Structural Synthesis based on PCA: Methodology and Evaluation

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Regularization.

Abstract: In recent surface inspections systems, interactive training of fault classification is becoming state of the art.

While being most informative for both training and explanation, fault samples at the decision boundary are rare in production datasets. Therefore, augmenting the dataset with synthesized samples at the decision boundary could greatly accelerate the training procedure. Traditionally, synthesis methods had proven to be useful for computer graphics applications and have only been applied for generating samples with stochastic and regular texture patterns. Presently, the state of the art synthesis methods assume that the test sample is available and are feature independent. In the context of surface inspection systems, incoming samples are often classified to several defect classes after the feature extraction stage. Therefore, the goal in this research work is to perform the synthesis for a new feature vector such that the resulting synthesized image visualizes the decision boundary. This paper presents a methodology for structural synthesis based on principal components analysis. The methodology expects the samples of the training set as an input. It renders the synthesized form of the input samples of training set through eigenimages and its computed coefficients by solving a linear regression problem. The methodology has been evaluated on an industrial dataset to validate its performance.

1 INTRODUCTION

Synthesis is defined as the process of generating a sample which is close to the visual appearance of the input sample. In the context of surface inspection systems, synthesis plays a vital role in rendering fault instances that lie on the decision boundary. Assuming the surface inspection system is embedded with a dynamic classifier. The dynamic classifier uses the extracted features of the incoming samples and performs the classification to existing or new defect classes. The dynamic classifier updates the decision boundary through which it communicates its decision to the end-users (i.e. machine operators).

Communicating classifier decision to end-users often requires the synthesis of fault instances to visualize the decision boundary between classes. This led to the motivation of generating synthesized images which are located either close to or on the decision boundary within the n-dimensional feature space. Therefore, a methodology for structural

synthesis based on principal components analysis is presented in this research work.

1.1 Related Work

In general, synthesis methods assumes that the test sample is available where small patch region is obtained from it and then synthesized to render a sample of arbitrary size with texture similar to that of the test sample or fill the missing regions of the test sample. Parametric texture synthesis methods explicitly measure some set of statistics in the input sample in an analysis step. In the synthesis step, an arbitrary image (e.g. initialized with random noise) is altered according to constraints derived from the previously extracted statistics. (Heeger and Bergen, 1995) uses a random noise image which is modified to match filter response histograms, obtained at different scales in the analysis step. Good results for stochastic textures were reported, but the method failed to capture structured or regular texture patterns. More recently (Portilla and Simoncelli, 2000) proposed a texture synthesis method based on

the first and second order statistics of joint wavelet coefficients. Similar to Heeger and Bergen, these coefficients are obtained for different scales of the input texture. While the synthesis quality for regular textures could be improved, this approach still lacks capabilities to reproduce structured textures. The major advantage of parametric texture synthesis methods is that the statistics of the synthesized image can be explicitly controlled. For this reason parametric texture synthesis is highly relevant for experimental design in the field of human perception (Balas, 2006) (Balas et al., 2009). All parametric texture synthesis approaches share the disadvantage, that several iterations are required until the multiscale representation of the output texture is altered to match the statistics of the input sample. This optimization process is time consuming, in particular when compared to pixel based or patch based synthesis methods (Moore and Lopes, 1999).

Current pixel-based synthesis methods are based on (Efros and Leung, 1999) which follow a very simple idea. Starting from an input exemplar, the output is first initialized by copying a small seed region from the input. The synthesized region is then gradually grown from the initial seed by assigning the output pixels one by one in an inside-out, onion layer fashion. The colour of each output pixel is determined by a neighbourhood searching for similar neighbourhoods in the input image. From this candidate set of matching neighbourhoods the output pixel is chosen from the centre of a randomly selected neighbourhood from the candidate set. This process is repeated until all the output pixels are assigned. In order to speed up this process, the search for matching neighbourhoods (Wei and Levoy, 2000) may be made more efficient using tree search (Kwatra et al., 2005), jump-maps (Zelinka and Garland, 2004) or k-coherence (Tong et al., 2002); (Thumfart, 2012).

