singularity' the point in history that computers equal and then surpass human intelligence.

## ACKNOWLEDGEMENTS

This work was supported by the MYCINN's Research Project and Group of the Spanish UNED (Universidad Nacional de Educación a Distancia), entitled *"Polemics and Controversies"* (El papel de las polémicas en la producción de las prácticas teóricas...). Being its former Principal Researcher Prof. Quintín Racionero, which recently had passed away, and from then it is directed by our new Principal Researcher, Prof. Cristina de Peretti, both from UNED.

## REFERENCES

- Ackermann, R., 1967, *An Introduction to Many-Valued Logics*. London, Routledge and Kegan Paul.
- Belnap, N.D., 1977, "How a computer should think", in G. Ryle (ed.), *Contemporary Aspects of Philosophy*, Stockfield: Oriol Press, pp. 30–56.
- Bochvar, D.A., 1938, "Ob odnom trechznacnom iscislenii i ego primenenii k analizu paradoksov klassiceskogo rassirennogo funkcional'nogo iscislenija", *Matematiceskij Sbornik*, 4 (46): 287–308. [English translation: Bochvar, D.A., "On a three-valued logical calculus and its application to the analysis of the paradoxes of the classical extended functional calculus", *History and Philosophy of Logic*, 2, pp. 87–112.]
- Bolc, L., and Borowik, P., 1992, *Many-Valued Logics 1: Theoretical Foundations*. Springer Verlag, Berlin, New York.
- Ibid., 2003, *Many-Valued Logics 2: Applications*. Springer Verlag, Berlin, New York.
- Chang, C.C., 1958, "Algebraic analysis of many valued logics", *Transactions American Mathematical Society*, 88, pp. 476–490.
- Ibid., 1959, "A new proof of the completeness of the Łukasiewicz axioms", *Transactions American Mathematical Society*, 93, pp. 74–80.
- Cignoli, R., Esteva, F., Godo, L. and Torrens, A., 2000, "Basic Fuzzy Logic is the logic of continuous t-norms and their residua", *Soft Computing*, 4, pp. 106–112.

- Cignoli, R., d'Ottaviano, I. and Mundici, D., 2000, *Algebraic Foundations of Many-Valued Reasoning*, Dordrecht, Kluwer Acad. Publ.
- Cintula, P. and Hájek, P., 2010, "Triangular norm based predicate fuzzy logics", *Fuzzy Sets and Systems*, 161 (3), pp. 311–346.
- Cintula, P., Hájek, P. and Noguera Ch. (eds.), 2011, *Handbook of Mathematical Fuzzy Logic* (Studies in Logic, Volumes 37–38), College Publications, London.
- Deaño, A., 2004, *Introducción a la Lógica Formal*. Alianza Universidad, Madrid.
- Domínguez Prieto, P., 1995, *Indeterminación y Verdad. La polivalencia lógica en la Escuela de Lvov-Varsovia*. Nossa y Jara Editores, Móstoles.
- Dubois, D., Prade, H., 1980, *Fuzzy sets and systems-Theory and applications*. Academic Press, New York.
- Ibid., 2000, *Fundamentals of Fuzzy Sets*. With a Foreword by Lofti Asker Zadeh. Kluwer Academic Publishers, Basel, Springer-Verlag, New York.
- Esteve, F. and Godo, L., 2001, "Monoidal t-norm based logic: towards a logic for left-continuous t-norms", *Fuzzy Sets and Systems*, 124, pp. 271–288.
- Ibid., and Montagna, F., 2004, "Equational characterization of the subvarieties of BL generated by t-norm algebras", *Studia Logica*, 76, pp. 161–200.
- Feferman, A. E. and S., Alfred Tarski. *Life and Logic*. Cambridge University Press.
- Fitting, M. and Orłowska, E. (eds.), 2003, *Beyond Two*, Heidelberg, Physica Verlag.
