

An Interbank Market Network Model based on Bank Credit Lending Preference

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Abstract: An interbank market network model based on bank credit lending preference is built in this paper to explain the formation mechanism of interbank market network structure. As well, we analyze the impact of credit lending risk preference on network topology structure, which includes degree distribution, network clustering coefficient, average shortest path length and network efficiency. Simulation results demonstrate that the accumulation degree follows dual power law distribution with credit lending risk preference parameter value equal or greater than 1, while the accumulation degree follows power law distribution with credit lending risk preference parameter value smaller than 1. The interbank market network shows small world topology property. With the increasing of bank credit lending risk preference, the average shortest path length decreases but network efficiency improves, which enhances the stability of the network.

1 INTRODUCTION

In the banking system, complex network relationships are formed through interbank lending, payment and settlement, discount and guarantee. The interbank market allows liquidity exchanges among financial institutions through facilitating the allocation of the liquidity surplus to illiquid banks, but also provide channel for risk contagion, which might trigger a domino effect. The subprime crisis broke out in the US financial market in August 2007, which quickly evolved into global financial crisis, resulting in a large number of bank failures and great damaged to the stability of the financial system.

Complex network theory is an important tool for complex systems modelling, and has been applied to statistical physics, life sciences, social sciences and many other fields. Random network (Burda et al., 2004), small-world networks (Watts and Strogatz, 1998; Newman and Watts, 1999) and scale-free networks (Barabási and Albert, 1999) are common complex network topologies. In the economic system, the complex network theory has been used for modelling in the fields of e-commerce (Reichardt and Bornholdt, 2005), network transactions (Garlaschelli and Loffredo, 2004), the stock market (Boginski et al., 2005; Bonanno et al., 2004; Huang et al., 2009) and other areas of modelling. The

interbank market exhibits high degree of complexity, with different network structures, such as money centre structure (Freixas et al., 2000), complete market and incomplete market (Allen and Gale, 2000), etc.

There have been larger number of empirical research literature on interbank market network structure topology, such as degree distribution, average path length of the network, clustering coefficient, etc. Souma et al. (2003) modelled Japanese business network and found scale-free property through empirical results. Boss et al. (2004) analyzed Austrian interbank market and found that the degree distributions followed power law distribution, interbank liability network showed a community structure, a low clustering coefficient and a short average path length. Iori et al. (2008) found the structure of Italian interbank market was fairly random and changed with time. Iori et al. (2007) showed that the Italian interbank consists of two communities, one mainly composed by large and foreign banks, the other composed by small banks. Cajueiro and Tabak (2008) found that the Brazilian interbank network structure had a weak community structure and high heterogeneity. Tabak et al. (2009) built Brazilian interbank market with minimum spanning tree method and showed that the private and foreign banks tended to form clusters within the network and that banks with different

sizes were also strongly connected and tended to form clusters.

In recent years, researchers began to explore the interbank market network structure formation mechanism. Inaoka et al. (2004) presented a procedure to extract a network structure described by a power-law degree distribution from a set of records of transactions. Li et al. (2010) introduced a network model of an interbank market based on interbank credit lending relationships and found some typical structural features such as a low clustering coefficient and a relatively short average path length, community structures, and a two-power-law distribution of out-degree and in-degree.

In summary, the present simulation methods for constructing an interbank market can be divided into the following categories: (1) Establishing an interbank market network by setting a fixed link probability; (2) By setting a linking threshold, credit links are created if the given threshold is exceeded; (3) By assuming that the interbank network of a particular network architecture (such as scale-free networks, dual power rate networks, small-world networks, etc.). From the above analysis, we can see that the current model construction methods have not taken bank behaviours such as assets and liabilities into consideration. But empirical results demonstrate that the formation of credit lending links between banks is related to the banks behaviours. Banks with different credit lending scales are strongly connected and tend to form clusters (Tabak et al., 2009). In this paper, an interbank credit lending network model is constructed through designing a probability associated with bank lending scale and risk preference. Then, we analyze the topology property of network and the influence of risk preference on network structure.

The remainder of this paper is organized as followed. The model is presented in part 2, simulation analysis is shown in part 3, and finally conclusions are conducted in part 4.

2 THE MODEL

In this paper, a directed graph $G=(V,E)$ is used to denote interbank market network, where the vertex set V represents the set of all banks and the set E is a collection of edges which represent the interbank credit lending relationships. A directed edge $e_{i,j}$ exists between nodes $i, j \in V$, if and only if bank i is the creditor bank of bank j . Assuming

that the total bank number $|V|=N$, and N_i denotes the set composed by neighbours of bank i . l represents the total interbank lending scales, and l_i is the lending scale of bank i , satisfying $l = \sum l_i$.

