A Hypercube Queuing Model Approach to the Police Units Allocation Problem

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Abstract: Providing security requires efficient police services. Considering this, we deal in this paper with the police units allocation problem. To describe the problem a probabilistic model based on Hypercube Queuing Model is proposed. Considering an action radius and constraints for minimal coverage and mandatory closeness, the model aims to allocate police units on several points of an urban area to minimize the expected distance traveled by these units when they are answering calls for service. A VND heuristic is used to solve the model, and we analyse the improvement of using a Tabu Serach method instead of a random initialization. We experiment the methods in scenarios with different parameters values to verify the robustness and suitability of the proposed model. The results presented a high influence of service time on solutions quality, some difficulties in getting feasible solutions.

1 INTRODUCTION

Public safety is one of the main concerns in the modern society and has direct impact on the quality of life. Brazil is a notable case where the violence actions affect the wellfare of all social classes.

As presented in (Waiselfisz, 2013), the homicide rate per 100 thousand people was 11,7 in 1980, reaches 28,9 in 2003, and has barely lowered to 27,1 in 2011. In 2013, over 50 thousand people were killed in the country, besides around 1,2 million robberies and 50 thousand rapes (Fórum Nacional de Segurança Pública, 2013). These numbers put Brazil as one of the most violent countries in the world, even when considering those involved on wars (Cerqueira, 2005).

Some of important causes of this situation are strategic and technical failures on the area of public safety management. We can highlight the lack of training of police force in many aspects, as those related with use of external resources, statistics, historical informations or softwares and digital devices for helping their work.

Considering the context described, in this paper we propose a mathematical model to describe and optimize the problem of police unit allocation. This model aims to work on two factors which directly affects the perception of violence and satisfaction with police services (Cihan et al., 2012): police units visibility and speedy response to calls for service.

The model aims to reduce the expected distance traveled within an action radius for responding calls for service and to provide a minimal expected coverage beside ensuring a mandatory expected proximity among all points within an urban area to at least one patrol unit.

The model presented is based on the Minimum Expected Response Location Problem (MERLP), a stochastic model proposed originally for emergency medical services by Rajagopalan and Saydam (Rajagopalan and Saydam, 2009) that has good results on this field. This model, in turn, was built over fundamentals of the Hypercube Queuing Model (Larson, 1974), which describes queuing systems where the servers go towards the clients in a determined location, such as in emergence services.

The contribution brought by our paper is a model that corrects and turns the MERLP more general, by adding constraints of mandatory expected response coverage and also support for servers of different kinds (cars, motorcycles, police agents on foot, etc...). The last feature does not change directly the model, but impacts the solutions evaluation. Furthermore, the number of vertex and edges on the graph used in this paper is much bigger than in previous works.

In order to solve the presented model, we use a heuristic based on the VND (*Variable Neighborhood Descent*) metaheuristic. The main goal, however, is not to test the efficiency of the heuristic proposed in

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solving the model, but to show the suitability of using this model to describe the problem. In other words, we argue in this work that once the model proposed is quickly solvable by using the VND heuristic and presents solutions that satisfy the scenarios tested, it may be useful for real contexts.

In the next sections, we present the details of the proposed model and the solving approaches experimented. On section 2, we present a review of previous works about problems related with this one. Next, in section 3, the Hypercube Queuing Model and the approximative method for estimating the parameters of this model, the Jarvis Approximation, are presented. On section 4, we present the models used as base for the model proposed and the VND heuristic for solving it. After that, section 5 explains how the experiments were performed ending up on section 6 with a discussion of results.

2 LITERATURE REVIEW

According to Larson (Larson, 1974), one of the first researches that focus the police work was published in a book by Smith (Smith, 1961) in 1961. The work was about the design of sectors for patrol beats, aiming to minimize the mean intersector travel time.

In the late 60's, the New York City Rand Institute (NYCRI) was founded, conducting a profund impact research on the area of emergence services models (Green and Kolesar, 2004), as such the Hypercube Queuing Model (Larson, 1974), explained on next section; and the Patrol Car Allocation Model (PCAM) (Chaiken and Dormont, 1976): a software that uses results from Queue Theory to assist police's departments in determining the number of patrols cars and their locations while on duty (Larson and Rich, 1987), that is still used in more recent researches, as in (D'Amico et al., 2002a).

On late 80's and early 90's, criminology experiments showed the high concentration of call or incidents at few places in a city (denominated *hot spots*) and the efficacy of a geographically focused police service (Telep and Weisburd, 2012) (Weisburd and Eck, 2004), generating several studies around this fact, as those described on (Keskin et al., 2012), (Chawathe, 2007) and (Li et al., 2011). The first aims to determine, for several police cars, patrol routes on highways to visit *hot spots*, while they are "hot", in other words, during a specific period of the day (on one or more days) when more car crashes happens. The second builds a model where a city is a graph, each street considered an edge and each corner a vertex. With each street having a weigh, corresponding to its "hotness", and a length, the author defines a strategy to get the route with higher density. The last uses an algorithm based on cross-entropy for building a randomized route, point by point, through a Markov Chain Model.