Restoring of structure in image with missing/damaged regions is investigated e.g. in (Guo et al., 2008) using morphological erosion and structure feature replication or (Chen and Xu, 2010) which builds upon a primal sketch representation model for the reconstruction of missing structure. In contrast to (Guo et al., 2008), the methodology presented here describes the use of eigenimages to synthesize the test sample. In the absence of test sample, the proposed methodology tries to restore the structure of the test sample using the neighbouring samples which are either close to or on the decision boundary. Therefore, the principal components analysis approach of utilizing the eigenimages was more suitable for our application.

1.2 Paper Organization

Section 2 describes the methodology for structural synthesis. Section 3 will explain the evaluation procedure, simulation setup and its results will be discussed. A few details about the dataset on which the evaluation procedure was performed will also be given in Section 3. Section 4 concludes the research work and provides an outlook regarding the future work.

2 METHODOLOGY

The methodology proposed in this section focuses on the structural synthesis of classified images. The methodology assumes the following pre-requisites to be fulfilled for performing structural synthesis:

- Acquire a training set of classified images
- Resize the training set images with constant width and height
- Extracted features of the training set images must be available

2.1 Principal Components Analysis

Principal components analysis (PCA) is one of the methods applied for dimensionality reduction of a dataset while retaining most of the information in the dataset. The principal components analysis method is based on the following assumptions:

- Dimensionality of the dataset can be efficiently reduced by linear transformation
- Information of the dataset is concentrated in those directions where input data variance is maximum

Assume 'N' number of resized images are contained in the training set. Every resized image I(x, y) is a matrix of size $n \times n$ of 8-bit gray scale intensity values. Initially, the training set images are formed into a single rectangular matrix $\mathbf{I_{ts}}$ of size $n^2 \times N$.

$$[\mathbf{I_{ts}}]_{n^2 \times N} = [I_1 \quad I_2 \quad \cdots \quad I_N] \tag{1}$$

Using Eq. (1) as an input, eigenimages are then computed by applying the PCA method (Belhumeur, Hespanha, and Kriegman, 1997). The result of principal components method is given by Eq. (2).

$$[\mathbf{E}\mathbf{I}_{n^2xn^2} \quad \mathbf{S}_{Nxn^2} \quad \mathbf{e}\mathbf{v}_{n^2x1}] = PCA([\mathbf{I}_{ts}]^T) \tag{2}$$

where **EI**, **S** and **ev** represent eigenimages matrix, score matrix and eigenvalues vector respectively. In the next step, images are projected onto the subspace

of eigenimages to compute the coefficient matrix (Turk and Pentland, 1991). The subspace of eigenimages is selected from the number of non-zero elements contained in the eigenvalues vector. The coefficient matrix is represented by Eq. (3) and Eq.

$$\mathbf{C} = \mathbf{E}\mathbf{I}_{\mathbf{S}}^{\mathbf{T}}[\mathbf{I}_{\mathbf{ts}} - \mathbf{\mu}] \tag{3}$$

$$\mathbf{C} = \begin{bmatrix} c_1 & c_2 & \cdots & c_m \end{bmatrix}^T, \tag{4}$$

where C, EI_s and μ represent coefficient matrix, subspace of eigenimages and column wise mean of I_{ts} respectively. m corresponds to the number of non-zero elements of ev. The subspace of the coefficient matrix is dependent upon the selection of desired number of eigenimages in EI_s. The selection of desired number of eigenimages in EIs is given by

$$n_{dc} = \gamma \sum_{i=1}^{m} \mathbf{ev}_i, \quad where \ 0 < \gamma < 1$$
 (5)

where n_{dc} and γ represent the scalar parameters responsible for deriving the subspace of coefficient matrix. The preferable choice for parameter γ should be close to its higher limit (Noortiawati et al., 2006). The subspace of coefficient matrix and the desired number of eigenimages in **EI**_s are represented by Eq. (6) and Eq. (7).

