

# Classification of Dust Elements by Spatial Geometric Features

A. Proietti, M. Panella, E. D. Di Claudio, G. Jacovitti and G. Orlandi

*Department of Information Engineering, Electronics and Telecommunications,  
University of Rome "La Sapienza", Via Eudossiana 18, 00184 Rome, Italy*

**Keywords:** Dust Analysis, Classification, Feature Extraction, CMOS Sensor.

**Abstract:** Management of air quality is an important task in many human activities. It is carried out mainly by installing ventilation and filtering facilities. In order to ensure efficiency, these systems must be designed after the knowledge of key environmental parameters, such as size and type of particles and fibres present in the air. In this paper, we propose a new method for the classification of dust particles and fibres based on a minimal set of geometric features extracted from binary images of dust elements, captured by a very cheap imaging system. The proposed technique is discussed and tested. Experimental results obtained by real-measured data are presented, showing satisfactory performance by using several well-known classifiers.

## 1 INTRODUCTION

Maintenance of air quality in indoor environments is relevant for safety and health of people, but also for the protection of property and works. The local authorities and the international organizations have enacted specific regulations and instruments for controlling the indoor air quality and the levels of pollutants and harmful agents, e.g., the environmental and air quality laws by International Organization for Standardization (ISO, 2010).

Agents affecting the air quality level can be classified on the basis of three characteristics: origin, nature and effects. As regard for the origin, we can distinguish between indoor contaminants (arising from people, combustion, machining by-products and material degradation) and outdoor contaminants (particles collected through the windows, ventilation ducts, etc.). Based on the nature, we can distinguish among gases and vapours, organic fragments of plants, animal sources, micro-organisms, mineral particulate (dust, fumes, machining residuals) and liquids (fog and suspensions). From effects viewpoint, we can distinguish among pollutants causing physiological stimuli (such as smells), stress (such as headaches or fatigue) and diseases (such as irritation, allergies, mutations, or cancer).

In order to obtain the desired quality, a suitable artificial or natural ventilation can be provided to remove or dilute the contaminants by mixing and re-distributing air. To this purpose, it is necessary to

consider the type of pollutants to be removed, their size, shape and other physical properties, the air flow speed, etc. In the case when ventilation is insufficient, due to high level of pollutants, the air quality must be controlled by filtering to minimize the concentration of particles and micro-organisms. Pollutants are trapped by filters (i.e., porous partition) according to four basic mechanisms:

- sieve effect, occurring when the space between the filter fibres (or netting, wire mesh, etc.) is smaller than the diameter of the particles;
- direct interception, taking place when particles striking the filter surface are trapped;
- inertial impact, occurring when particles are blocked within the tortuous channels of the filter media;
- diffusion deposit, which occurs when the particles are so small that they spread by Brownian motion and remain in the filter.

The combination of these mechanisms establishes the overall efficiency of an air filter. It is evident that the correct design of the filtering equipment relies on a correct evaluation of the size and nature of the dust elements. In particular, it is important to distinguish between *particulate* and *fibres* to prevent and correct the pollution phenomenon.

Classification is a common task in many scientific fields (Rizzi et al., 2008; Panella and Martinelli, 2011; Maisto et al., 2013; Scardapane et al., 2015). Regarding particle shape recognition, a classifier based on

a mixture of Gaussian models (Panella et al., 2003; Parisi et al., 2007) was employed for automatic recognition of biological particles in microscopic images (Ranzato et al., 2007), starting from digital images of airborne pollen. An approach based on harmonic wavelet transform (Drolon et al., 2000) was applied for the analysis of wear and erosion phenomena acting on particles. An approach based on fuzzy kernel-based membership functions (Panella et al., 2001) was applied for shape classification (Proietti et al., 2016). Other traditional methods for monitoring the dust (Baron, 2001; Camuffo, 2013; Ghedini et al., 2011) rely on physical and chemical analyses (spectrometry, spectroscopy, etc.).