- Florio, C. de, *La forma della verità. Logica e filosofia nell'opera di Alfred Tarski*. Ed. Mimesis.
- García Bacca, J. D., 1936, *Introducción a la Lógica Moderna*. Ediciones Labor, Madrid.
- Garrido, A., 2006, "Special functions in Fuzzy Analysis". *Opuscula Mathematica*, vol. 26(3), pp. 457–464. AGH University of Science and Technology, Krakow.
- Ibid., 2011, "Searching the arcane origins of Fuzzy Logic". *BRAIN Research in Artificial Intelligence and Neuroscience*, vol. 2(2), pp. 51–57.
- Ibid., 2013, *Filosofía y Matemáticas de la Vaguedad y de la Incertidumbre*, PhD. Thesis in Mathematical Logic, qualified with Summa Cum Laude. Extraordinary Doctorate Award. Madrid, UNED, 2013.
- Ibid., 2014, *Lógicas de nuestro tiempo*. Madrid, Editorial Dykinson.
- Ibid., 2014, *Lógica Aplicada. Vaguedad e Incertidumbre*. Madrid, Editorial Dykinson.
- Ibid., 2015, *Lógica Matemática e Inteligencia Artificial*. Madrid, Editorial Dykinson.
- Giles, R., 1974, "A non-classical logic for physics", *Studia Logica*, 33, pp. 397–415.
- Ibid., 1975, "Łukasiewicz logic and fuzzy set theory". In: *Proceedings 1975 Internat. Symposium Multiple-Valued Logic* (Indiana Univ., Bloomington/IN), Long Beach/CA: IEEE Computer Soc., pp. 197–211.
- Ibid., 1976, "Łukasiewicz logic and fuzzy set theory". *Internat. Journal on Man-Machine Studies*, 8, pp. 313–327.
- Ibid., 1979, "A formal system for fuzzy reasoning". *Fuzzy Sets and Systems*, 2, pp. 233–257.
- Ibid., 1988, "The concept of grade of membership". *Fuzzy Sets and Systems*, 25, pp. 297–323.
- Gödel, K., 1932, "Zum intuitionistischen Aussagenkalkül", *Anzeiger Akademie der Wissenschaften Wien* (Math.-naturwiss. Klasse), 69: 65–66; – reprinted: (1933), *Ergebnisse eines mathematischen Kolloquiums*, 4, pp. 40 ss.
- Goguen, J. A., 1969, "The logic of inexact concepts". *Synthese* 19(3/4), pp. 325–373.
- Gottwald, S., 1999, "Many-valued logic and fuzzy set theory", in U. Höhle, S.E. Rodabaugh (eds.), *Mathematics of Fuzzy Sets: Logic, Topology, and Measure Theory* (The Handbooks of Fuzzy Sets Series), Boston, KluwerAcad. Press, pp. 5–89.
- Ibid., 2001, *A Treatise on Many-Valued Logics* (Studies in Logic and Computation, vol. 9), Baldock, Research Studies Press Ltd.
- Ibid., 2007, "Many-valued logics", in D. Jacquette (as ed.) *Philosophy of Logic* (Handbook of the Philosophy of Science Series), Amsterdam, North-Holland, pp. 675–722.
- Ibid., 2008, "Mathematical fuzzy logics", *Bulletin Symbolic Logic*, 14, pp. 210–239.
- Hähnle, R., 1993, *Automated Deduction in Multiple-Valued Logics*, Oxford : Clarendon Press.
- Ibid., 1999, "Tableaux for many-valued logics", in M. d'Agostino et al. (eds.) *Handbook of Tableau Methods*, Dordrecht, Kluwer, pp. 529–580.
- Ibid., 2001, "Advanced many-valued logics", in D. Gabbay, F. Guenther (eds.), *Handbook of Philosophical Logic* (Volume 2), 2nd ed., Dordrecht, Kluwer, pp. 297–395.
