

Grey Prediction on Cage Dynamic Behavior of Cylindrical Roller Bearing

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Abstract: Dynamic behaviour of a cage in cylindrical roller bearing is a nonlinear kinetic and it is a key factor which influences applying performance of the bearing. Its displacements are forecasted by means of the grey dynamic model GM (1, 1). Residual test and posteriori error test are conducted to verify the reliability of the results of prediction. The experiment shows that the method proposed has the high precision and satisfy the engineering demand.

1 INTRODUCTION

Radial cylindrical roller bearings are designed to carry heavy radial loads and are suitable for high speed applications (Moore, R., Lopes, J, 1999). When cylindrical bearing operated, they generate vibrations and noise. The principle forces, which drive these vibrations, are time varying nonlinear contact forces, which exist between the various components of the bearings: raceways, rollers and cage (Smith, J., 1998).

The importance of energy efficiency has been increasing and has become a quality criterion for bearing producers and users in recent years. Hence, more and more researchers drew their attention on dynamic behaviours on the cage. Houpert developed simulation software to simulate cage behaviour and relative experimental validation was carried out (Houpert, L., 2010). Harsha analysed the nonlinear dynamics analysis of ball bearings due to cage run-out and number of balls (Harsha, S. P., 2006). The conclusion of his work showed that obtained FFT due to non-uniform spacing the ball passage frequency was modulated with the cage frequency. In some special applications, the data responding cage dynamic behaviour in future can prevent the disaster when the bearing is applied in key equipment's. In past years, many researchers applied the theory in predicting future data. For rolling bearing, the friction torque drew a lot of attentions by researches. For example, Xia et al. researched a dynamic prediction model for rolling bearing friction torque using the grey bootstrap fusion

method and chaos theory. Xia et al., forecasted rolling bearing friction torque by dynamical GM [1,1] model (Xia, X. T., Lv, T. M, 2012). In this paper, dynamic behavior of cage in cylindrical roller bearing is involved in the research as another performance parameter. The prediction values are compared with experiment values. The small deviations between them confirm the validity of the calculation model.

2 GREY PREDICTION MODEL

GM (1, 1) model of Grey System Theory is widely used in prediction realm. It is a time serious forecasting model, encompassing a group of differential equations adapted for parameter variance, rather than a first order differential equation.

The original data state sequence $X(0)$ can be given by

$$X(0) = (x(0)(1), x(0)(2), \dots, x(0)(i), \dots, x(0)(n)) \quad (1)$$

Where $x(0)(i)$ is the i th datum in $X(0)$ and n is the number of the data in $X(0)$.

The AGO information of $X(0)$ can be defined as

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(k), \dots, x^{(1)}(n)) \quad (2)$$

Where

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), k=1, 2, \dots, n \quad (3)$$

The GM (1, 1) model can be whiten by establishing

a first order differential equation $x^{(1)}(k)$ as

$$\frac{dx^{(1)}}{dt} + ax^{(1)}(t) = b \quad (4)$$

The solution to the whitening differential equation is

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a}, k=1,2,\dots,n-1 \quad (5)$$

In which

$$(a,b)^T = (B^T B)^{-1} B^T Y \quad (6)$$

with

$$Y = (x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n))^T \quad (7)$$

and

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \quad (8)$$

The reduction sequence can be given by

$$\hat{X}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \dots, \hat{x}^{(0)}(k), \dots, \hat{x}^{(0)}(n)) \quad (9)$$

where

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \quad (10)$$

3 TEST MODEL

A grey model needs to be tested to determine whether it is reasonable. Only through the model test it can be used to predict. Two test methods are proposed in this work.

3.1 Residual Test

The residual sequence is defined as

$$\mathcal{E} = \mathcal{E}(1), \mathcal{E}(2), \dots, \mathcal{E}(n) = (x^{(0)}(1) - \hat{x}^{(0)}(1), x^{(0)}(2) - \hat{x}^{(0)}(2), \dots, x^{(0)}(n) - \hat{x}^{(0)}(n)) \quad (11)$$

The relative error sequence is defined as

$$\Delta = (\Delta(1), \Delta(2), \dots, \Delta(n)) = \left(\left| \frac{\mathcal{E}(1)}{x^{(0)}(1)} \right|, \left| \frac{\mathcal{E}(2)}{x^{(0)}(2)} \right|, \dots, \left| \frac{\mathcal{E}(n)}{x^{(0)}(n)} \right| \right) \quad (12)$$