An original and cheap method for the imaging of dust samples was introduced in (Proietti et al., 2014), by using a high resolution optical CMOS sensor directly exposed to the air flow (i.e., without a lens). Using this device, dust elements adhering to the sensor and backlit by a LED source (see Figure 1) appear as *shadows*, that can be employed for effective dust analysis. The sensor is periodically cleaned before each acquisition cycle.



Figure 1: A typical dust deposition image collected by the CMOS sensor.

This basic technique enables to perform analysis of dust elements, according to their size and two fundamental typologies, herein referred to as *particles* and *fibres*, respectively characterized by a round shape or an elongated structure. In this paper the task is accomplished using just three geometric features.

The selected features reflect properties of backlit dust shadows. The size and the elongation of the particle are jointly quantified by a pair of features, i.e., the *perimeter length* and the perimeter/area ratio (*isoperimetric index*). The third feature is the *varimax norm*, a novel and powerful feature recently presented in (Di Claudio et al., 2015), which measures the regularity of the shadow contour. It appears determinant to obtain high discrimination power.

As proven by the experiments, the three selected features are largely independent and generate well-

separable clusters in the feature space, leading very robust classification performance when applied to state of the art trained classifiers.

The proposed approach differs from similar tasks reported in the literature (e.g., character recognition), since it does not perform template matching with pre-set shapes.

In Sect. 2 we describe the feature extraction strategy starting from the dust deposition image, as well as the resulting feature space for training a classifier. The setup of the classification experiment on the considered dataset is illustrated in Sect. 3. In Sect. 4 the numerical results obtained from the application of several well-known classifiers on the measured dataset are discussed, in order to assess the effectiveness of dust characterization and compare the performances of the selected approaches.

## 2 FEATURE EXTRACTION

The proposed method employs pattern recognition techniques based on a set of geometric features extracted from the images collected by the above mentioned CMOS sensor-based acquisition system (Proietti et al., 2014), as illustrated in Figure 2.

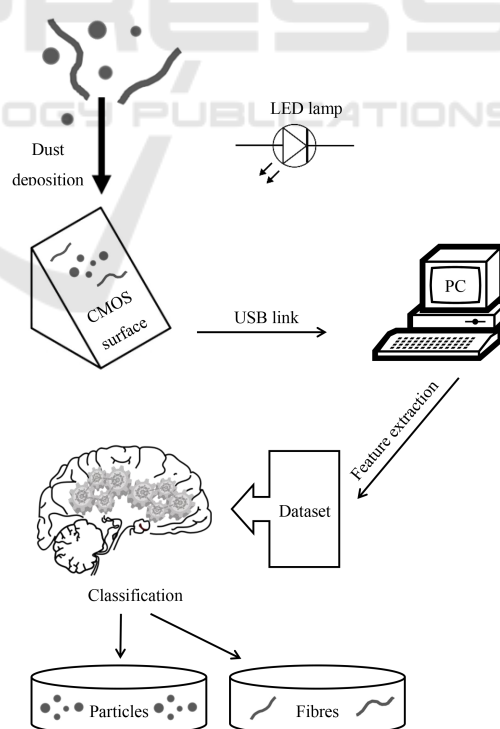


Figure 2: A schematic overview of the employed acquisition system based on CMOS imaging sensor.

Dust directly deposits on a naked imaging sen-

sor. The shadows projected by the illuminating LEDs (showed in Figure 3) are processed to yield binary images (see Figure 4), using auxiliary processing tools, such as background subtraction, equalization, filtering and thresholding (Proietti et al., 2014). Each element is extracted using a morphological analysis and subsequently classified on the basis of some well-defined features.



Figure 3: A collection of dust samples extracted from an acquired image.



Figure 4: The binary image resulting from the preprocessing and binarization of dust deposition in Figure 1.

First, *connected components* of the binary image are isolated (see Fig. 5). These components are sets of pixels of value 1 surrounded by pixels of value 0. The search for connected components is performed by a classical procedure (Gonzalez et al., 2004).