Based on the empirical results (Boss et al., 2004), it is assumed that bank credit lending scale follows power-law distribution: $P(l) \sim l^{-\gamma}$, where γ is power law parameters. So, the interbank market network we build is composed by a large number of small banks and a few large banks. The specific process to construct the interbank market network is listed as follows:

- 1) Initialization: Generating the total number of N banks and lending scales followed by power-law distribution.
- 2) The construction process of interbank credit lending relationships: The connection probability p_{ij} of bank $i (1 \leq i \leq N)$ and bank $j (1 \leq j \leq N, j \neq i)$ depends on their lending scales: $p_{ij} = 1 - \exp(-\lambda(l_i/l_j + l_j/l_i - 2))$, where λ denotes bank credit lending risk preference coefficient, where $\lambda \in [0, +\infty)$.

The interbank relationship connection probability $p_{ij} = 1 - \exp(-\lambda(l_i/l_j + l_j/l_i - 2)) \in (0,1]$, as the inequality $l_i/l_j + l_j/l_i \geq 2\sqrt{l_i/l_j \times l_j/l_i} = 2$ and $\lambda \geq 0$. The probability gets the minimum when $l_i = l_j$ and gets the maximum when $l_i \gg l_j$ or $l_i \ll l_j$. Obviously, the connection probability p_{ij} increases monotonically with parameter λ .

3 SIMULATION RESULTS

In this paper, the parameters are initialized as follows: The total bank number $N = 200$, the power-law parameter $\lambda = 1.87$ (Boss et al., 2004), and the bank credit risk preference coefficient $\lambda = 0.01, \lambda = 0.1, \lambda = 1, \lambda = 10$ and $\lambda = 100$, respectively.

The interbank market structure calculated by the model with $\lambda = 0.01$ is given in Figure 1, in which nodes denote banks and edges represents credit lending links between banks. It can be found from figure 1 that the banks which have large credit lending scales get bigger degree than those with small ones. This is because the number of banks with large credit lending scales is much smaller than those with small ones and the credit lending scales

of banks are generated by power-law distribution. From the model, it can be known that banks with different credit lending scales establish connections more easily than those with the similar ones. So big banks show large degree and small banks get small degree.

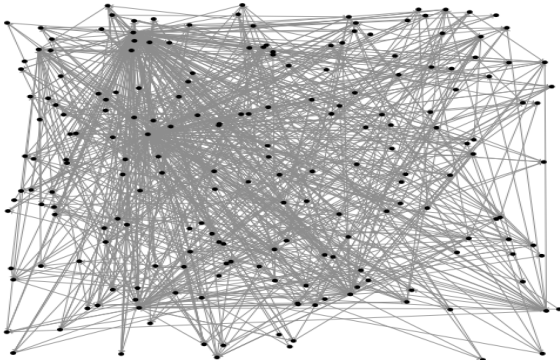


Figure 1: An interbank market network structure.

3.1 Degree Distribution

In this paper, we let d_i, d_i^{in}, d_i^{out} denote the degree, in-degree and out-degree of bank i respectively, $d_i = d_i^{in} + d_i^{out}$. The degree distribution $p(k)$ is defined as the proportion of the nodes with degree equals k in the network. Cumulative distribution $P_{cum}(k) = \sum_{k \geq k'} p(k')$, represents the proportion of the

nodes with degree no less than k in the network. Figure 2-1 to figure 2-5 is the simulation results of cumulative distribution with bank credit lending parameter $\lambda = 0.01, 0.1, 1, 10, 100$, respectively.

It can be found from figure 2-1 to figure 2-5 that the cumulative distribution of the constructed interbank network follows power-law distribution with $\lambda < 1$, but the cumulative distribution obeys to dual power-law distribution with $\lambda \geq 1$, which demonstrate that few number big banks which have large credit lending scales own the majority interbank credit lending business while the large number small banks with small credit lending scales have the minority interbank credit lending business. With the increment of credit lending risk preference, the maximum of in-degree and out-degree improves simultaneously. The dual power-law distribution obeyed by cumulative distribution through simulation experiments are consistent with the empirical findings in Austria and Japanese interbank (Boss et al., 2004; Souma et al., 2003) when $\lambda \geq 1$. As well, with $\lambda < 1$, the power-law distribution of cumulative distribution is in accordance with

Inaoka's empirical results (Inaoka et al., 2004).

