

Decentralized Gradient-based Field Motion Estimation with a Wireless Sensor Network

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Abstract: Information on the advection of a spatio-temporal field is an important input to forecasting or interpolation algorithms. Examples include algorithms for precipitation interpolation or forecasting or the prediction of the evolution of dynamic oceanographic features advected by ocean currents. In this paper, an algorithm for the decentralized estimation of motion of a spatio-temporal field by the nodes of a stationary and synchronized Wireless Sensor Network (WSN) is presented. The approach builds on the well-known gradient-based optical flow method, which is extended to the specifics of WSNs and spatio-temporal fields, such as spatial irregularity of the samples, the strong constraints on computation and communication and the assumed motion constancy over sampling periods. A specification of the algorithm and a thorough analytical analysis of its communicational and computational complexity is provided. The performance of the algorithm is illustrated by simulations of a sensor network and a spatio-temporal moving field.

1 INTRODUCTION

Information on the advection of dynamic features in the atmosphere or in the ocean is an important input to estimation algorithms. Examples include algorithms for precipitation field interpolation (Fitzner and Sester, 2015), nowcasting (Bowler et al., 2004) or the prediction of the evolution of oceanographic features advected by ocean currents. In this paper we investigate, how the advection of spatio-temporal dynamic features, modelled as spatio-temporal fields of a scalar attribute value, can be estimated decentralized by the nodes of a Wireless Sensor Network (WSN) deployed within and sensing the field.

For estimating the motion, a gradient-based optical flow algorithm is used as a basis and adjusted to the specifics of WSNs and spatio-temporal fields, i.e. the irregularity of samples, the strong constraints on communication and computation and the assumed motion constancy over sampling periods (see Section 4 for a more thorough elaboration on the specifics of WSNs and spatio-temporal fields). The main contributions of the work can be summarized as follows:

- *Approach for Estimation of Optical Flow Gradient Constraints in the Network.* We provide an approach for the estimation of the

required partial derivatives (i.e. the gradient constraint, see Section 3.2 on optical flow) of the field from irregular sensor data as well as a methodology for error estimation inspired by the calculation of error measures from satellite configurations for GPS positioning.

- *Incremental Field Motion Estimation.* A kalman filter based formalization of the optical flow equations is provided, in order to account for motion vector correlation over sampling steps and to allow efficient real-time processing in the network by each node.
- *Decentralized Algorithm Specification and Complexity Analysis.* A decentralized pseudo-code specification of the algorithm is provided, using the structure and formalisms provided by (Duckham, 2012), as well as a thorough analysis of its communication and computational complexity.

The proposed algorithm requires the specification of only three parameters with clear-cut interpretations: the maximum communication distance in between the sensor nodes, which is, in a real deployment of a WSN, determined by the physical hardware. Further parameters are the kalman filter prediction and measurement noise covariances.

The paper is structured as follows. In Section 2,

the relevant related work is introduced, mainly those concerned with the estimation of properties of spatio-temporal fields, such as peaks, pits or boundaries. In Section 3, the employed WSN and field model as well as the basics of image-based optical flow are provided. Section 4 contains the theory of the proposed approach and Section 5 the algorithm specification. In Section 6, the performance of the algorithm is illustrated by means of a simulated dynamic field and sensor network. Section 7 discusses possible extensions of the proposed method and concludes.

2 RELATED WORK

There exists a significant body of work on the estimation of properties of dynamic spatio-temporal fields with WSN. Problems include the estimation of field boundaries (Sester, 2009), identification of critical point such as peaks and pits (Jeong et al., 2014) or even spatial interpolation in the network (Umer et al., 2010). The book of (Duckham, 2012) provides a thorough overview on this topic as well as a description of the advantages of decentralized computation in the network, which also apply to the work presented in this paper.

Another research line related to our work is the tracking (here, in the meaning of *following*) of advected spatio-temporal features by mobile nodes (Brink and Pebesma, 2014), (Das et al., 2012). While in these works mobile nodes are assumed that can either move by themselves (Brink and Pebesma, 2014) or are advected with the field, e.g. buoys advected by ocean currents (Das et al., 2012), our work assumes a network of stationary sensors and aims at estimating the motion from the time series of sensor measurements.

Further, there is a significant amount of work on object tracking with WSNs, i.e. generating information on the trajectory of a mobile object without necessarily following it, such as (Tsai et al., 2007). However, to the best of our knowledge, the problem of estimating field (not object-) motion of a spatio-temporal field within a network of stationary sensor nodes has not been tackled so far.

3 BACKGROUND

3.1 Network and Field Model

Throughout the paper, a sensor network is modelled

as a graph $G = (V, E)$ where V is the set of sensor nodes distributed on the plane and E is the set of communication links between nodes. The allowed bidirectional communication links are solely determined by a maximum Euclidean communication distance r in the plane and hence, G is a unit disk graph (UDG). We assume that a node $n_i \in V$ knows its position $s_i = (x_i, y_i)$ on the plane, e.g. by GPS. Further, the nodes are able to sense a real-valued scalar spatio-temporal field $Z(u): \mathbb{R}^2 \times \mathbb{R} \rightarrow \mathbb{R}$ where $u = [x, y, t]^T$ is a location in the space-time cube (and $[]^T$ indicates matrix transposition). A particular sensor measurement of sensor n_i at timestep t is denoted with $z(u_{i,t})$ where $u_{i,t} = [x_i, y_i, t]^T$.