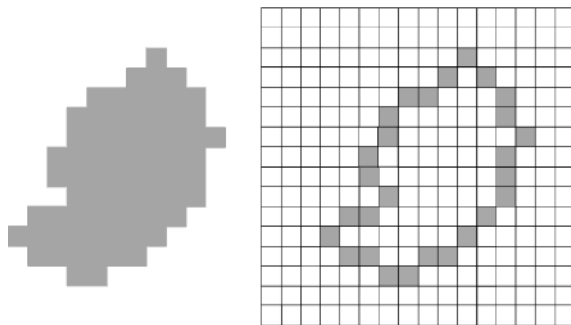


Figure 5: An example of a silhouette of a dust element and of its boundary.

In substance, the connected components are the *silhouettes* of each single dust element. Due to the image nature and to the short time between clean operations, the occurrence of the superposition of two silhouettes is a rare event, so that its effect on the global statistics is neglected.

Specifically, silhouette is separately classified based on the following three features.

1. *Length*  $b$  of the silhouette boundary. It is evaluated by a special version of the Moore neighbour tracing algorithm, adopting Jacob's stopping criteria (Reddy et al., 2012). This procedure returns a new binary image composed by only the boundary pixels  $q_i$ , wherein the inner ones are set to zero (see Fig. 5). Hence, the boundary length is calculated as:

$$b = \sum_{i=0}^{L-1} q_i. \quad (1)$$

where  $L$  is the total number of the one valued pixels in the connected component. Note that  $b$  is not a geometrical measure, since its value only approximates the length of the silhouette perimeter.

2. *Isoperimetric index*  $g$  defined as:

$$g = \frac{b^2}{4\pi L}. \quad (2)$$

The meaning of this index is related to the isoperimetric theorem (Bogomolny, 1987). It assumes the minimum value (equal to one) for a circular shape. In substance, it is an estimate of the silhouette *roundness*.

On the end,  $L$  approximates the effective area of the dust silhouette if multiplied by the area  $\sigma^2$  of each pixel, where  $\sigma$  is the pixel pitch of the used CMOS optical sensor. The  $\sigma$  parameter is critical for the minimum detectable particle size. In the present setting, the sensor is characterized by  $\sigma = 3 \mu m$ , allowing to barely detect the PM10 particulate.

Since the value of  $b$  does not exactly represent the silhouette perimeter, the sample value of  $g$  is also approximated.

3. *Varimax norm*  $v$  of the differences  $\delta_{\phi_l}$  between the orientation consecutive tangents of the silhouette boundary, estimated using a two-stage fast smoothing procedure as described in (Di Claudio et al., 2015). The varimax norm is then defined as follows:

$$V = b \frac{\sum_{l=1}^b [\delta_{\phi_l} - \mu]^4}{\left\{ \sum_{l=1}^b [\delta_{\phi_l} - \mu]^2 \right\}^2}, \quad (3)$$

where

$$\mu = \frac{1}{b} \sum_{l=1}^b \delta_{\varphi l}. \quad (4)$$

represents the average curvature of the particle. The varimax norm measures the *roughness* of the silhouette boundary and, in particular, assumes high values in the presence of abrupt changes of its orientation, typical of fibres. This original feature is determinant for the discrimination power of the whole procedure.

### 3 SETUP FOR CLASSIFICATION

In order to evaluate the effectiveness of the proposed features system, some classification tests were performed. Starting from a real dust images acquisition similarly to the example in Fig. 4, several dust particles and fibres using the procedure discussed in Sect. 2 were collected. As a result, 400 real dust elements (54 fibres and 346 particles, representing a realistic distribution of dust deposition elements), annotated with a classification by experts (i.e., particle or fibre), were produced in order to build a preliminary archive for training and testing classification procedure (this database will be available for testing other methods after request to the first author). Thanks to the wide variety of collected dust elements (in terms of size, shape, convexity/concavity, irregularity, etc.), the adopted dataset seems a valid basis for the assessment and the performance evaluation of classification techniques.