- Hájek, P. and Zach, R., 1994, "Review of Many-Valued Logics 1: Theoretical Foundations, by Leonard Bolc and Piotr Borowik", *Journal of Applied Non-Classical Logics*, 4 (2), pp. 215–220.
- Hájek, P., 1998, *Metamathematics of Fuzzy Logic*. Kluwer Academic Publishers, Boston-Dordrecht.
- Ibid., 2005, "Making fuzzy description logic more general", *Fuzzy Sets and Systems*, 154, pp. 1–15.
- Jaskowski, S., 1936, "Recherches sur le système de la logique intuitioniste", in *Actes du Congrès Internationale de Philosophie Scientifique 1936*, vol. 6, Paris, pp. 58–61. [English translation: *Studia Logica*, 34 (1975), pp. 117–120.]
- Kleene, S.C., 1938, "On notation for ordinal numbers", *Journal of Symbolic Logic*, 3, pp. 150–155.
- Ibid., 1952, *Introduction to Metamathematics*. Amsterdam, North-Holland Publ. Co., and New York, Van Nostrand, 1952.
- Kripke, S.A., 1975, "Outline of a theory of truth", *Journal of Philosophy*, 72, pp. 690–716.
- Łukasiewicz, J., 1963, *Elements of Mathematical Logic*. Państwowe Wydawnictwo Naukowe, Warsaw.
- Ibid., 1970, *Selected Works*. L. Borkowski, as editor. Amsterdam, North Holland C. Publ. Co.
- Ibid., 2013, *Écrits logiques et philo-sophiques*. Librairie Philosophique Jacques Vrin, Paris.

- Malinowski, G., 2007, "Many-valued Logic and Its Philosophies", in *Handbook of History of Logic*, ed. by D. M. Gabbay and J. Woods. Vol. 8, pp. 13-94. Elsevier-North Holland Publ. Co., Amsterdam.
- Marcus, S., 2006, "Grigore C. Moisil: A life becoming a myth", *IJCCC (International Journal of Computers, Communications & Control)*, vol. 1, no. 1, pp. 73-79.
- Ibid., 2015, *Order vs What?* Presentation to ICTAMI 2015, September 2015. "1 Decembrie 1918" Universitatea din Alba Iulia, Transilvania.
- Metcalfe, G., Olivetti, N. and Gabbay, D., 2009, *Proof Theory for Fuzzy Logics*, New York, Berlin, Springer Verlag.
- Mira, J., et al., 1995, *Aspectos Básicos de Inteligencia Artificial*. UNED.
- Moisil, G., 1965, *Încercări vechi și noi în logica neclasică*. Bucuresti.
- Ibid., 1968, *Elemente de logică matematică și teoria mulțimilor*, Bucuresti.
- Montagna, F. (ed.), 2015, *Petr Hájek on Mathematical Fuzzy Logic (Outstanding Contributions to Logic, vol. 6)*, Cham etc., Springer Verlag, Berlin-New York.
- Mosterín, J., 2007, *Los lógicos*. Austral. Editorial Espasa-Calpe, Madrid.
- Murawski, R., 2007, *The Philosophy of Mathematics and Logic in the 1920s and 1930s in Poland*. Birkhäuser Verlag, Basel, Switzerland.
- Ibid., 2010, *Essays in the Philosophy and History of Logic and Mathematics*. Poznan Studies in the Philosophy of the Sciences & the Humanities. Rodopi Verlag.
- Novák, V., Perfilieva, I. and Močkoř, J., 1999, *Mathematical Principles of Fuzzy Logic*, Boston, Kluwer Acad. Press.
- Novák, V., 2008, "A formal theory of intermediate quantifiers", *Fuzzy Sets and Systems*, 159, pp. 1229-1246.
- Patterson, D., 2008, *New Essays on Tarski and Philosophy*. Oxford University Press, OUP.