3.2 Gradient-based Motion Estimation

Optical flow methods such as (Lucas et al., 1981) or (Horn and Schunck, 1981) are usually employed for estimating pixel displacement (motion) in between two images and use image derivatives for estimation. They have successfully been applied for computing the motion of spatio-temporal fields, e.g. from weather radar images (see e.g. (Bowler et al., 2004) or (Fitzner and Sester, 2015)), mainly for nowcasting purposes. The underlying assumption of optical flow is that the intensity (pixel / field values) remains constant in between the sampling periods and a change in values for a particular location solely comes from field motion. Formally, this means that there exists a vector in the space-time cube $h = [\Delta x, \Delta y, \Delta t]^T$ such that Equation (1) holds.

$$Z(u) = Z(u + h) \quad (1)$$

Gradient-based optical flow methods are further based on the assumption that a first-order Taylor series expansion of the field values is adequate, as displayed in Equation (2).

$$Z(u + h) \cong Z(u) + Z'_x(u)\Delta x + Z'_y(u)\Delta y + Z'_t(u)\Delta t \quad (2)$$

where Z'_x, Z'_y and Z'_t are the partial derivatives in space and time directions resp. Equation (2) is called the linearity assumption of optical flow, as higher order terms are ignored. Combining (1) and (2) and dividing by Δt then results in the *gradient constraint* equation of (3).

$$Z'_x(u)\Delta x/\Delta t + Z'_y(u)\Delta y/\Delta t + Z'_t(u) \cong 0 \quad (3)$$

Instantiating the gradient constraint equation (3) requires estimates of Z'_x, Z'_y and Z'_t . Usually, they are estimated with numerical differentiation using neighboring (in space and time) pixel values. Estimating motion $v_x = \Delta x/\Delta t$ and $v_y = \Delta y/\Delta t$

then requires at least two gradient constraints to be integrated. More than two constraints are usually integrated over a pixel neighborhood by least squares adjustment (Lucas et al., 1981), (Fleet and Weiss, 2006).

4 MOTION ESTIMATION WITH A WSN

Estimating spatio-temporal field motion from irregular sensor data differs from image based optical flow, in that:

1. The data is irregular, i.e. the samples are not aligned along the coordinate axes and direct estimation of the partial derivatives, e.g. via forward-, backward-, or even central numerical differentiation, as e.g. proposed by (Horn and Schunck, 1981) is not possible. The methodology for estimating the required partial derivatives from irregular data is described in Sections 4.1 and 4.2.
2. For estimating field motion with a WSN, it is likely that the motion per sampling timestep is smaller than the spacing in between sensor nodes (this corresponds to subpixel motion in images). For example, considering two sensors with a sampling interval of 1-minute, a spacing of 1 km in between and a spatio-temporal field moving with a velocity of 60 km/h. In this case, the spacing is equal to the field motion per sampling time step. However, for the use cases imagined, we consider 60 km/h as being already rather fast and 1 km spacing as rather dense. For example, the average density of rain gauges in Germany is one station per 1800 km² (Haberlandt and Sester, 2010). Therefore, in the remainder of this paper, it is assumed that the motion is small compared to sensor spacing. For such cases of “subpixel” motion, a gradient-based method is adequate, as e.g. described by (Huang and Hsu, 1981). This assumption of small motion is directly implemented in the weighting scheme for the estimation of partial from directional derivatives, described in Section 4.4.
3. In contrast to image-based optical flow, it is likely that the motion is correlated over several sampling periods. For example, (Zawadzki, 1973) concludes that the Taylor hypothesis on frozen turbulence, which is closely related to the optical flow assumption of (1), is valid for precipitation for time periods shorter than 40 minutes. Therefore, it is likely that past field

motion can be used to improve the estimate of current field motion. A Kalman filter formalization of this assumption is presented in 4.5.

4. Finally, in contrast to motion in images, a-priori knowledge on the motion properties can be assumed. For example, wind speed statistics exist or information on the advection of rainclouds. In a real-world application of the algorithm, this domain knowledge can be used to specify the required parameters, mainly concerning the Kalman prediction and measurement noise covariances.

4.1 Estimating Directional Derivatives

When the data is irregularly sampled in space and time, estimating the partial derivatives directly is impossible, as there might be no samples aligned along the coordinate axes. Therefore, the partial derivatives need to be estimated from the directional derivatives in the space-time cube, defined as displayed in Equation (4).

$$Z_d'(u) = \lim_{|d| \rightarrow 0} \frac{Z(u+d) - Z(u)}{|d|} \quad (4)$$

where $d = [d_x, d_y, d_t]^T$ is a separation vector in the space-time cube, u is a spatio-temporal location and

$$|d| = \sqrt{d_x^2 + d_y^2 + d_t^2}.$$

For any pair of sensor samples $z(u_{i,t})$ and $z(u_{j,r})$ of sensors n_i and n_j and timesteps t and r (where $i = j$ is also allowed when $t \neq r$ and vice versa), an estimate of the directional derivative of Equation (4) can be calculated using forward differentiation along the particular direction, as displayed in equation (5).

$$\hat{z}_{j_i,rt}'(u_{i,t}) \approx \frac{z(u_{j,r}) - z(u_{i,t})}{|d_{j_i,rt}|} \quad (5)$$

where $d_{j_i,rt}$ is the spatio-temporal distance vector between the spatio-temporal locations $u_{i,t}$ and $u_{j,r}$ and $z(u_{j,r})$ and $z(u_{i,t})$ are field measurements at these locations.