The values of the features used in this work are statically summarized in Table 1, where the minimum, maximum, mean and the standard deviation are reported.

In Figure 6, the distribution of the size of the particles, calculated as the diameter of the equivalent circle having the same area  $L$  of the silhouette, is displayed. Likewise, in Figure 7, the distribution of the size of the fibres, approximated by its half length ( $b/2$ ), is reported.

As far as particles/fibres classification is concerned, the discussed feature set was employed. In order to represent data within the unitary 3-D space, the whole dataset was normalized on each feature independently between 0 and 1 as follows:

$$x_m \leftarrow \frac{x_m - \alpha}{\omega - \alpha}, \quad m = 1 \dots M, \quad (5)$$

where  $x_m$  is the generic feature (i.e.,  $b$ ,  $g$  or  $v$ ),  $M$  is the number of patterns in the dataset and

$$\omega = \arg \max_{m=1 \dots M} \{x_m\} \quad , \quad \alpha = \arg \min_{m=1 \dots M} \{x_m\}, \quad (6)$$

are the maximum and minimum value that the features  $x_m$  can assume. This normalization is essential to balance the respective amplitudes at the classifier input to improve numerical conditioning.

As a result, the dataset consists of a matrix containing 400 rows and 3 columns (three features for each element), plus an additional column representing the class label annotated by experts (i.e., particle or fibre) and used as a benchmark. The classification was performed in the overall space  $(b, g, v)$  and in the subspaces generated by the three single features ( $b$ ), ( $g$ ) and ( $v$ ) and by the features pairs  $(b, g)$ ,  $(b, v)$  and  $(g, v)$ , resulting in seven different datasets.

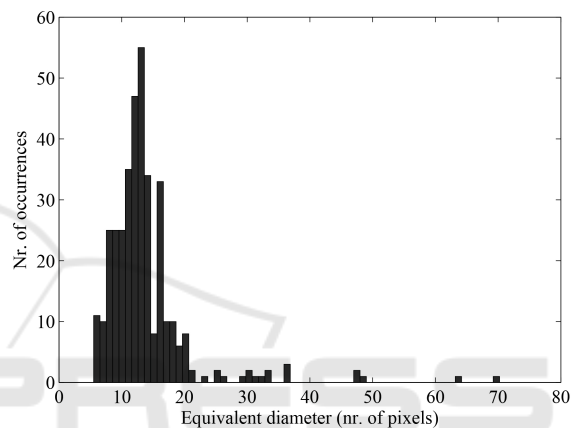


Figure 6: The particles distribution versus the diameter of the equivalent circle of area  $L$ .

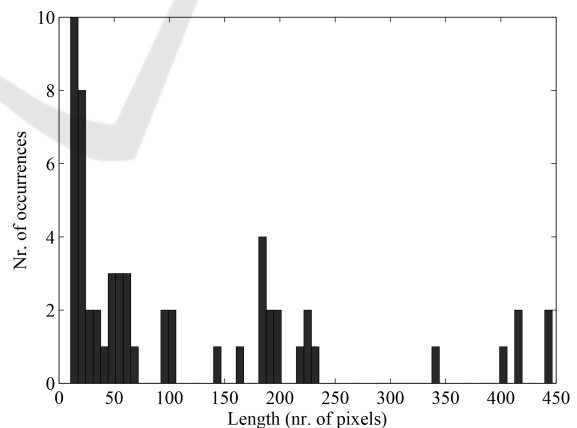


Figure 7: The fibres distribution versus the length (computed as the semi-boundary).

As a preliminary analysis, a singular value decomposition (SVD) was performed (Golub and Loan, 1989). Overall, the SVD analysis of the considered datasets demonstrated that the chosen features were nearly mutually uncorrelated over the dataset itself. Moreover, the use of SVD-rotated features in classifi-

Table 1: Numerical overview of the features extracted from the considered dust dataset.