Some papers swap the concept of *hot spot* by the concept of *target*, that are specific points on a area. We cite in this direction (Basilico et al., 2012) and (Perrucci, 2011), which uses a game theory to guide a robot in a graph. The first looks for a leader-follower equilibrium and the second aims to find the shortest path to visit all targets.

The above mentioned papers which deal with *hot spots* or *target* concepts are routing problems. Patrolling routes are also object of study of other papers, not using those concepts. We cite some that build non-deterministic routes based on Markov Chains Model (Chen, 2013) (Ruan et al., 2005) (Lin et al., 2013), and (Vasconcelos, 2008) that uses a genetic algorithm for calibrating routing simulation parameters.

Papers dealing with the issue of dividing a area in several disjoint police districts are also relevant. We can cite (D'Amico et al., 2002b) that uses a simulating annealing approach; a comparative work developed by Zhang et. al. (Zhang et al., 2013), where three methods for district design had their result put side by side. It is also notable the series of studies conducted by Zhang and Brown. The first described at (Zhang and Brown, 2013) a simulation and a geographic information system were used to register the call of service data as a base to create districts with workload balance; the second (Zhang and Brown, 2014a) describes an adjusted simulated annealing and finally (Zhang and Brown, 2014b) uses a sophisticated method of response surface for the same objectives.

More related with this paper subject, on police duty context, coverage problems are studied by Saladin (Saladin, 1982), who created a goal programming model to police patrol allocation, using PCAM to evaluate the solutions found; Curtin et. al. (Curtin et al., 2010), that deal with a coverage and a backup coverage model, where in the last, the objective is to get the maximum coverage by at least two police units on each point; And Mendes et. al (Mendes et al., 2014) and Mendes and Santos (Mendes and dos Santos, 2015), that proposed a model for maximizing a profit related with a coverage, with mandatory closeness constraints using patrol units with different action radius.

More inovative models were proposed by Dell'Olmo et. al. (Dell'Olmo et al., 2014) and Araz et. al. (Araz et al., 2007). The first models safety camera allocation, for traffic surveillance, where a set of cameras changes their positions through time to avoid to be memorized by drivers who try to hide their infractions. The second model is a fuzzy multi-objetive model, for dealing with the uncertainty of emergency services.

For those who want a more deep view of researches developed in this subject, we suggest the surveys written by Simpson and Hancook (Simpson and Hancock, 2009), Maltz (Maltz, 1996) and Green and Kolesar (Green and Kolesar, 2004).

3 THEORICAL BACKGROUND

In this section we present a brief description of the theorical background of our model. A detailed explanation of the themes presented here can be found at the original papers (Larson, 1974), (Jarvis, 1985) and in the tutorial written by Chiyoshi et. al. (Chiyoshi et al., 2011).

The Hypercube Queuing Model (henceforth referenced as HQM) is a queuing model proposed by Larson at 1974 to address problems of facility location and design of action areas (Larson, 1974). HQM has showed itself a powerful tool to describe any queuing system where the servers need to go where clients are.

In the queuing system described by HQM, a finite set V of points $j \in V$ represents the location of clients and and a subset of V represents the position of servers. With this configuration, is calculated the time required to any server arrive at any client from its positions. Then, for each client a server dispatch order is fixed, with the closer servers being first.

Once a call for service arrives on the system from a point j, the first idle server following the dispatch order is selected to answer the call.

In our implementation, when all servers are busy, the call is ignored and not answered (loss systems).

To represent a state s, HQM uses a n-uple of 0's and 1's, where each digit represents the current state of a single server (0 if a server is idle and 1 otherwise). For instance, a system with 4 servers can assume states such as: $s_1 = (1,0,0,1)$ or $s_2 = (0,0,1,1)$, where this last represents a state with servers 1 and 2 busy. Designing state trasition graph we get a hypercube. This result gives to HQM its name.

To estimate the performance measures of this queuing system, it is necessary to calculate the steady state probabilities of being in each state. For each state one equation is built. This generates a linear system that has 2^n equations and unknowns to be solved for getting all steady state probabilities. For any reasonable number of servers, this resolution becomes costly in terms of computer time.

To deal with this problem we use the Jarvi's Approximation, a generalization of the approximation proposed by Larson (Larson, 1975), where the service time distribution can be a general distribution dependent on both client and server (Rajagopalan and Saydam, 2009).