4.2 Estimating the Gradient Constraint

Estimating the partial derivatives required for optical flow using a set of estimates of directional derivatives of the form of (5) requires some form of adjustment, e.g. least squares adjustment, to be performed by each sensor. The functional relationship between the estimate of a particular

directional derivative and the partial derivatives along the space-time cube coordinate axes is displayed in Equation (6). For easing readability, we skip the spatio-temporal position $u_{i,t}$, indicating the position of the particular sensor n_i at time t .

$$\hat{z}_{ij,rt'} = \hat{z}'_X \hat{u}_X + \hat{z}'_Y \hat{u}_Y + \hat{z}'_T \hat{u}_T \quad (6)$$

where $\hat{u}_{ji,rt} = (\hat{u}_X, \hat{u}_Y, \hat{u}_T)$ is the unit vector in the particular spatio-temporal direction $d_{ji,rt}$, $\hat{z}_{ij,rt'}$ is the directional derivative estimated with (5) and \hat{z}'_X , \hat{z}'_Y and \hat{z}'_T are the partial derivatives to be estimated. A set of such linear equations available at a specific spatio-temporal sensor position $u_{i,t}$ then allows calculating \hat{z}'_X , \hat{z}'_Y and \hat{z}'_T . In matrix form, the linear system to be solved is:

$$A\hat{x} = \hat{b} \quad (7)$$

where A is the $m \times 3$ matrix of unit vectors at $u_{i,t}$, $m \geq 3$ is the number of directional derivatives available and $\hat{x} = (\hat{z}'_X, \hat{z}'_Y, \hat{z}'_T)^T$ is the 3×1 vector of partial derivatives to be estimated. \hat{b} is the $m \times 1$ vector of known estimates of directional derivatives. An approximate solution to the equation is given by the least-squares estimator displayed in Equation (8).

$$\arg \min_x \|A\hat{x} - \hat{b}\| \quad (8)$$

where $\|\cdot\|$ is the l^2 - or Euclidean norm. The minimization is then performed by solving the normal equations for the vector of partial derivatives \hat{x} , as displayed in Equation (9).

$$\hat{x} = (A^T W A)^{-1} A^T W \hat{b} \quad (9)$$

The weight matrix W contains a weight for each estimate of a directional derivative. Certainly, the weight should be a function of field properties and the distance $|d_{ji,rt}|$. In Section 4.4., the methodology for the estimation of weights is described.

4.3 Requirements on Node Stationarity and Sampling Synchronicity

As the distance vector d is a vector in space and time, the calculation of the length $|d|$ requires the a-priori specification of a spatio-temporal anisotropy factor such as a decision on the unit of measures. If this knowledge is available a-priori, the sensors are allowed to move and sample the field asynchronously, i.e. at different time steps. However, in this work, it is assumed that the anisotropy factor is not known. Therefore, the assumptions of *node stationarity* and *time synchronicity* are essential. In this case, space can be

treated separated from time and there are always neighboring sensor samples in space only and time only available, while those in space-time can be ignored. In Figure 1, a visualization of this assumption is displayed.

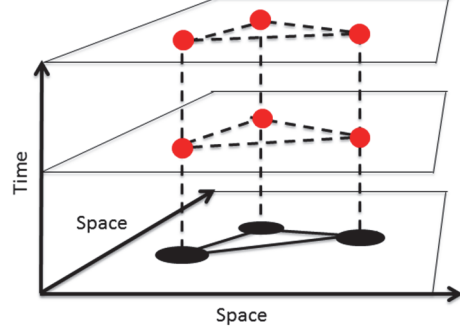


Figure 1: Assumption of node stationarity and sampling synchronicity. Red dots represent sensor measurements, which are taken at fixed spatial sensor positions at synchronized time steps / periods.

Another advantage is that the amount of computation within and communication between the nodes can be reduced, since the sensors need to communicate their position only once, as it (and hence matrix A as well) is constant. Constancy of W depends on the chosen weight. In Section 4.4, a weight is proposed that is solely derived from the spatial configuration. Therefore, each sensor can compute $M = (A^T W A)^{-1} A^T W$ once, reducing computational and communication costs per timestep. Further, the estimation of the temporal derivative does not have to be part of the linear system of (7). The temporal derivative can be estimated by each sensor using simple backward differencing along the time axis. Further, the temporal difference between sampling steps can be set to be equal to 1, i.e. $|d_{ii,t t-1}| = 1$. This way, the estimated temporal derivative is equal to the difference between current and previous value, i.e. $z(u_{i,t}) - z(u_{i,t-1})$ for a particular sensor n_i and no division is required.

4.4 Derivative Weighting and Gradient Constraint Error Estimation

From the assumptions described in 4.3 follows that the partial derivative along the time axis can be directly estimated. Therefore, no error for the estimated temporal derivative is assumed. For estimating the error (and therefore the weight) associated with the estimate of a particular directional derivative in space, the heuristic

displayed in Equation (10) is used:

$$\varepsilon_{ij} = \frac{|d_{ij}|}{r} \quad (10)$$

where r is the maximum possible communication range between sensors, $|d_{ij}|$ is spatial distance between the two sensors n_i and n_j . The weight matrix W then contains the inverse distance $1/\varepsilon_{ij}$ as the weight.