- Pawlak, Z., and Skowron, A., 2007, "Rudiments of Rough Sets". *Information Sciences*, Elsevier, 177, pp. 3-27.
- Post, E. L., 1920, "Determination of all closed systems of truth tables", *Bulletin American Mathematical Society*, 26, pp. 437 ss.
- Ibid., 1921, "Introduction to a general theory of elementary propositions", *American Journal Mathematics*, 43, pp. 163-185.
- Rasiowa, H. and Sikorski, R., 1963, *The Mathematics of Metamathematics*. Warszawa.
- Rescher, N., 1969, *Many-Valued Logic*, New York, McGraw Hill Publ. Co.
- Rosser, J.B. and Turquette, A.R., 1952, *Many-Valued Logics*, Amsterdam, North-Holland Publ. Co.
- Sacristán, M., 1990, *Introducción a la Lógica y al Análisis Formal*. Colección Circulo Universidad, Circulo de Lectores, Barcelona.
- Scarpellini, B., 1962, "Die Nichtaxiomatisierbarkeit des unendlichwertigen Prädikatenkalküls von Łukasiewicz", *Journal of Symbolic Logic*, 27, pp. 159-170.
- Skolem, Thoralf, 1957, *Bemerkungen zum Komprehensionsaxiom*, *Zeitschrift mathematische Logik Grundlagen Mathematik*, 3, pp. 1-17.
- Stegmüller, W., 2013, *Das Wahrheitsproblem und die Idee der Semantik: Eine Einführung in die Theorien von A. Tarski und R. Carnap*. Springer Verlag.
- Tarski, A., 2013, *Introduction to Logic: and to the Methodology of Deductive Sciences* (Dover Books on Mathematics).
- Ibid., 1977, *Einführung in die mathematische Logik* (Raabe, Samtliche Werke). Vandenhoeck und Ruprecht.
- Ibid., 1983, *Logic, Semantics, Metamathematics: Papers from 1923-38*. Hackett Publishing Co, Inc; Edición: 2nd Rev. ed. John Corcoran as editor.
- Ibid., with Mostowski, A., and Robinson, R. M., 2010, *Undecidable Theories: Studies in Logic and the Foundation of Mathematics* (Dover Books on Mathematics).
- Trillas, E., Alsina, C., and Terricabras, J. M., 1995, *Introducción a la Lógica Borrosa*. Ediciones Ariel, Barcelona, 1995.
- Wojcicki, R. and Malinowski, G. (eds.), 1977, *Selected Papers on Łukasiewicz Sentential Calculi*, Wrocław, Ossolineum Verlag.
- Wolenski, J., 1989, *Logic and Philosophy in the Lvov-Warsaw School*. Kluwer Academic Publishers, Dordrecht.
- Ibid., "Lvov-Warsaw School", entry at *The Stanford Encyclopedia of Philosophy*. Edward N. Zalta ed.
- Ibid., *Alfred Tarski and the Vienna Circle: Austro-Polish Connections in Logical Empiricism* (Vienna Circle Institute Yearbook). Springer Verlag.
- Ibid., 2013, *Historico-Philosophical Essays*, vol. I. Copernicus Cen-ter, Krakow.
- Wybraniec-Skardowska, U., 2009, *POLISH LOGIC. Some lines from a personal perspective*.
- Yager, R. R., 1987, *Fuzzy Sets and Applications: Selected Papers by L. A. Zadeh*. Yager et al. (eds.), John Wiley, New York.
- Zadeh, L. A., 1965, "Fuzzy Sets", *Information and Control*, 8, pp. 338-55.
- Ibid., 1975, "Fuzzy Logic and approximate reasoning". *Synthese*, 30, pp. 407-428.
- Ibid., 1987, *Fuzzy Sets and Applications. Selected Papers*. R. R. Yager et al. as editors. New York, Wiley.
- Zinovev, A.A., 1963, *Philosophical Problems of Many-Valued Logic*, Dordrecht: Reidel Verlag.