The Jarvi's Approximation consists basically on a iterative method for calculating the busy probability of servers. To do this, it relaxes the servers interdependence, treating servers busy probabilities as being independent. Then, for trying to correct errors caused by this assumption, a $Q(m,\rho,k)$ factor is defined as bellow:

$$Q(m,\rho,j) = C \sum_{k=j}^{m-1} \frac{(m-k)(m^k)(\rho^{k-j})}{(k-j)!} \quad \forall j \in V \quad (1)$$

$$C = \frac{(m-j-1)!}{m!(1-P_m)^j} * \frac{P_0}{1-\rho(1-P_m)} \quad (2)$$

The $Q(m,\rho,j)$ factor value (Equations (1) and (2)), is a function of the number of servers m, probability of the all nodes being idle P_0, the probability of all nodes being busy P_m, and the average system busy probability $\rho = \lambda\tau/m$, where τ is the system wide mean service time and λ the system wide arrival rate.

The values involved in the evaluation of $Q(m,\rho,j)$ factor, together with the number of servers m, as well as ρ and τ have their value fixed through an iterative method, that converges in few iterations. This method begins with the initialization of ρ_i and τ, made by Equation (3) and (4) respectively.

$$\rho_i = \sum_{j:\alpha_{j1}=i} \lambda_j \tau_{ij} \qquad \forall i \quad (3)$$

$$\tau = \sum_{j=1}^{n} \frac{\lambda_j}{\lambda} \tau_{\alpha_{j1},j} \quad (4)$$

These equations basically make this estimation through the values of the total demand λ and individual node demand λ_j, as well as the expected service time of server i on point j, represented by τ_{ij}. Note that this evaluation uses yet a variable α_{jl} that defines the index of this l^{th} preferred server for responding a demand at node j.

After this first step, we can estimate ρ (as aforementioned), P_0 and P_m, following the *Erlang's Loss Formula* for $M/M/m/K$ queues, with $K = m$.

Then, we use the Q factor value to update ρ_i values, making it equal to $\frac{V_i}{V_i+1}$, where V_i is defined as bellow:

$$V_i = \sum_{k=1}^{m} \sum_{j:a_{jk}=1} \lambda_j \tau_{ij} Q(m,\rho,k-1) \prod_{l=1}^{k-1} \rho_{a_{jl}} \quad \forall i \quad (5)$$

If the maximum variation between the previous and the current values of ρ_i is lower than a specified small

$\varepsilon > 0$ (in this implementation 0.10), the method stops. Otherwise, f_{ij}, the probability of a server i to be assigned to respond a demand at node j, is evaluated, through Equation (6) and normalized with Equation (7). For calculating this probability, it is necessary to know that the server i is the k^{th} preferred server to respond a demand at node j and the value of its busy probability ρ_i.

$$f_{ij} = Q(m,\rho,k-1)(1-\rho_i)\prod_{l=1}^{k-1}\rho_{\alpha_{jl}} \qquad \forall i,j \tag{6}$$

$$f'_{ij} = f_{ij}\frac{1-P_m}{\sum_{i=1}^{m} f_{ij}} \tag{7}$$

Finally, τ in Equation (8) value is updated, and the method goes to a new iteration, beginning from the update of $Q(m,\rho,k)$ factors.

$$\tau = \sum_{j=1}^{n}\frac{\lambda_j}{\lambda}\left[\sum_{i=1}^{m}\frac{\tau_{ij}f_{ij}}{1-P_m}\right] \tag{8}$$

Once the method converges, the $Q(m,\rho,k)$ factor and ρ_i values are used as a good approximation to the HQM.

4 MATERIAL AND METHODS

4.1 Police Units Allocation Model

The Police Units Allocation Model used here was previously proposed by Mendes e Santos (Mendes and dos Santos, 2015) for defining an allocation of diverse kinds of police units in an urban area for maximizing the profit related with the allocation. It is a version of the Maximal Coverage Problem with mandatory closeness constraints, defined by Church e ReVelle (Church and ReVelle, 1974) that supports a set Q of unit types with different coverage radius.

In this model, the city is defined as a graph with a set of segment streets E and corners V. Each segment $r \in E$ has a length and a covering profit. The time spent to travel from a corner to other depends of factors like distance, speed of the type unit, traffic sense, period of the day, passage forbidden for the units, among others, provided in advance, or obtained in a real time flow through of softwares that could colect all this data.

The decision variable x_{ij} defines how many units of type i are located at corner j. The auxiliary binary variables a_r and a'_r define if the street segment r is reached by any assigned police unit in a time lower than T_{MAX} and $2T_{MAX}$ respectively, where T_{MAX} is a travel time limit from the position of any unit position to a point to be considered covered, and $2T_{MAX}$ is the maximal distance allowed from a point to the closest unit.