The individual error associated with a gradient constraint can then be estimated from individual direction derivative errors using the law of propagation of error (see e.g. (Langley, 1999) for an application to GPS or any textbook on adjustment theory adjustment for the derivation of the formula):

$$C_{\hat{x}} = (A^T W A)^{-1} \quad (11)$$

where $C_{\hat{x}}$ is the 2×2 covariance matrix of the 2×1 vector of estimated partial derivatives \hat{x} . The overall quality of a gradient constraint is then computed as the sum of the diagonal elements of $C_{\hat{x}}$, similar to the calculation of the positional error variance in GPS positioning (Langley, 1999).

$$\sigma_i^2 = \sigma_x^2 + \sigma_y^2 \quad (12)$$

This way, a gradient constraint error variance σ_i^2 reflects the spatial neighborhood configuration of the node n_i generating the constraint. Since field properties have been ignored in Equation (10), the value σ_i^2 is not a proper representation of a single gradient constraint error but only a proper representation of the error relationships between gradient constraints. Therefore, when the error is to be used in the motion estimation algorithm described in the next section, it has to be transformed to the proper level of error for the specific field under consideration. The approach used here is to scale a-priori known gradient constraint error variance by σ_i^2 . In the evaluations, it is shown that this, which we call configuration-based error, indeed provided improved motion estimates compared to a uniform weighting and uniform error.

4.5 Kalman Filter for Incremental Motion Estimation

Depending on sampling rate and field properties, the motion vector at a particular position might be considered constant or at least highly correlated in between sampling periods. A particular model that fits nicely to this problem of real-time incremental estimation of a real-valued vector is the kalman filter (Kalman, 1960). In the following, the theoretical

model of the kalman filter is described with a specific focus on the problem of motion estimation. The implementation then follows standard implementations as described e.g. in (Greg Welch, 2006).

The kalman filter assumes a hidden state in the form of a real-valued column-vector that can not directly be measured, in our case, the field motion at a particular time step k , displayed in (13).

$$v_k = [v_{X,k}, v_{Y,k}]^T \quad (13)$$

where $v_{X,k}$ and $v_{Y,k}$ are the motion vector components in direction X and Y , resp. (again, subscripts concerning sensor node are omitted as each node is assumed to implement the filter).

The error of the state is formalized in an error covariance 2×2 matrix P_k . The filter then assumes that a current state is a linear function of a past state plus some mean-zero Gaussian noise w with a certain covariance Q , i.e. $w \sim N(0, Q)$, as displayed in Equation (14).

$$v_k = F v_{k-1} + w \quad (14)$$

where F is the matrix that transforms past to current state and hence describes motion evolution at the particular sensor position. To keep the model and computations simple, motion constancy in between sampling periods is assumed and therefore, F is the identity matrix. In addition to motion evolution, the kalman measurement model describes, how noisy measurements are related to the state:

$$z_k = H_k v_k + u \quad (15)$$

where z_k is the measurement, H_k is the linear function that maps v_k to the measurement and $u \sim N(0, R)$ is zero-mean Gaussian noise with covariance R . In order to use (15) for updating the motion state with a new gradient constraint instance, the constraint of (3) has to be recast into the linear regression form (see Särkkä (2013) or Watson (1983) for the theory on kalman filtering for linear regression problems). Then, the temporal derivative is considered a linear function of the motion components:

$$Z'_T = -Z'_X v_{X,k} + -Z'_Y v_{Y,k} \quad (16)$$

Here, Z'_T is considered the measurement, i.e. $z_k = Z'_T$, $H_k = (-Z'_X, -Z'_Y)$ is the measurement matrix and $v_{X,k}$ and $v_{Y,k}$ is as above. This way, the motion state is updated with new gradient constraints. Since z_k is scalar, R is scalar as well and therefore, solving the kalman update equations does not require matrix inversion (Särkkä, 2013), which is advantageous for

the amount of processing required. Essential is the specification of an a-priori state, which can only be set to the zero vector. The a-priori uncertainty P_{init} should contain large values on the diagonal, indicating the low confidence in the initial state. The uncertainty associated with prediction Q , depends on the temporal sampling rate, i.e. the time difference between kalman filter steps, as well as the assumed motion constancy of the spatio-temporal field. The scalar measurement noise variance R is a direct function of gradient constraint accuracy. In this work, it is derived from the accuracies of the directional derivatives, as described in Section 4.4. Further, it depends on the spatial distance between the sensor estimating the motion and the sensor estimating the gradient constraint. The assumption is: the closer the two sensors are the more similar is the motion.

In addition, as described in work on image-based optical flow such as (Lucas et al., 1981), the gradient constraint accuracy depends on field properties at the site where the gradient constraint is constructed. Including these properties (spatial distance and field) into the approach is subject to future extensions of the algorithm, which are further discussed in Section 7.

5 ALGORITHM PROTOCOL

For specifying the algorithm, the protocol structure and pseudo-code elements of (Duckham, 2012) are used.

Protocol: Field Motion Estimation

Restrictions: Graph $G = (V, E)$ with nodes $n_i \in V$ with constant positions and comm. links E . Function $nbr: V \rightarrow V$ returning neighbours of a node. $nbr_{>1}: V \rightarrow V$ returning neighbours with >1 neighbours.

Initialization: All nodes in state INIT

Data: Each node $n_i \in V$ implements the kalman filter (Section 4.5) and stores matrix M and the errors associated with the gradient constraints provided by itself or neighbors.

Parameters: Max. communication range r , kalman parameters Q and R .