$$\mathbf{C_s} \subseteq \mathbf{C} = \begin{bmatrix} c_1 & c_2 & \dots & c_r \end{bmatrix}^T, \qquad r \le m$$

$$\mathbf{EI_{rs}} \subseteq \mathbf{EI_s} = \begin{bmatrix} ei_0 & ei_1 & \dots & ei_r \end{bmatrix}$$
(6)

$$\mathbf{EI_{rs}} \subseteq \mathbf{EI_{s}} = [ei_0 \quad ei_1 \quad \dots \quad ei_r] \tag{7}$$

where C_s and EI_{rs} represent the subspace of coefficient matrix and desired number eigenimages respectively. C_s and EI_{rs} are then applied in the later stages of the methodology for synthesizing an image for a given new feature vector. In the subsequent section, solving of linear regression problem of C_s will be described.

2.2 **Linear Regression**

Linear regression is a statistical approach for modelling the behaviour of dependent variables denoted by Y and independent variables denoted by X. Here, the subspace of coefficient matrix C_s resulting from the Section 2.1 is considered as the subspace of the dependent variable Y and the Feature matrix is considered as the independent variable X. A linear regression model is assumed to fit the dependent variable Y. The linear regression model is represented by Eq. (8).

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} \tag{8}$$

where β is the parameter matrix. X, Y and β are represented by Eqs. (9) - (11).

$$\mathbf{X} = \begin{bmatrix} 1 & f_{1,1} & \cdots & f_{p,1} \\ 1 & f_{1,2} & \cdots & f_{p,2} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & f_{1,N} & \cdots & f_{p,N} \end{bmatrix}$$
(9)

$$\mathbf{Y} = \begin{bmatrix} c_{1,1} & c_{2,1} & \cdots & c_{r,1} & a_1 & b_1 \\ c_{1,2} & c_{2,2} & \cdots & c_{r,2} & a_2 & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ c_{1,N} & c_{2,N} & \cdots & c_{r,N} & a_N & b_N \end{bmatrix}$$
(10)

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_{0,1} & \beta_{0,2} & \cdots & \beta_{0,r+2} \\ \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,r+2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n,1} & \beta_{n,2} & \cdots & \beta_{n,r+2} \end{bmatrix}$$
(11)

where a and b represent the original width and height of the resized images of the training set. In the first instance, least squares method was applied to estimate the parameter matrix β . The least squares method seeks to minimize the sum of squared residual errors e_i .

$$\min_{\beta_{i} \in \mathbb{R}^{(p+1)x_{1}}} e_{i} = \sum_{i=1}^{r+2} (\mathbf{y}_{i} - \mathbf{X}\boldsymbol{\beta}_{i})^{2}$$
 (12)

where y_i and β_i represent the column vectors of matrices \mathbf{Y} and $\boldsymbol{\beta}$ respectively. The unique solution for the estimate of parameter vector $\hat{\beta}_i$ is given by the Eq. (13).

$$\widehat{\boldsymbol{\beta}}_{i} = (\mathbf{X}^{\mathsf{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathsf{T}}\mathbf{y}_{i} \tag{13}$$

To avoid overfitting situations where the ratio of model complexity to the training set size is too high, Elastic Net regularization method is chosen (Hastie et al., 2001). Elastic Net regularization method solves the cost function represented in Eqs. (14) – (16).