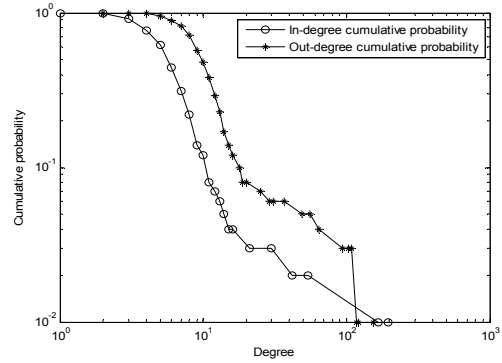


Figure 2-1: Cumulative distribution with $\lambda = 0.01$.

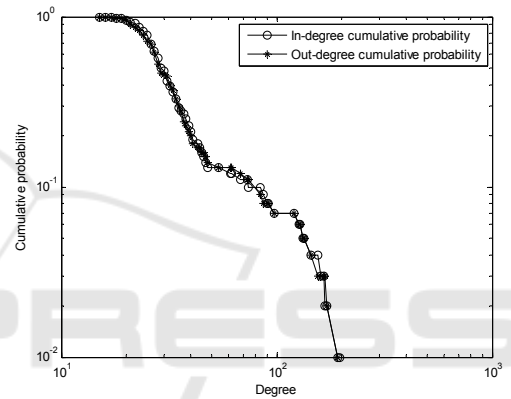


Figure 2-2: Cumulative distribution with $\lambda = 0.1$.

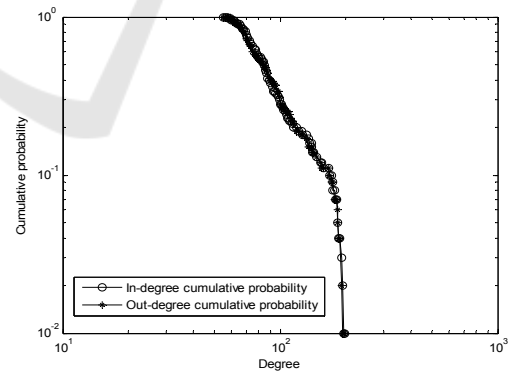


Figure 2-3: Cumulative distribution with $\lambda = 1$.

3.2 Network Clustering Coefficient

The clustering coefficient of a node is used to measure the connected probability of two neighbours of the node in an undirected graph. The network clustering coefficient is the average of the clustering coefficient of all nodes in the network.

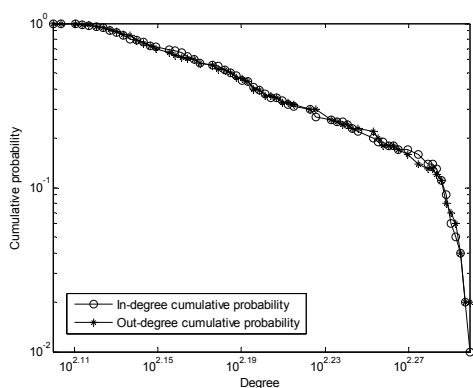


Figure 2-4: Cumulative distribution with $\lambda = 10$.

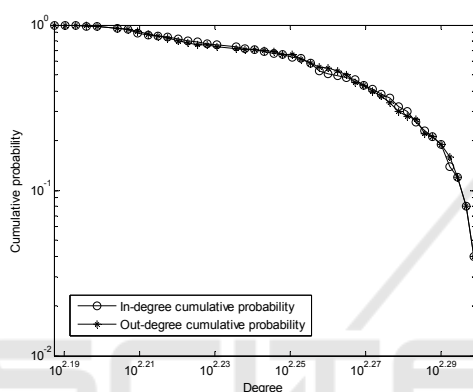


Figure 2-5: Cumulative distribution with $\lambda = 100$.

We let C denote network clustering coefficient, and c_i represent the clustering coefficient of node i , thus $C = \frac{1}{N} \sum_{i=1}^N c_i$. From the definition of node

clustering coefficient, we can get $c_i = \frac{2E_i}{d_i(d_i - 1)}$,

where E_i represents the number of connected edges between neighbours of node i . The directed graph should be transformed to be an undirected one before computing network clustering coefficient, since the clustering coefficient is defined in an undirected graph. Figure 3 shows logarithmic plot of network clustering coefficient relationship and bank credit lending risk preference.

It can be known from figure 3 that the clustering coefficient of interbank credit lending market network monotonically increases with bank credit lending risk preference. The explanation for this is that the probability to build credit lending relationships improves with the increment of bank credit lending risk preference. From the definition of network clustering coefficient, it is easy to

understand that network clustering coefficient increases monotonously with the connection probability.