INIT

broadcast (x_i, y_i)

if $|nbr(n_i)| < 2$

become PROCESSING

 Receiving other node position (x_j, y_j)

if $|nbr(n_i)| > 1$

compute $|d_{ij}|$

compute unit vector \hat{u}_{ij}

 Add \hat{u}_{ij} to A

 Add $1/\varepsilon_{ij}$ to W

if $|rows\ of\ A| = |nbr_{>1}(n_i)|$

compute $M = (A^T W A)^{-1} A^T W$

compute $C_{\hat{x}} = (A^T W A)^{-1}$

broadcast σ_i^2 to $nbr_{>1}(n_i)$

become PROCESSING

PROCESSING

(1) *Whenever* new sample $z(u_{i,t})$ is available

broadcast $(t, z(u_{i,t}))$

if $|nbr(n_i)| > 1$

compute kalman prediction step

compute temporal derivative

$\hat{z}_T' = z(u_{i,t}) - z(u_{i,t-1})$

(2) *Receiving* sample $(t, z(u_{j,t}))$ from neighb. n_j

if $|nbr(n_i)| > 1$

compute \hat{z}_d'

 Add \hat{z}_d' to \hat{b}

if $|columns\ of\ \hat{b}| = |nbr_{>1}(n_i)|$

compute $\hat{x} = M \times \hat{b}$

compute kalman update with \hat{x}

broadcast \hat{x} to $nbr_{>1}(n_i)$

(3) *Receiving* gradient constraint (t, \hat{x})

if $|nbr(n_i)| > 1$

compute kalman update with \hat{x}

In the INIT step, a sensor distributes its position to all neighbors. If it has only zero or only a single neighbor, it is not able to estimate partial derivatives and hence, proceeds to state PROCESSING. Otherwise, it waits for receiving the positions of all neighbors. If it has done so, the matrix M can be computed, the error σ^2 associated with the node can be broadcast and the node is ready for motion estimation (i.e. state PROCESSING).

At each new sampling step (1) of a sensor, a kalman prediction on the motion is performed and the temporal derivative is calculated. Further, the sensor measurement is transmitted to the neighbors. When a new sample from a neighboring sensor arrives (2), the current directional derivative is estimated and added to the derivative vector \hat{b} . If \hat{b} is filled with all data from participating neighbors, the gradient constraint is computed, which is then used for kalman update and transmitted to all neighbors participating in motion estimation. When a gradient constraint from a neighboring sensor arrives (3), the kalman update step is executed as well.

5.1 Algorithm Analysis

For analyzing the complexity of the proposed algorithm, we follow the approach of (Duckham, 2012) and focus on the overall amount of communication required (Section 5.1.1). Further, the amount of computation performed by each node is analyzed in Sections (5.1.2) and (5.1.3). For doing this, the widely used notion of a *flop* is employed, representing a floating point operation such as addition, multiplication or division (see e.g. (Golub

and Loan, 1996) for an introduction to *flops* and their usage in analyzing matrix computations).

5.1.1 Communication Complexity

In terms of communication complexity, the algorithm requires an initial broadcast of the sensor positions and the error associated with the gradient constraints provided by a sensor. Therefore, $|V| + |V'|$ messages are sent at the `INIT` state, where V is the whole set of sensor nodes and $V' \subseteq V$ is the subset of sensor nodes with at least 2 neighboring nodes. Then, each timestep, each sensor sends the sensed value and each sensor with at least 2 neighboring nodes sends the estimated partial derivatives in addition, i.e. the *gradient constraint*. Therefore, the number of sent messages per timestep is $|V| + |V'|$.

5.1.2 Complexity of Derivative Estimation

Estimating the partial derivative along the temporal axis requires a single subtraction of values and no multiplications / divisions at each timestep. Estimating the partial derivatives in space from neighboring sensor measurements requires the computation of $M = (A^T W A)^{-1} A^T W$ once at the `INIT` state of the algorithm, plus multiplication with the vector of the estimates of directional spatial derivatives at each timestep.

Processing Costs at `INIT` Step:

Building the Matrices: The computation of a directional derivative requires computing the Euclidean distance to each neighboring node. This requires 2 subtractions, 2 multiplications and a square root. Building the unit vectors then requires another 2 divisions. In addition, building the weight matrix W requires m divisions. Hence, in total $m \times 6 + m$ *flop* plus a square root are required. For example, a sensor with 5 neighbors requires 35 *flop* and a square root computation for building the required matrices and vectors

Computing $M = (A^T W A)^{-1} A^T W$: A is a $m \times 2$ matrix, where m is the number of neighboring sensors. W is a $m \times m$ diagonal matrix of weights. Therefore, due to W being diagonal, matrix multiplication $A^T W$ requires $2 \times m$ multiplications and no additions and results in a $2 \times m$ matrix. Subsequent multiplication with A requires additional $2 \times 2 \times m$ multiplications and $2 \times 2 \times (m - 1)$ additions. Inverting the resulting 2×2 matrix $(A^T W A)$ requires 2 multiplications and one subtraction for the determinant plus 4 divisions.

Multiplication with the already computed $A^T W$ then requires $2 \times 2 \times m$ multiplications and $2 \times m$ additions and results in the $2 \times m$ matrix M to be available for partial derivative estimation at each timestep. Computing the gradient constraint error from the already computed $(A^T W A)^{-1}$ requires a single addition, as shown in Equation (12). For example, a sensor with 5 neighbors requires 84 *flop* for computing M and σ_i^2 .