$$\min_{\boldsymbol{\beta}_i \in \mathbb{R}^{q \times 1}} \frac{1}{2N} \sum_{i=1}^{r+2} (\mathbf{y}_i - X \boldsymbol{\beta}_i)^2 + \lambda P_{\alpha}(\boldsymbol{\beta}_i)$$
 (14)

where
$$P_{\alpha}(\beta_i) = \frac{(1-\alpha)}{2} \cdot \|\beta_i\|_2^2 + \alpha \|\beta_i\|_1$$
 (15)

where
$$\lambda \ge 0$$
, $0 \le \alpha \le 1$ (16)

where λ is the complexity parameter. The publicly available package glmnet (Qian et al., 2013) is applied for estimating the parameter vector β_i . Assuming N images are available with the training set. Let's say N-1 images are used in the training phase for computing l number of solutions in λ for estimating β_i and k^{th} image which is left out in the training phase is used for validation phase (Friedman et al., 2010). During the validation phase, a measure risk (i.e. absolute error) is computed for each value of λ . Then, the λ where the measure risk is minimal will be selected and its resulting parameter vector $\boldsymbol{\beta}_i$ is chosen. In the subsequent section, synthesizing of images for new feature vectors will be described.

2.3 **Synthesis**

Synthesizing an image for a given new feature vector $\mathbf{f}_{\mathbf{w}}$ is generated using the estimated parameter matrix $\hat{\beta}$ from Section 2.2 and the desired number of eigenimages EIrs from Section 2.1. Initially, a dependent variable is derived for generating new coefficient vector and estimating the size of the synthesized image. The new dependent variable $\mathbf{y}_{\mathbf{w}}$ and the resultant coefficient vector $\mathbf{c}_{\mathbf{w}}$ are represented by Eq. (17) and Eq. (18).

$$\mathbf{y_w} = \mathbf{x_w} \widehat{\boldsymbol{\beta}} \tag{17}$$

$$\mathbf{y}_{\mathbf{w}} = \mathbf{x}_{\mathbf{w}} \mathbf{\beta}$$

$$\mathbf{y}_{\mathbf{w}} = [\mathbf{c}_{\mathbf{w}} \quad a_{\mathbf{w}} \quad b_{\mathbf{w}}], \quad \mathbf{x}_{\mathbf{w}} = [1 \quad \mathbf{f}_{\mathbf{w}}]$$
(18)

Finally, the synthesized image I_{syn} is obtained from the matrix multiplication of EIrs and cw.

$$\mathbf{I_{syn}} = \mathbf{E}\mathbf{I_{rs}}\mathbf{c_w^T} \tag{19}$$

The synthesized image is reshaped back to an image of $n \times n$ size. This reshaped image is then resized using the estimated width and height which can be retrieved as the last two entities of the new dependent variable y_w .

EVALUATION 3

In this section, the evaluation procedure of the proposed methodology will be explained in detail. A few details about the dataset on which the evaluation was done will be mentioned. The simulation setup and subsequent results will also be presented here.

3.1 **Sheaves Dataset**

The inspected sheaves with a diameter of around 100mm are applied in wire rope-based elevators. It comprises a steel wheel body coated with polyurethane on the circumference that carries the rope notch. Coating is exercised through a casting process with the final notch shape produced in a turning lathe. The following are the event types of the inspected sheaves:

Type 1: Surface spots that occurred during lathing also known as lathing damages

- Type 2: Events detected by the segmentation processes that do not correspond to surface faults also known as false positives from segmentation
- Type 3: Delamination of the coatings
- Type 4: Closed cavities
- Type 5: Open cavities
- Type 6: Swarf (turnings) that adhere to the circumference

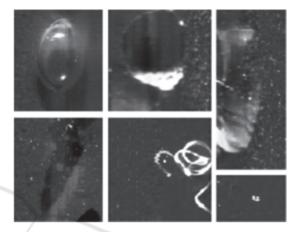


Figure 1: Examples of event types on elevator sheaves. Row-wise, left-to-right: closed cavities, open cavities, lathing damages, delamination, swarf, false positives from segmentation.