The model is presented bellow:

$$\max Z = \sum_{r\in E} l_r a_r \tag{9}$$

$$\sum_{j\in V} x_{ij} \le U_i, \qquad \forall i \in Q \tag{10}$$

$$a_r \le \sum_{i\in Q}\sum_{j\in V} p_{rji}x_{ij} \qquad \forall r \in E \tag{11}$$

$$a'_r \le \sum_{i\in Q}\sum_{j\in V} p'_{rji}x_{ij} \qquad \forall r \in E \tag{12}$$

$$\sum_{r\in E} a'_r = |E| \tag{13}$$

$$x_{ij} \in \mathbb{Z}^+, \qquad \forall i \in U, j \in V \tag{14}$$

$$a_r \in \{0,1\}, \qquad \forall r \in E \tag{15}$$

$$a'_r \in \{0,1\}, \qquad \forall r \in E \tag{16}$$

The objective function (9) maximizes the sum of the profits l_j of covered street segments a_j. Constraint (10) states that the number of units of each type allocated cannot exceed U_i, the number of units of type i available. In constraints (11) and (12), the parameters p_{rji} and p'_{rji} defines if the time spent to reach a street segment r with an unit of type i located at j is lower than T_{MAX} and $2T_{MAX}$ respectively, determining which streets segments are covered. In constraint (13), the mandatory closeness is defined, stating that all nodes should be reachable from an unit location in a time lower or equal than $2T_{MAX}$. Finally, the others constraints (14), (15) and (16) define the valid values of the variables.

4.2 Minimum Expected Response Location Problem

The term Minimum Expected Response Location Problem (MERLP) is a denomination created by Rajagopalan and Saydam to the models presented by them for addressing the problem ambulance allocation in an urban area (Rajagopalan and Saydam, 2009). In this section we show a version of the model called by them MERLP_2 in their paper, chosen due to the fact of being the queuing model with greater resemblance with the police units allocation model described on the previous section among the models known by the authors. Some corrections are also presented.

In MERLP, m non distinguishable ambulances must be allocated in n points of an area. This allocation is used to define, for each point, a preference order to dispatch an ambulance from its position when a service is required at that point.

Once are available: the number of ambulances, the position of those ambulances, the dispatching preferences for each client and the expected response service times for each pair of server/client; the Jarvis Approximation can estimate the values of the Q factor and the busy probability p_k of ambulance k. The model can be thus defined as following:

$$\min Z = \sum_{j=1}^{n} \sum_{k=1}^{m} d_{jk} h_j y_j Q(m, \rho, k-1)(1-\rho_{\alpha_{jk}}) \prod_{l=1}^{k-1} \rho_{\alpha_{jl}} \tag{17}$$

$$\left[1 - \prod_{k \in N_j} \rho_{\alpha_{jk}} Q(m, \rho, \Gamma_j - 1)\right] \geq \alpha y_j \quad \forall j \tag{18}$$

$$\sum_{l=1}^{n} \sum_{k \in N_j} x_{lk} = \Gamma_j \quad \forall j \tag{19}$$

$$\sum_{j=1}^{n} h_j y_j \geq c \tag{20}$$

$$\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = m \tag{21}$$

$$x_{ij} \in \{0,1\} \quad \forall i,j \tag{22}$$

$$y_j \in \{0,1\} \quad \forall j \tag{23}$$

$$\Gamma_j \in \mathbb{Z}^+ \quad \forall j \tag{24}$$

Two decision variable sets are used in MERLP. The first one, y_j, defines, for each point j if it is covered (set as 1) or not (set as 0). The second one, x_{jk}, defines if a server k is located at point j.

In the objective function (17), the distance from an ambulance to a point is multiplied by the probability that the ambulance will respond a call from that point and the fraction of calls comming from that point, represented by the parameter h_j. This product is accounted in the objective value function if the point is considered covered, which is defined by the value of variable y_j.

By its turn, in (18), for each point, the product of busy probability of all ambulances that can cover that point is calculated, multiplied by the Q factor. The result corresponds to the probability of not covering that point. Then, if, and only if, the probability of covering is greater than an α reliability level predetermined, the point is said covered.

The constraint (19) defines the number of servers that can cover each node from their position Γ_j. In the constraint (20) the minimum fraction of calls c that must be answered with reliability rate α is set. Finally, the constraint (21) defines that exactly m servers will be used.