Processing Costs at `PROCESSING` Step:

At each timestep, the resulting $2 \times m$ matrix M is to be multiplied with directional derivative $m \times 1$ vector \hat{b} , requiring $2 \times m$ multiplications and $2 \times (m - 1)$ additions. For example, a sensor with 5 neighbors requires 18 *flop* per timestep for estimating the required partial derivatives in space.

5.1.3 Complexity of Motion Estimation

The computations in the kalman prediction and update steps are independent from the number of neighbors m . However, while the prediction step is executed once per timestep, the update step is executed m times per timestep.

Prediction Step: The kalman prediction step requires a single matrix addition of the two 2×2 matrices Q and P . As Q is diagonal, this requires two additions or 2 *flop*.

Update Step: The following analyses rely on a standard kalman filter implementation with the matrices as described in Section 4.5. The main part is the computation of the kalman gain matrix. Computing $P_k^- H_k^T$ requires 4 multiplications and 2 additions. Subsequent multiplication with H_k requires 2 multiplications and a single addition and results in a scalar value to be added to measurement error variance R . The resulting value is inverted, requiring a single division. Multiplying with the already computed $P_k^- H_k^T$ requires 2 multiplications and results in the 2×1 kalman gain matrix K . In total, 13 *flop* are required for kalman gain computation. Updating with a new measurement requires 2 multiplications and one addition for $H_k \hat{v}_k^-$, a subtraction and the subsequent scalar multiplication with the kalman gain vector. The resulting vector is added to the state, requiring two additions. In total, 8 *flop* are required for updating the state with a new measurement. Updating the error covariance matrix requires 4 multiplications, 4 subtractions for $(I - K_k H_k)$ plus another 8 multiplications and 4 additions for the multiplication with the previous error covariance matrix P_k^- . In total, 20 *flop* are required for updating P_k^- to P_k^+ .

Therefore, the motion estimation update step requires 41 *flop* per neighbour per timestep.

6 EMPIRICAL EVALUATION

6.1 Evaluation Methodology

6.1.1 Simulation of Moving Field and Sensor Network

For evaluation, a set of 25 sensor nodes is distributed on the unit square, with a maximum communication range of $r = 0.25$. In addition, a gaussian field of the form of (17) is simulated on a larger square.

$$Z(x, y) = \sum_{i=1}^n e^{-\frac{(x-\mu_{i,x})^2+(y-\mu_{i,y})^2}{2\sigma_i^2}} \quad (17)$$

where n is the number of gaussians, $\mu_{i,x}$ is the x-coordinate, $\mu_{i,y}$ is the y-coordinate of the center of gaussian i and σ_i^2 is the variance. Due to the linearity assumption of optical flow of Equation (2) and (3), it is clear that the motion estimation works well, when the field is approximately linear at the sites where the gradient constraints are constructed. The degree of linearity of the simulated field can be controlled by the parameters n and σ_i^2 : the larger the number of gaussians and the smaller σ_i^2 , the less linear is the simulated field. Therefore, for evaluation, the field is computed from a large number of $n = 300$ gaussians using Equation (17), each with a variance randomly chosen from the small interval $[0, 0.01]$ and a center position $\mu_{i,x}, \mu_{i,y}$ randomly chosen on the larger square.

The simulations are executed over 50 timesteps and each of the 25 nodes samples the field at each timestep. Further, at each timestep, the whole field is displaced along a particular motion vector, i.e. the variances of the gaussians remain constant while the center coordinates $(\mu_{i,x}, \mu_{i,y})$ are shifted uniformly. In some of the experiments presented below, the motion vector is also manipulated at each timestep with a constant acceleration in both directions. Further, in some experiments, the motion is spatially inhomogeneous, which means each gaussian moves differently depending on its current location in the unit square. In Figure 2, a snapshot of the field, the sensor network with node *ids* and true motion vectors (per timestep) is displayed.

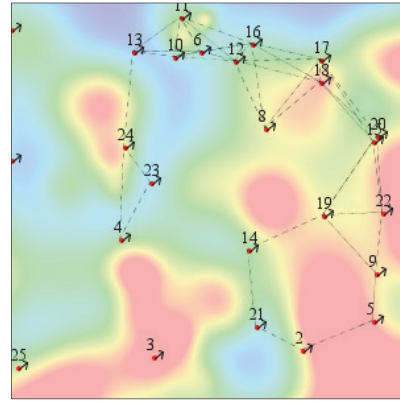


Figure 2: Snapshot of the simulated field, sensor network and true motion vectors at the beginning of a simulation run.

Figure 3 displays the situation 20 timesteps later, with a displaced field and the current (changed) true motion vector.

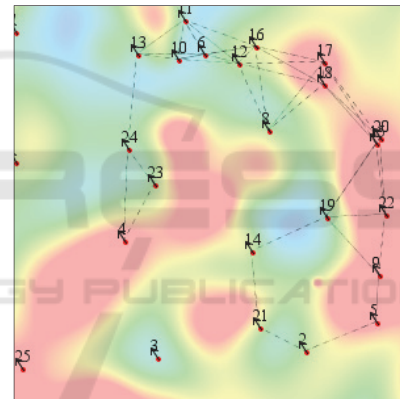


Figure 3: Snapshot of the simulated field, sensor network and true motion vectors, 20 timesteps after the snapshot of Figure 2.