Typical examples of aforementioned event types are shown in Figure 1 (Weigl et al., 2014). Inspection of the sheaves is confined to the coated circumference which is scanned by a line-scan camera with a resolution of $20\mu m$ per pixel. Classification of event types which were detected on the elevator sheaves is performed using a random forest classifier. The classifier is trained on a fully expert-annotated database. The output of the classifier consists of the extracted feature vectors and class probability vectors of the respective images. The features computed for the images which correspond to the event types describe the shape, size, intensity level, histograms of the events.

3.2 **Simulation Setup**

Given 6 classes of Sheaves dataset, there will be 15 unique possible class pairs. For a given class pair (CP), images are selected in such a way that the first two highest class probabilities present in their respective class probability vector are close to each other. The steps for acquiring the training set images for all class pairs are the following:

Step 1: Class probabilities present in the class probability vector of all the images of Sheaves dataset are sorted in ascending order

- Step 2: Gradient between the first two highest class probabilities using the sorted class probability vector from step 1 is computed
- Step 3: Five percent of the maximum class probability is assigned as a threshold value. This threshold value is then compared with the gradient from step 2 for selecting specific images from the dataset which belong to the 15 class pairs
- Step 4: Assuming the evaluation procedure has been initiated for a given class pair. Then, the images resulting from step 3 for the given class pair are sorted in such a way that the gradient (computed from step 2) values are in ascending order
- Step 5: The sorted out images from step 4 are then used as the training set for the given class pair

Except for 'delamination/open cavities', 'delamination/swarf' and 'closed cavities/swarf' class pairs, images close to the decision boundary for the remaining 12 class pairs were found with respect to the above selection procedure.

Let's assume 'N' test samples (i.e. images) are contained in the training set of j^{th} class pair and each test sample is of different size. For evaluation, we are holding out the i^{th} instance (test sample) from the training set and this one instance is then synthesized. Likewise, this process is repeated for all N number of instances in the training set.

Synthesized form of the test samples resulted from the evaluation are then fed to feature extraction software for computing their respective feature vectors.

3.3 Results

In this section, the evaluation results will be discussed. Figure 2 represents the Euclidean distance ${}^{c}ED_{r,s}$ computed between feature vectors of the test samples (i.e. training set of j^{th} class pair) and that of the synthesized form of test samples. Here, the distance between the feature vectors of the k^{th} test sample with the highest probability indicating that it belongs to the first class and l^{th} test sample with the highest probability indicating that it belongs to the second class of the training set of j^{th} class pair is considered as the performance metric. This performance metric is denoted by ${}^{c}Ed_{om}$ in Figure 2.

Assuming an ideal scenario where the computed euclidean distance is close to zero, this will give an indication that the synthesized form of test samples are a close approximation of the test samples in terms of feature vectors and visual appearance. Here, the evaluation result is far from the ideal scenario.

However, euclidean distance of majority of the synthesized form of test samples of 'lathing damages/delamination', 'lathing damages/closed cavities', 'lathing damages/swarf', 'false positives from segmentation/delamination', 'false positives from segmentation/closed cavities', 'false positives

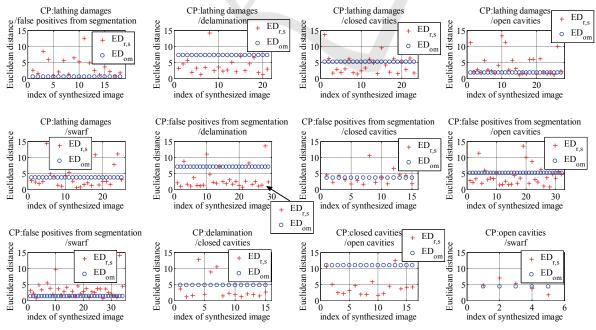


Figure 2: Euclidean distance versus index of synthesized test samples of feasible 12 class pairs.