Attribute	Feature		
	$b$	$g$	$v$
min	15.00	0.68	1.07
max	893.00	16.18	33.22
mean	74.23	1.63	2.51
std. dev.	129.14	2.57	3.34

cation tasks did not provide performance advantages over the raw feature set and therefore was not pursued further.

## 4 EXPERIMENTAL RESULTS

To test the classification capabilities of the proposed features, they were fed as inputs to the standard classifiers described in the following:

- the Linear Discriminant Analysis (Fisher, 1938), which tries to characterize a dataset using a linear polynomial in order to separate patterns in two or more classes;
- the Quadratic Discriminant Analysis (Krzanowski, 1988), similarly to the LDA, it tries to characterize a dataset using a quadratic polynomial;
- the Nearest Neighbour ( $k$ -NN) approach (Cover and Hart, 1967), which assigns a class based on the most frequent class among the pattern neighbourhood;
- the Naive Bayes classifier (Langley et al., 1992), which is a classification algorithm based on the Bayes' theorem, supposing a strong independence among features;
- the Classification And Regression Tree (CART) classifier (Lawrence and Wright, 2001), which operates by recursively splitting the data until ending points are achieved using some preset criteria;
- the Probabilistic Neural Network (PNN) classifier (Specht, 1990), based on a four layers neural network employing Bayesian decision-making theory;
- the Fuzzy Inference Systems (FIS) classifier (Takagi and Sugeno, 1985), which relies on the use of fuzzy logic and fuzzy sets theory to achieve classification tasks (Liparulo et al., 2013).

The tests were performed using the above mentioned datasets and a stratified ten-fold cross-validation (Purushotham and Tripathy, 2012), using a uniform patterns split into groups of the same size.

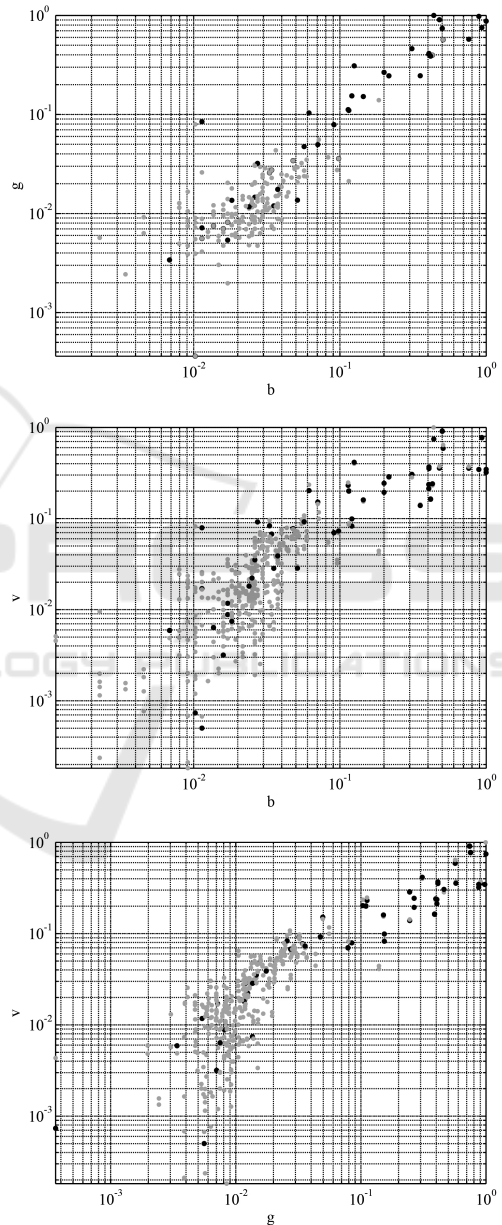


Figure 8: The 2-D projections of the classified ( $b, g, v$ ) dust dataset using the Naive Bayes classifier: blacks dots are patterns classified as fibres; grey dots are patterns classified as particles.



Table 2: Average performance over 50 trials for every classifier applied to the proposed set/subset of features.