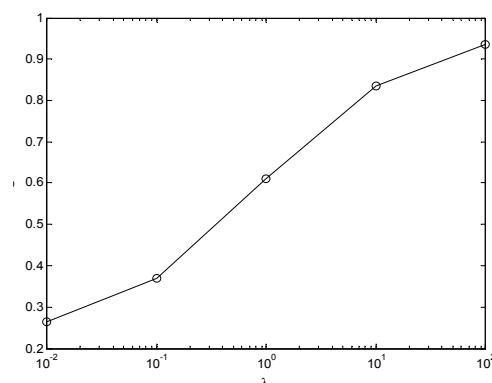


Figure 3: Relation between network clustering coefficient and λ .

3.3 Average Shortest Path Length

The shortest path length between two nodes is used to measure the distance of the two nodes. The shortest path d_{ij} from node i to node j is defined as a simple path starting from node i , and sinking in node j , which has the shortest nodes number. The shortest path length d_{ij} from node i to node j is the edge number of the shortest path. Obviously, in an undirected graph, the shortest path length d_{ij} from node i to node j equals the shortest path length d_{ji} from node j to node i . The average path length of the network can be calculated as $L_G = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}$. To simplify the calculation

of the average shortest path length of the network, the interbank market network is transformed into an undirected graph. The relationships between the average path length of the interbank market network and bank credit lending preference is shown in figure 4.

From figure 4, it can be found that the constructed interbank market network is a small world network with average path length less than 2. The average path length decreases monotonously with bank credit lending preference coefficient, since the potential paths between any two nodes increase with the growth of credit links number in the network. The results of interbank markets network structure in Mexico, USA and the Great Britain (Martínez-Jaramillo et al., 2010; Soramäki et al., 2007; Becher et al., 2008) also shows small

world property, which are the same as the finding of this paper.

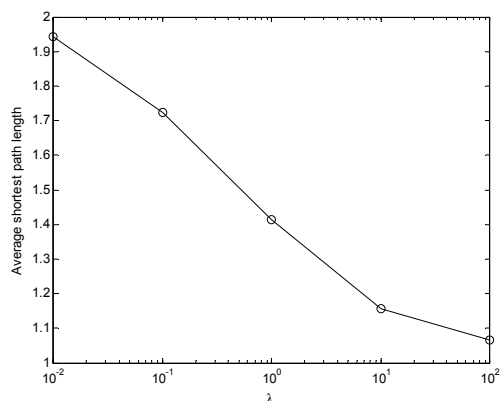


Figure 4: Relations between average path length and λ .

3.4 Network Efficiency

Network efficiency is another approach to measure the capacity of a network, and can be computed by

$$E_G = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}},$$

where d_{ij} is the shortest path length from node i to node j . From the definition of network efficiency, we can conclude that network efficiency can apply not only to describe a connected graph but also represent a non-connected graph connections. Similarly as calculating the average shortest path length, the interbank market network should also be transformed into an undirected graph when computing network efficiency. Simulation results are shown in figure 5.

It can be seen from figure 5 that network efficiency increase monotonously from 0.55 to 0.97 with the increment of bank credit lending preference and network efficiency approaching to 1 when $\lambda \rightarrow 1$. The reason for this is that the connection probability increases with the improvement of bank credit lending preference, which results in more interbank linkages in the interbank network. Then, there will be more potential paths between any two nodes as the addition of network edges and the shortest path length of them may be shorter in the meanwhile.

4 CONCLUSIONS

In this paper, an interbank market network is constructed based on bank lending credit scales followed by power-law distribution. Simulation

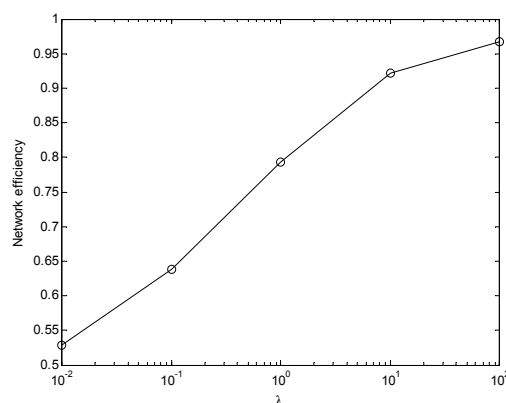


Figure 5: Relations between network efficiency and λ .

experiments demonstrates that interbank credit lending market network has small world property and follows power-law distribution with bank credit lending risk preference parameter value smaller than 1, while follows dual power-law distribution with the same parameter equal or greater than 1. With the increment of bank credit lending preference, the average shortest path length decreases and network efficiency increases, which improves the capacity and stability of the network.

The results of this paper have some policy guidance. On one hand, when liquidity shortage occurred in the interbank market, bank regulators can adopt positive policies to guide banks to increase credit lending risk preference for prospering interbank market. On the other hand, when interbank market exhibits excessive prosperity, bank regulators should strengthen the interbank market supervision, and guide banks to reduce risk preferences in order to prevent potential systemic risks.

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