For $k \leq n$, the average relative error is given by

$$\bar{\Delta} = \frac{1}{n} \sum_{k=1}^n \Delta(k) \quad (13)$$

with

$$\Delta(k) = \left| \frac{\mathcal{E}(k)}{x^{(0)}(k)} \right| \quad (14)$$

The average precision is defined as

$$p^0 = (1 - \bar{\Delta}) \times 100\% \quad (15)$$

3.2 Posteriori Error Test

Posteriori error test is a statistical concept, which is in accordance with the probability distribution of the residuals, to evaluate the accuracy of a model. Posteriori error test can be divided in 4 steps as following.

The 1st step is calculating the mean \bar{x} and the variance S_1^2 of $X^{(0)}$

$$\bar{x} = \frac{1}{n} \sum_{k=1}^n x^{(0)}(k) \quad (16)$$

$$S_1^2 = \frac{1}{n} \sum_{k=1}^n (x^{(0)}(k) - \bar{x})^2 \quad (17)$$

The 2nd step is calculating the mean $\bar{\mathcal{E}}$ and the variance S_2^2 of \mathcal{E} :

$$\bar{\mathcal{E}} = \frac{1}{n} \sum_{k=1}^n \mathcal{E}(k) \quad (18)$$

$$S_2^2 = \frac{1}{n} \sum_{k=1}^n (\mathcal{E}^{(0)}(k) - \bar{\mathcal{E}})^2 \quad (19)$$

The 3rd step is calculating the posterior error ratio C and the small error probability P :

$$C = \frac{S_2}{S_1} \quad (20)$$

$$P = \left\{ \left| \mathcal{E}(k) - \bar{\mathcal{E}} \right| \leq 0.674 S_1 \right\} \quad (21)$$

The 4th step is determining the precision of the model according to the precision grade shown in Table 1.

Table 1: Precision scale.

Precision grade	<i>P</i>	<i>C</i>
Good	>0.95	<0.35
Qualified	>0.80	<0.5
Barely qualified	>0.70	<0.65
Unqualified	≤0.70	≥0.65

4 CASE STUDY

A cylindrical roller bearing typed as NU2310G1 is chosen as an example for case study. All the coordinates are expressed in the right-handed Cartesian coordinate system. While the outer ring is fixed in space, the inner ring rotates clockwise about the -X axis and the radial load is in the +Z direction. Gravity is in the -Z direction. Tab.2 shows the specifications of the test bearing and running conditions.

Measurement of cage behavior is carried out using two eddy-current displacement gauges for both Y and Z directions. The reason for using two gauges for the X direction is to check the cage for absence of conical oscillation.

Fig.1 to Fig.4 are the comparison of experiment data and simulation results. Fig.1 and Fig.2 are the comparison of data at +X and -X sides from Y direction, respectively. Similarly, Fig.3 and Fig.4 are the comparison of data at +X and -X sides from Z direction, respectively.

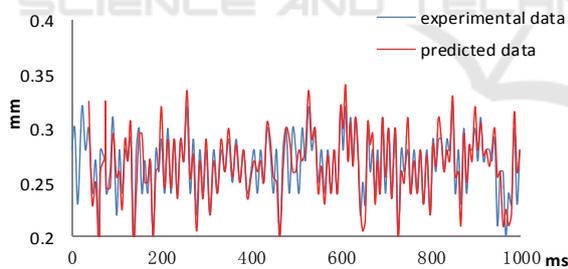


Figure 1: Comparison of experimental data and predicted data from Y direction at +X side.

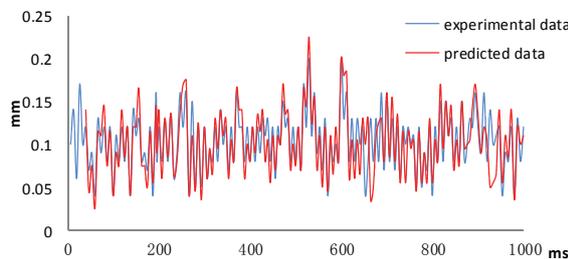


Figure 2: Comparison of experimental data and predicted data from Y direction at -X side.