6.1.2 Kalman Filter Initialization and Parameters

The filter for motion estimation implemented by each node is initialized with an initial motion state estimate of $\hat{v}_{init} = [0,0]^T$ and an initial error covariance matrix P_k containing arbitrarily chosen large values on the diagonal, indicating the low confidence in the initial motion state. The remaining parameters Q and R have been specified based on visual evaluations of the motion estimation results. It has turned out that the motion estimation performance increased with significantly lower assumed error for the prediction than for measurement. This can be explained by the rather low accuracy of the individual gradient constraints.

6.2 Results

In the following, some results of the generated motion information are provided in order to explain the performance of the proposed algorithm. In the first experiment of Section 6.2.1, constant motion in space and time is simulated. Then, in Section 6.2.2, a spatially varying motion field is generated, that is constant in time. In another experiment described in Section 6.2.3, the motion vectors are manipulated by a constant acceleration. Finally, in Section 6.2.4, it is shown that a configuration-based kalman measurement error indeed provides an improvement compared to a fixed error that is independent of the spatial configuration.

6.2.1 Constant Motion in Space and Time

In Figure 4, the difference between estimated (upper row) and true motion (lower row) is displayed for a particular experiment, some sensor nodes and constant motion.

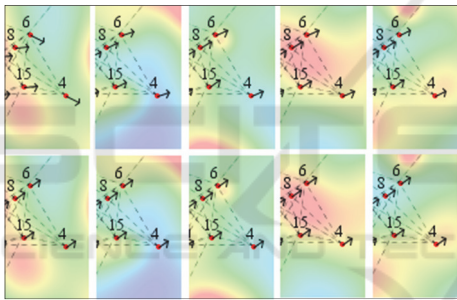


Figure 4: Constant Motion example. Estimated motion time series (upper row) vs. true motion time series (lower row). Temporal snapshots (columns) separated by 10 timesteps.

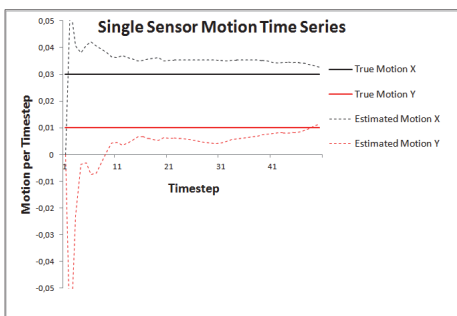


Figure 5: Constant Motion Example. Time series of motion estimates (kalman state vector, dotted lines) of a single sensor (sensor nr. 4 of Figure 4) vs. true motion (solid lines).

At the beginning (first column), the motion estimates are rather inaccurate. Over time, the

accuracy of the kalman motion estimates increases. Figure 5 displays the whole time series of motion estimates for a particular sensor node (node 4 of Figure 4).

The inaccurate estimates eventually converge to the true motion.

6.2.2 Spatially-varying Motion

As motion is estimated locally in the sensor neighborhood, it is clear that the approach is able to cope with locally spatially varying motion. In Figure 6, a snapshot of spatially inhomogeneous true motion vectors is shown, where the vectors differ with location.

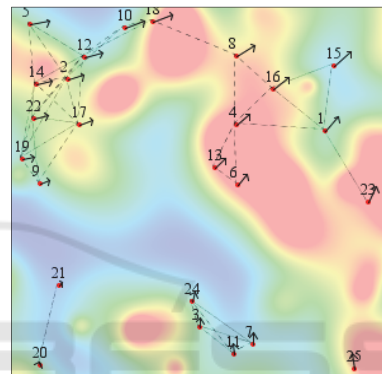


Figure 6: Snapshot of a spatially inhomogeneous motion field.

In Figure 7, the field of Figure 6 after 50 timesteps and estimated motion vectors are displayed. Due to locality of motion estimation in the sensor neighborhoods, the motion vectors adjust to the local motion, e.g. there is a difference between motion estimated in the upper left of the unit square and the motion estimated in the lower part or upper right part.

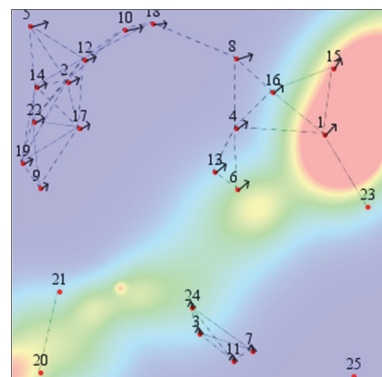


Figure 7: Snapshot of the experiment of Figure 6, 50 timesteps later and estimated motion vectors.

6.2.3 Dynamic Motion

In Figure 8, examples of an experiment with dynamic motion is presented. Again, the upper row shows estimated, the lower row true motion. The true motion is manipulated each timestep. Again, the motion estimates exhibit large errors at the beginning and are refined over time. However, as the kalman filter assumes constant motion, the motion changes are only slowly adjusted and always lag behind the true motion.

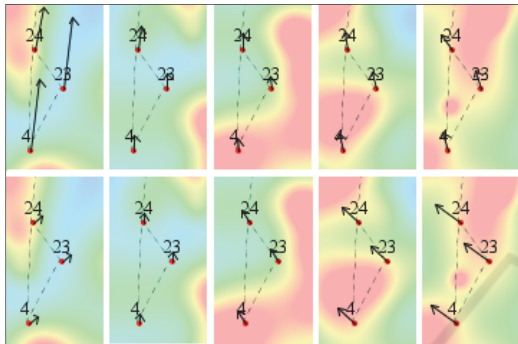


Figure 8: Dynamic Motion. Estimated motion times series (upper row) vs. true motion time series (lower row). Temporal snapshots (columns) separated by 10 timesteps.