Classifier	Error rate (%)						
	$(b, g, v)$	$b$	$g$	$v$	$(b, g)$	$(b, v)$	$(g, v)$
LDA	5.2	5.2	5.2	4.8	5.2	5.2	4.8
QDA	3.3	3.3	2.9	4.8	3.8	3.3	3.8
$k$ -NN	2.4	2.4	4.5	4.3	2.4	2.4	2.9
Naive Bayes	<b>1.8</b>	3.3	2.1	3.3	2.9	2.1	2.1
CART	3.3	3.8	2.4	4.7	2.9	3.3	2.9
PNN	5.2	6.7	5.2	8.5	5.2	5.2	5.2
FIS	4.3	16.2	3.8	9.5	4.8	4.3	3.3

Table 3: Confusion matrix resulting from the Naive Bayes classifier using the  $(b, g, v)$  dataset. F is the fibres class, P is the particles class.

True class	Estimated class		Intra-class error (%)
	F	P	
F	52	2	3.7
P	5	341	1.5

The numerical parameters to be set in advance for the used classifiers have been determined using an inner ten-fold cross-validation on each training subset: the value of  $k$  in  $k$ -NN varied in the range (2, 10), using Euclidean distance. The value of the spread in PNN varied in the interval (0.05, 1) with a step of 0.05; the number of rules within the FIS classifier varied in the range (1, 10). The following choices were made within the used classifiers: for  $k$ -NN we set  $k = 3$ . For the Naive Bayes classifier, the normal distribution with diagonal covariance was adopted as prior. For the PNN the spread of the radial basis functions was set to 0.1. The FIS classifier consisted of five Mamdani-type fuzzy rules (Mamdani and Assilian, 1975).

The results in terms of misclassification errors for each classifier and for every dataset, among all the possible combinations of the proposed features, are reported in Table 2. Since the training of some classifiers depends upon a random initialization of model parameters, the averages over 50 different trials of the above ten-fold cross-validation were performed.

The proposed approach for dust characterization achieves valuable performances in terms of error rate (i.e., below 5% of error) for most classification models and independently of the combination of features in the dataset. A remarkable exception is constituted by the FIS classifier if the single features  $b$  or  $v$  or the pair  $(b, v)$  is involved.

In particular, the best performance was obtained by the Naive Bayes classifier using the overall features space  $(b, g, v)$ , with an error rate smaller than 2%. The high performance of this classifier was not unexpected, considering the low correlation among the employed features. The confusion matrix related to the results obtained through the Naive Bayes classifier is reported in Table 3. Two fibres are misclassified

as particles, with an intra-class error of 3.7% within the total 54 samples of fibres. Five particles are misclassified as fibre, with an intra-class error of 1.5% within the total 346 samples of particles.

A 3-D plot of the classification results is shown in Figure 9. The principal directions resulting from the previously discussed SVD analysis were also reported. In Figure 8, all the 2-D projections of the classified  $(b, g, v)$  dust dataset onto the three coordinate planes are shown. It is interesting to note that the two considered classes are well separable, even if it is impossible to employ for this purpose a simple spherical or ellipsoidal surface.

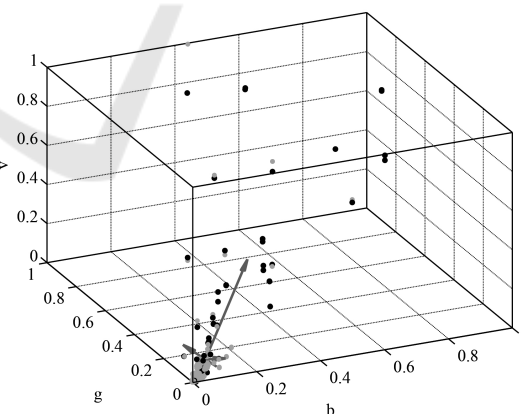


Figure 9: A 3-D plot in the normalized  $(b, g, v)$  feature space of the classified dust dataset using the Naive Bayes classifier. Black dots are patterns classified as fibres and grey dots are patterns classified as particles.