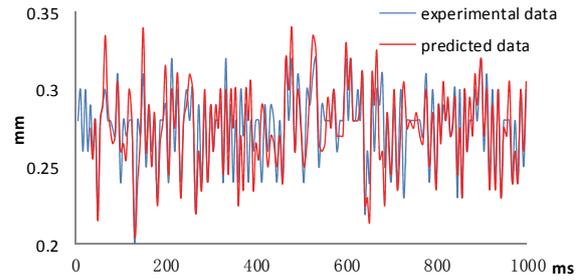


Figure 3: Comparison of experimental data and predicted data from Z direction at +X side.

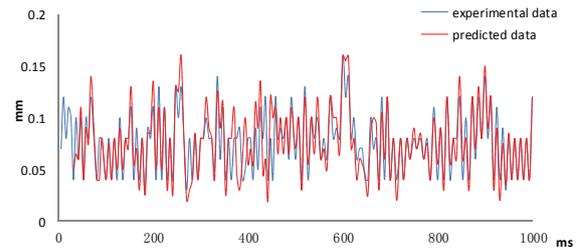


Figure 4: Comparison of experimental data and predicted data from Z direction at -X side.

The simulation results begin at the 6th datum, and the 6th datum is predicted by the 1st to the 5th datum of original data sequence. Similarly, the 7th datum is predicted by the 2nd to 6th datum of original data sequence, so on and so forth. As can be seen in the four figures, the variation tendency of the two curves is parallel, and the difference between data is relatively small.

Table 2: Test bearing and operating conditions.

Parameter /Unit	Value
Bearing Size /mm	Ø50×Ø 100×40
Number of rollers	12
Basic static load rating /N	131000
Cage type	Machined, Outer ring land riding
Radial internal clearance/ μm	40
Cage guide clearance, mm	0.445
Lubricant	VG56, Air-oil lubrication
Rotational speed/ RPM	3000
Radial load /N	4900
Temperature of outer ring at O.D. /°C	35 ± 3

Fig.5 and Fig.6 are relative error sequences of predicted data versus experimental data in Y and Z direction, respectively. According to the figures, most of the relative error are concentrated less than 20 percent. Obviously, the relative error sequence of Y direction is more concentrated than that of Z direction. The cause of the phenomenon is that the original data of Z direction are relatively smaller than that of Y direction.

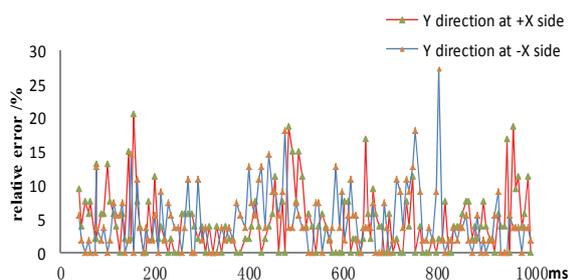


Figure 5: Relative error sequence of Y direction.

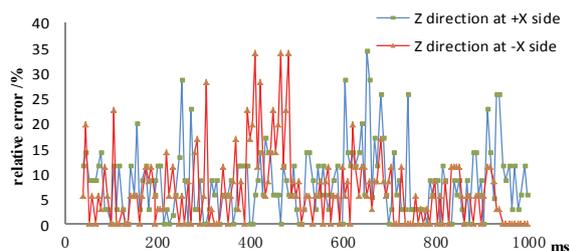


Figure 6: Relative error sequence of Z direction.

5 CONCLUSIONS

The posteriori error test results are listed as Tab.3. According to the results listed in the Tab, the variance ratio C of four original data serious are 0.52, 0.19, 0.48 and 0.44, receptively. Then the precision grade can be defined as qualified based on the precision scale listed in Tab.1. Factually, the small error probability P is set as 0.85 in prediction process. The posteriori error test results illustrate that the reliability of predicted data.

Table 3: Posteriori error test results.

	S_1^2	$\bar{\varepsilon}$	S_2^2	C
Y +X side	0.00103	0.000172	0.000275	0.49
-X side	0.00078	0.000316	0.000028	0.19
Z +X side	0.00149	0.001033	0.000353	0.48
-X side	0.00112	0.000126	0.000213	0.44

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