4.3 MERLP With Mandatory Expected Closeness Constraints

The model proposed here is called Minimum Expected Response Location Problem with Mandatory Expected Closeness Constraint (MERLP-MECC). This model basically is the MERLP with the set of constraints (26) to (31) shown bellow and replacement of third parameter of Q factor on constraint (18) by $\Gamma_j - \kappa_j$, generating the equation (25). In other words, is a merge of the models presented at sections 4.1 and 4.2, being more similar to this last. This model improves MERLP by providing a better general area coverage, instead of a coverage with lower distance between the points and lower demand, just the enough to satisfy the constraint of minimal coverage (20).

$$\left[1 - \prod_{k \in N'_j} \rho_{\alpha_{jk}} Q(m, \rho, \Gamma_j - \kappa_j)\right] \geq \alpha y_j \quad \forall j \tag{25}$$

$$\left[1 - \prod_{k \in N'_j} \rho_{\alpha_{jk}} Q(m, \rho, \Gamma_j - \kappa_j)\right] \geq \beta y_j \quad \forall j \tag{26}$$

$$\kappa_j \leq \Gamma_j \quad \forall j \tag{27}$$

$$\Gamma_j \leq M \kappa_j \quad \forall j \tag{28}$$

$$\sum_{j=1}^{n} y'_j = n \tag{29}$$

$$\Gamma_j \in \mathbb{Z}^+, \quad \forall j \tag{30}$$

$$\kappa_j, y'_j \in \{0,1\}, \quad \forall j \tag{31}$$

Now, besides of constraint (18) of MERLP there is a second constraint (26) for defining which vertex are covered. The covering verified at (26) is equivalent to that verified in (12) and is used for stating which node have an expected coverage greater than a β value (such as $\beta \leq \alpha$) in a time lower than $2T_{MAX}$. With the constraint (26), it is defined that for all j, y'_j must be equal to one, creating a mandatory expected closeness constraint (29).

Finally, constraints (27) and (28) define the value of the variable κ_j, created for avoiding the third argument of Q factor assume a negative value when the sum of constraint (19) is equal zero. On equation (28) a parameter M is set as being any big number greater than m.

4.4 VND Heuristic

This section presents the algorithm used for solving the MERLP-MECC. As mentioned in (Rajagopalan and Saydam, 2009), commercial solvers such as CPLEX are unable to solve MERLP or models derived from it. To deal with this, we use a *Variable Neighboorhood Descent - (VND)* heuristic (Mladenović and Hansen, 1997) (Talbi, 2009, p .150).

In our approach, three local searches are available and are selected according to the number of iterations without improvement of the best global objective value. These local searches were inspired on those tested by (Mendes and dos Santos, 2015), which have reached good results in exploring the solutions space. In the best of our knowledge and belief, there are no studies using this metaheuristic to stochastic coverage problems similar to this one, although it was successful used on some location problems as cited in (Glover and Kochenberger, 2003).

The Algorithm 1 describes the overall scheme of the VND algorithm here implemented. As can be seen at line 1, the execution starts with a function for initializing the solution. This function may build a solution in a randomized way, choosing where each unit will be located, unit by unit. A variant proposed uses the tabu search heuristic proposed by (Mendes and dos Santos, 2015) for solving the coverage model of section 4.1, in a strategy similar to that employed by (Rajagopalan and Saydam, 2009) for finding an initial feasible solution.

Algorithm 1: VND Heuristic Pseudocode.

```
1:  s* ← initializeSolution()
2:  s ← s*
3:  NoImprove ← 0
4:  bestObj ← Evaluate(s*)
5:  while NoImprove < Max_No_Improvements do
6:      NoImprove++
7:      if NoImprove < 0.8*m then
8:          LSearchOne(s, s*, NoImprove, bestObj)
9:      else if NoImprove ≤ 1.4*m then
10:         LSearchTwo(s, s*, NoImprove, bestObj)
11:     else
12:         LSearchThree(s, s*, NoImprove, bestObj)
13: return s*
```

At line 5, there is a loop controlled by the number of iterations without improvement of the global best objective value. The *Max_No_Improvements* value used on final tests is equal to $2m$.

For selecting which local search to execute on each iteration, the number of iterations without improvements is used again, as can be seen at lines 7, 9 and 11. The values chosen as limit for changing the local search used, as the stop criteria described on previous paragraph, were defined after some preliminary runs. The objective was to get low run times, without getting significantly worse solutions.

The local search execution order was also defined with the objective of reducing the run time, verified on preliminary runs. It does not follow the common rule of the VND, that uses broader and more intensive local searches in advanced steps. The Local Search Two, for instance, explores a more tight neighborhood than Local Search One. It is useful for bringing improvements on the good solutions already found by Local Search One, that has no mechanisms to correct small imperfections. In another hand, the Local Search Three is a more intensive and broader local search than the previous, following the VND traditional behavior.