The approach proposed therein was compared to other techniques: the early standard approach based on grey-valued image analysis (Proietti et al., 2014) and the approach based only on contour analysis (Di Claudio et al., 2015). The results are reported in

Table 4: Comparison of the proposed approach (geometric features) with respect to other techniques: an early standard approach based on grey-valued images analysis (Proietti et al., 2014), the approach based only on contour analysis (Di Claudio et al., 2015) and an approach based on binary template matching (Proietti et al., 2015).

Algorithms	Error rate
Geometric features	2%
Grey-valued images	15%
Contour analysis	4%
Template matching	9%

Table 4. Observe that the performance of the present approach exceeds the performance of the said ones by a sizeable margin.

As far as the computational costs are concerned, the time required by all these algorithms for performing the classification task is comparable. They are less than 100 $\mu$ s, referring to a x64 Intel(R) Core(TM) i7-2600K CPU running at 3.40 GHz with 8 GB, 1333 MHz RAM.

Moreover, let us mention that an approach based on binary template matching (Proietti et al., 2015) was also tried in previous works (Proietti et al., 2015). However, this technique yielded a misclassification error greater than 9% with a much higher computational time.

## 5 CONCLUSIONS

A new approach for the classification of dust on the basis of their size and typology (particles and fibres) based on geometric features extracted from binary images was presented. The approach represents an effective choice in terms of speed and accuracy and requires a very simple acquisition device.

Since the involved algorithms are essentially multiplication-free, the global classification technique is very fast and energy saving. Hence, it is ideal in distributed sensor networks and especially in wireless scenarios, where the processing power consumption is a major problem, since the classification task competes with the energy spent for communication among sensing devices and image acquisition.

The aim of this paper is to propose a novel approach to features selection for fibres classification. In the paper, encouraging results on real dataset by using well-known classification models have been presented. A deep assessment of more complex scenarios, with different datasets and different classifiers, will be considered in future contributions. Also, future works will entail the use of enhanced optics over the CMOS sensors to capture even smaller particles and more detailed classification.

## REFERENCES

- Baron, P. (2001). Measurement of airborne fibers: A review. *Industrial Health*, 39(2):39–50.
- Bogomolny, A. (1987). On the perimeter and area of fuzzy sets. *Fuzzy Sets and Systems*, 23(2):257 – 269.
- Camuffo, D. (2013). *Microclimate for Cultural Heritage: Conservation, Restoration, and Maintenance of Indoor and Outdoor Monuments*. Elsevier Science, Boston.
- Cover, T. and Hart, P. (1967). Nearest neighbor pattern classification. *Information Theory, IEEE Transactions on*, 13(1):21–27.
- Di Claudio, E., Jacovitti, G., Orlandi, G., and Proietti, A. (2015). Fast classification of dust particles from shadows. In *ICPRAM 2015 - 4th International Conference on Pattern Recognition Applications and Methods, Proceedings*, volume 2, pages 241–247.
- Drolon, H., Druaux, F., and Faure, A. (2000). Particles shape analysis and classification using the wavelet transform. *Pattern Recognition Letters*, 21(67):473 – 482.
- Fisher, R. A. (1938). The statistical utilization of multiple measurements. *Annals of Eugenics*, 8(4):376–386.
- Ghedini, N., Ozga, I., Bonazza, A., Dilillo, M., Cachier, H., and Sabbioni, C. (2011). Atmospheric aerosol monitoring as a strategy for the preventive conservation of urban monumental heritage: The florence baptistry. *Atmospheric Environment*, 45(33):5979 – 5987.
- Golub, G. and Loan, C. V. (1989). *Matrix Computations*. John Hopkins University Press, Baltimore, USA, 2nd edition.
- Gonzalez, R. C., Woods, R. E., and Eddins, S. L. (2004). Digital image processing using matlab. *Upper Saddle River, N. J: Pearson Prentice Hall*.
- ISO (2010). Cleanrooms and associated controlled environments - part 1: Classification of air cleanliness by particle concentration. *ISO/DIS 14644-1*.
- Krzanowski, W. J., editor (1988). *Principles of Multivariate Analysis: A User's Perspective*. Oxford University Press, Inc., New York, NY, USA.
- Langley, P., Iba, and, W., and Thompson, K. (1992). An analysis of bayesian classifiers. In *Proceedings of the Tenth National Conference on Artificial Intelligence, AAAI'92*, pages 223–228. AAAI Press.
- Lawrence, R. L. and Wright, A. (2001). Rule-based classification systems using classification and regression tree (cart) analysis. *Photogrammetric engineering and remote sensing*, 67(10):1137–1142.