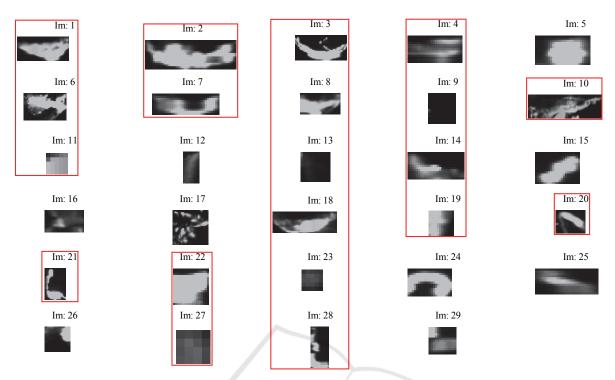


Figure 3: Test samples of 'lathing damages/open cavities' class pair.

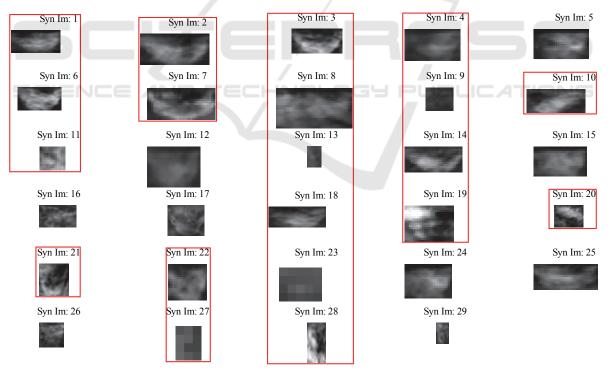


Figure 4: Synthesized form of test samples of the 'lathing damages/open cavities' class pair.

from segmentation/open cavities', 'delamination/closed cavities' and 'delamination/open cavities' class pairs are within

their respective performance metric value. This gives an indication that those synthesized form of test samples lie within the hypersphere (formed by

the test samples located as points in n-dimensional feature space) in which the corresponding test samples are contained.

For the remaining class pairs, the synthesized images are located away from the hypersphere in which the corresponding test samples are comprised. This is due to the fact that, each and every image present in the training set of these class pairs is very distinct in shape and structure. Nevertheless, an average of 58.5% of the synthesized images resulted from all 12 class pairs are within the average of the performance metrics of all 12 class pairs.

Figures 3 and 4 represent the test samples and the synthesized form of test samples of 'lathing damages/open cavities' class pair respectively. From Figures 3 and 4, it can be seen that the methodology performs the synthesis of specific test samples (which are shown in the red rectangular boxes) reasonably well. The cause behind such result is the structure and shape present in the ith specific test sample is still visible from the remaining samples of 'lathing damages/open cavities' class pair and are contained as the prominent principal components in the eigenimage matrix at PCA stage. Therefore, more than half of the synthesized samples (i.e. synthesized form of test samples) are close to the test samples in terms of visual appearance. Similarly for the remaining class pairs, the number of synthesized samples which were visually close with their respective test samples was found to be approx. in between 50% to 60%.

4 **CONCLUSION**

A methodology for performing structural synthesis based on principal components analysis is formulated here. Also, a framework for the feature based synthesis evaluation procedure has been presented. The methodology has been evaluated on a dataset from elevator sheaves inspection. Test samples on which the methodology performs the synthesis quite good is shown with respect to a certain class pair. Although satisfactory result was obtained for the remaining class pairs where test samples were fewer, the methodology might have worked better if the class pairs consisted of high number of test samples. In contract to state of the art texture and structural synthesis methods which assumes the availability of a test sample, the proposed methodology is able to generate synthesized form of the test sample using the neighbouring samples which lie close to the decision boundary. This enables an inspection system's

human operator to visualize the decision boundary where availability of test samples lying close to the decision boundary is rare. The next step in this research work is to enhance the PCA with kernel functions to compute principal components in high-dimensional space, related to the input space by some nonlinear map. In addition, we intend to factor the dominant fault orientation out of the principal component decomposition in a similar way varying fault sizes are handled now. Using a subspace of these principal components, the linear regression and synthesis steps from the methodology could be repeated to acquire much better visual closeness of the synthesized samples with that of test samples.

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