Local Search One does the following: selects a unit u by random and then selects 10 nodes from the set of those that can be reached in a time lower than T_{MAX} from the position of u. After to put the unit u in each of these then nodes, the node where the best objective function value was found is choosen as the new location of unit u. If the lowest value found in this local search is lower than the best global objective function value, this value is set as the new best global objective function, as well as the solution is set as the best one, and the variable counting the number of iterations without improvements is reset to zero.

The Local Search Two uses a similar strategy, but selecting just the adjacent nodes of current position of the server selected. It evaluates all adjacent nodes, instead of a subset of them, which differs from Local Search One.

The Local Search Three is similar to Local Search One. The differences are the number of nodes selected to the local search (15 now, instead of 10) and definition of node set from where these 15 nodes will be selected, which now is the node set reachable within $2T_{MAX}$.

Unfeasible solutions are penalized in two ways, depending of which constraint is not satisfied. When constraint of minimal coverage (20) is not satisfied, the objective value is multiplied by the ratio of desired coverage c and the reached coverage of the solution. When the mandatory closeness constraint (29) is not satisfied, the objective value is multiplied by the product $m*n$ and divided by number of nodes covered following the conditions of (26). This penalization is stronger because our intention is to give priority to satisfy more quickly the mandatory closeness constraint.

5 EXPERIMENT DESCRIPTION

For testing the efficiency and efficacy of the VND heuristic to solve the MERLP-MECC, we build an instance set based on real street track data of Brazilian city of Viçosa, Minas Gerais state, with a population around 90 thousand people.

The city has its street track data modeled as a graph, where each street segment is an edge and each corner a vertex, summing up 5100 edges and 2125 edges. Each edge received a random weight, meaning the demand on that edge. However, once the model proposed considers vertex demands instead of vertexes demands, they were defined as the sum of incident edges weight, divided by two, to avoid a doubled counting.

Regarding to the servers, some configurations of numbers of units available were defined. Aside this, their coverage radius was defined as being approximately the distance which they could reach in a time interval lower than four minutes, following statistical data obtained in (Coupe and Blake, 2005) about effects of quick responses in arrests after burglaries.

Considering the service time as non negligible at demand point, situations with service time fixed in 15 and 30 were tested. The default total demand was fixed from medium to low levels, and as a Poisson random variable, with $\lambda = 7$ and $\lambda = 15$.

For each instance and method tested, 40 repetitions of execution were performed. The tests were executed in a computer with processor Intel Core i5-3300 with 3.00GHz, 8GB of RAM and Windows 8.

6 RESULTS AND DISCUSSION

Three main measures were chosen for evaluation of the heuristic quality: objective function value, feasibility and run-time. It is important to say that there is a strong relation between objective function value and feasibility. This relation is discussed in along the text.

In the first subsection, the results with standard conditions are described, comparing the efficiency of each method for initializing the VND heuristic. After that, once that is confirmed a better performance of tabu search, we describe the solutions obtained in more realistic scenarios, with higher demands and services times.

6.1 Standard Conditions

In the tests described in this section the action radius of motorcycles is 2,6km; of cars is 2,0km and of pedestrians units is 0,8km.

Regarding the method of initialize solutions, a detailed comparison between the results found by using each one of them are presented on Table 1. The two first columns describe the instance, being the first the number of units available (pedestrians(P), motorcycles(M) and cars(C) respectively), the second defines the constants of MERLP-MECC, and the following the results associated with those instances.

The first point of comparison is the number of feasible solutions (columns V) found by each method. What can be seen in this sense is a proximity of performance. In ten of eighteen instances, the tabu search got a higher value on column V. The random initialization got the better value seven times and in only one had a tie. This equality can be explained by looking to the columns V_α and V_β, that represents the number of runs where solutions satisfying the constraint of minimum coverage and mandatory closeness were obtained, respectivelly. While the tabu search initialization has better results in almost all instances, considering the V_β column, it does not maintain this quality when the column V_α is observed.

The result described above was partially expected. Firstly, because the tabu search aims, in some way, to satisfy the mandatory closeness constraint (29) of the MERLP-MECC, once it needs to respect the mandatory closeness constraint of Police Units Allocation Model. When the algorithm do this, it can deliver a solution far from satisfying the constraint of minimum coverage (20).

We can see also in Table 1 an equilibrium among the best feasible solutions found by each method (columns $min_{OF}\ feasible$). Beside this, we observe that in the majority of the instances, in special on those with motorcycles available, the average run-time (μ_t) of VND with tabu search initialization was close to the version with random initialization. This indicates that the additional time spent on execution of tabu search heuristic is not significant and preserves the low average run-time.