- Liparulo, L., Proietti, A., and Panella, M. (2013). Fuzzy membership functions based on point-to-polygon distance evaluation. In *Fuzzy Systems (FUZZ), 2013 IEEE International Conference on*, pages 1–8. IEEE.
- Maisto, M., Panella, M., Liparulo, L., and Proietti, A. (2013). An Accurate Algorithm for the Identification of Fingertips Using an RGB-D Camera. *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, 3(2):272–283.
- Mamdani, E. and Assilian, S. (1975). An experiment in linguistic synthesis with a fuzzy logic controller. *International Journal of Man-Machine Studies*, 7(1):1–13.
- Panella, M. and Martinelli, G. (2011). Neural networks with quantum architecture and quantum learning. *International Journal of Circuit Theory and Applications*, 39(1):61–77.
- Panella, M., Rizzi, A., and Martinelli, G. (2003). Refining accuracy of environmental data prediction by MoG neural networks. *Neurocomputing*, 55(3-4):521–549.
- Panella, M., Rizzi, A., Mascioli, F. F., and Martinelli, G. (2001). ANFIS synthesis by hyperplane clustering. In *Proceedings of Joint IFSA World Congress and NAFIPS International Conference (IFSA/NAFIPS 2001)*, volume 1, pages 340–345. IEEE.
- Parisi, R., Cirillo, A., Panella, M., and Uncini, A. (2007). Source localization in reverberant environments by consistent peak selection. In *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP 2007)*, volume I, pages I–37–I–40. IEEE.
- Proietti, A., Leccese, F., Caciotta, M., Morresi, F., Santamaria, U., and Malomo, C. (2014). A new dusts sensor for cultural heritage applications based on image processing. *Sensors (Switzerland)*, 14(6):9813–9832.
- Proietti, A., Liparulo, L., Leccese, F., and Panella, M. (2016). Shapes classification of dust deposition using fuzzy kernel-based approaches. *Measurement*, 77:344–350.
- Proietti, A., Panella, M., Leccese, F., and Svezia, E. (2015). Dust detection and analysis in museum environment based on pattern recognition. *Measurement*, 66(0):62–72.
- Purushotham, S. and Tripathy, B. (2012). Evaluation of classifier models using stratified tenfold cross validation techniques. *Communications in Computer and Information Science*, 270 CCIS(PART II):680–690.
- Ranzato, M., Taylor, P., House, J., Flagan, R., LeCun, Y., and Perona, P. (2007). Automatic recognition of biological particles in microscopic images. *Pattern Recognition Letters*, 28(1):31–39.
- Reddy, P. R., Amarnadh, V., and Bhaskar, M. (2012). Evaluation of stopping criterion in contour tracing algorithms. *International Journal of Computer Science and Information Technologies (IJCSIT)*, 3:3888–3894.
- Rizzi, A., Buccino, M., Panella, M., and Uncini, A. (2008). Genre classification of compressed audio data. In *Proceedings of IEEE MMSP 2008*, pages 654–659. IEEE.
- Scardapane, S., Wang, D., Panella, M., and Uncini, A. (2015). Distributed learning for random vector functional-link networks. *Information Sciences*, 301:271–284.
- Specht, D. F. (1990). Probabilistic neural networks. *Neural Networks*, 3(1):109–118.
- Takagi, T. and Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man and Cybernetics*, 15(1):116–132.