This lag can also be recognized from the time series of motion estimates of a single sensor (Figure).

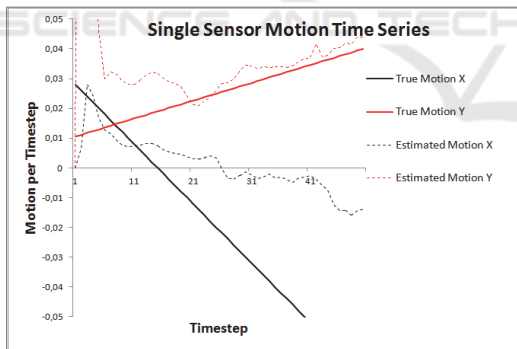


Figure 9: Dynamic Motion Example. Time series of motion estimates (i.e. kalman state vector, dotted lines) of a single sensor (sensor nr. 23 from Figure 8) vs. true motion (solid lines).

6.2.4 Fixed vs. Configuration-based Kalman Measurement Noise

In another experiment, the methodology for error estimation described in Section 4.4. is compared with an assumed fixed kalman measurement error variance, which is independent of the spatial configuration. The simulations show, that the

proposed methodology indeed provides improved motion estimates in most cases. Figure 10 shows an example of the time series of motion estimates of a single sensor for both methodologies. The performance of the configuration-based error estimation is, at most timesteps, improved (i.e. estimated motion is closer to the true motion).

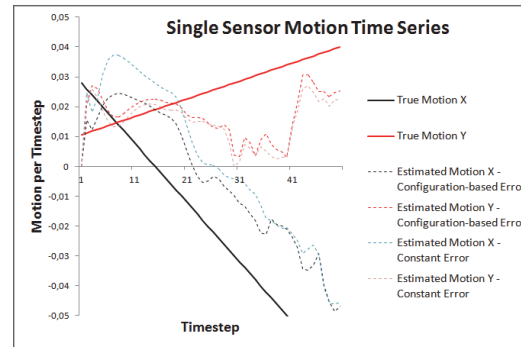


Figure 10: Comparison of the motion estimation performance for a fixed kalman measurement error variance vs. a configuration-based measurement error variance and dynamic motion.

In Figure 11, the average motion estimation error is displayed over all timesteps, sensor nodes and 10 different simulation runs. The error is calculated as the norm of the vector difference between true and estimated motion, normalized by the norm of true motion.

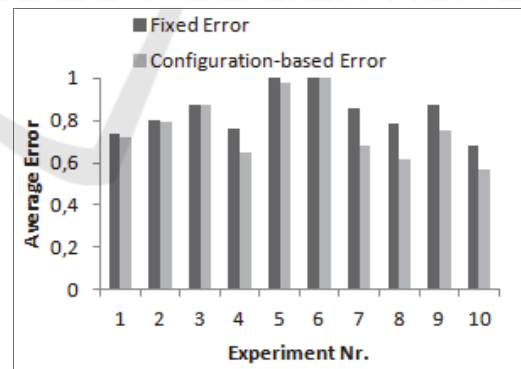


Figure 11: Average overall error for a fixed kalman measurement error variance (dark grey) and a configuration-based measurement error variance (light grey) for ten experiments.

The results show that the configuration-based error provides an improvement in almost all experiments. The magnitude of error depends on a plethora of factors, mainly field properties. Therefore, it is not further discussed here.

7 DISCUSSION & CONCLUSION

An approach for the decentralized estimation of the motion of a spatio-temporal field with a WSN has been presented. The performance of the algorithm has been illustrated by examples of a simulated dynamic field and sensor network. In the following, possible extensions of the proposed algorithm are discussed.

Sensor Network: Currently, stationary sensors and time-synchronized sampling is assumed, since this is considered the base case and eases the equations. However, moving sensors monitoring spatio-temporal fields, such as cars for measuring rainfall (Fitzner et al., 2013); (Haberlandt and Sester, 2010), exist and provide interesting possibilities for extension. Further, the approach uses single-hop communication and therefore assumes local translational motion within the sensor neighborhood. If motion is assumed to be constant over larger neighborhoods, motion estimation accuracy could be improved by multi-hop communication.

Accuracy of Gradient Constraint: Currently, the accuracy of a gradient constraint is solely determined by spatial configuration of the neighborhood generating the constraint. However, it is clear and already discussed in early work on optical flow such as (Lucas et al., 1981) that field properties such as the magnitude of the first or second derivative are indicators of the accuracy. Including these as well as weighting measures based on spatial distance is planned for future extensions.

Kalman Filter: The kalman filter proposed in this work comprises the motion vector only and therefore, constant motion over time is assumed. Possible motion changes are solely modeled by the prediction error variance, which is larger zero and hence, allows for state changes over time. A more realistic assumption is motion change constancy that could be implemented by adding motion change variables to the kalman state. Further, the kalman filter assumes white gaussian noise for both, prediction and measurement, a requirement that has to be tested in a real deployment of the algorithm. In addition, the values for the kalman filter noise parameters Q and R have been set rather arbitrarily based on a visual evaluation of the motion estimation results. In future work, methods for estimating these from the data will be investigated.

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