This scenario of equality seems to disappear when the average objective value found by each initialization method is observed. The values obtained using the tabu search initialization were significantly better in all instances with no motorcycles. In the remaining instances, the results were almost equal in four of them, with a slightly advantage to random initialization.

Both results presented on the last two paragraphs can be attributed to the better performance of tabu search in satisfying the mandatory closeness constraints (29), that is more penalized when not satisfied. It is notable yet that once the motorcycles are

Table 1: Comparison of results found by the VND heuristic with different initializations.

Instances		Random initialization						Tabu search initialization					
P/M/C	$\alpha;\beta;c$	μ_{OF}	min_{OF} $feasible$	μ_T	V_α	V_β	V	μ_{OF}	min_{OF} $feasible$	μ_T	V_α	V_β	V
7/0/7	0,90 ; 0,50 ; 60	9407,5	1265,8	6	16	22	13	3771,6	1422,8	13	20	32	18
	0,90 ; 0,50 ; 80	9065,8	1882,0	8	4	23	3	4185,5	1758,4	14	3	34	3
	0,95 ; 0,60 ; 60	9619,6	1427,1	6	9	21	8	4935,7	1058,7	11	11	30	11
	0,95 ; 0,60 ; 80	9039,7	1943,1	7	2	26	2	4143,4	-	13,7	0	32	0
	0,99 ; 0,75 ; 60	7121,0	1243,9	5	13	27	12	2848,5	1164,7	13	18	32	18
	0,99 ; 0,75 ; 80	11764,0	1819,2	7	2	22	2	3527,0	1741,5	13	3	36	3
8/0/8	0,90 ; 0,50 ; 60	6952,1	1356,8	8	20	19	17	1584,8	1207,4	13	19	40	19
	0,90 ; 0,50 ; 80	10797,2	2073,2	9	3	25	3	1796,2	1665,4	16	4	40	4
	0,95 ; 0,60 ; 60	6430,5	1258,2	7	24	30	19	2117,7	1252,0	14	16	39	16
	0,95 ; 0,60 ; 80	10742,3	1610,6	9	7	26	6	2717,1	1829,3	18	3	39	3
	0,99 ; 0,75 ; 60	7020,7	1128,4	7	15	29	15	1470,3	1154,7	14	17	40	17
	0,99 ; 0,75 ; 80	11839,7	1694,8	9	8	24	8	3041,3	1830,2	17	5	37	5
5/5/5	0,90 ; 0,50 ; 60	1877,0	1243,6	8	36	40	36	348,8	1263,6	14	37	40	37
	0,90 ; 0,50 ; 80	1967,0	1746,6	13	19	40	19	1998,0	1811,2	15	20	40	20
	0,95 ; 0,60 ; 60	1785,2	1511,6	9	35	40	35	1941,0	1284,6	13	33	40	33
	0,95 ; 0,60 ; 80	1954,5	1894,0	9	14	40	14	2045,8	1830,5	13	21	40	21
	0,99 ; 0,75 ; 60	1791,7	1351,6	6	30	40	30	2464,3	1511,7	10	28	39	28
	0,99 ; 0,75 ; 80	1931,6	1702,7	11	15	40	15	1997,6	1877,5	14	13	40	13

included there is a high improvement on quality of solutions obtained through the random initialization, while this does not happen with solutions provided by the VND when initialized with tabu search.

For assuring definitely the difference of performances among the two methods of initialization, a two-way ANOVA test with repetitions was performed. With these data, when compared the overall average objective function value of solution, better values were found when tabu search initialization was used. Beside this, a p-value around $5,0 * 10^{-29}$ was found, with a critical value of F equals to $3,85$ and F equals to $130,8$, conditions that are sufficient to discard the hypothesis of results equality.

6.2 Results with Demand and Service Time Variations

After we have certified the efficiency of tabu search initialization, in this section we describe the results of the sensitivity tests performed. These tests were done to observe the changes on solutions quality caused by variations on demand and service time values.

A first overview of results is presented on Table 2. In this table the solutions obtained with the default demand ($\lambda = 7$) and doubled demand ($\lambda = 15$) are compared.

One surprisingly result was referent to the number of feasible solutions obtained. In ten of eighteen instances more feasible solution on runs with demand doubled were found and, in another two there was a tie. Looking to the components of these numbers (columns V_α and V_β), we can note that the values of V_β have dropped, and mainly, the values of V_α have increased. It maybe did the likelihood of a solution to be feasible improves.

However, this result is not reflected on the average values of objective function. Observing this measure, the values obtained on runs with doubled demand were always worse. It is an effect of the higher distance traveled for responding the calls for service.

When we look to the minimum objective value among the feasible solutions we see that the increase of demand makes the objective values worse. This does not mean, necessarily, that the solutions are worse. The same allocation can have different objective values with different total demands due to the impact on the value of Q factor, that depends indirectly of value of λ. In the other hand, it is important to note that the parameter h_j continues with the same values if the edges have their demands multiplied by the same factor, as we done, because it represents a fraction of total demand instead of an absolute value.

In the following tests, we abandoned the simplification of considering the service time equals to the travel time from the unit location to the demand node. The doubled demand was kept for providing a description of a more realistic scenario.

Two values of average service time were used, 15 minutes ($1/4$ hour) and 30 minutes ($1/2$ hour). Those times were added to the travel time from the unit position to the demand node and was not accounted the time spent to return.

On the Fig. 1 a comparative of the number on feasible solutions, as such the number of solutions satisfying the constraints of minimal coverage (20) and

Table 2: Comparison of results found by the VND heuristic with different levels of demand.

Instances		Default demand						Doubled demand					
P/M/C	$\alpha;\beta;c$	μ_{OF}	min_{OF} $feasible$	μ_T	V_α	V_β	V	μ_{OF}	min_{OF} $feasible$	μ_T	V_α	V_β	V
7/0/7	0,90 ; 0,50 ; 60	3771,6	1422,8	13	20	32	18	9525,4	1573,5	11	18	21	18
	0,90 ; 0,50 ; 80	4185,5	1758,4	14	3	34	3	7885,5	-	12	0	29	0
	0,95 ; 0,60 ; 60	4935,7	1058,7	11	11	30	11	6873,5	1973,5	12	24	30	24
	0,95 ; 0,60 ; 80	4143,4	-	14	0	32	0	9772,3	3636,1	11	3	28	3
	0,99 ; 0,75 ; 60	2848,5	1164,7	13	18	32	18	11189,3	2831,8	12	21	25	19
	0,99 ; 0,75 ; 80	3527,0	1741,5	13	3	36	3	11395,4	2961,0	11	3	24	3
8/0/8	0,90 ; 0,50 ; 60	1584,8	1207,4	13	19	40	19	4485,5	1587,4	15	25	33	25
	0,90 ; 0,50 ; 80	1796,2	1665,4	16	4	40	4	6178,7	2966,7	14	5	29	5
	0,95 ; 0,60 ; 60	2117,7	1252,0	15	16	39	16	8347,1	1012,1	14	23	26	22
	0,95 ; 0,60 ; 80	2717,1	1829,3	18	3	39	3	7425,0	2764,3	15	7	30	7
	0,99 ; 0,75 ; 60	1470,3	1154,7	14	17	40	17	4111,0	1551,1	14	23	30	23
	0,99 ; 0,75 ; 80	3041,3	1830,2	17	5	37	5	8251,8	2572,0	13	8	27	8
5/5/5	0,90 ; 0,50 ; 60	1948,1	1263,6	14	37	40	37	3099,8	1174,4	15	36	39	36
	0,90 ; 0,50 ; 80	1998,0	1811,2	15	20	40	20	2978,3	2878,2	16	12	39	12
	0,95 ; 0,60 ; 60	1941,0	1284,6	13	33	40	33	2909,6	1862,0	16	31	38	31
	0,95 ; 0,60 ; 80	2045,8	1830,5	13	21	40	21	3099,5	2675,4	15	16	39	16
	0,99 ; 0,75 ; 60	2464,3	1511,7	10	28	39	28	3620,7	1094,3	14	36	37	35
	0,99 ; 0,75 ; 80	1997,6	1877,5	14	13	40	13	2785,4	2837,5	13	8	37	8

mandatory closeness (29) is presented. Each bar color represents a different average service time.

The instances without motorcycles (with IDs from 0 to 11) presented low levels of satisfactibility of minimum coverage constraint (20). These levels do not decreased much with the rise of service time, being the higher variation a decrease of $37,5\%$ when compared the first and third column, at instance 11. However, in the instances with motorcycles, the variations were bigger, with decreases of at least 25%, except to the instances 14 and 17, on the second column.

The three central columns of Fig. 1 present the number of solutions satisfying the mandatory closeness constraint (29). As we can see, these numbers are more regular than those related to the first three columns. They are no just higher, but neither have big oscillations between the instances. Although the scores were higher in instances with motorcycles (IDs 12 to 17) with the default service time, this difference become irrelevant in tests with 30 minutes of service time.

This result lead us to an intuitive conclusion: as longer the average service time, less relevant the type of vehicle used by the unit, because the travel time starts to be just a minor variable. It does not mean, however, that the location of units also becomes less relevant. It is desirable to put more units for covering places with higher demand, even when the speed of these units does not impact significantly the total service time.

The number of feasible solutions (showed on the last three columns) follows approximately the numbers of the three first columns and it is not much influenced by the three central columns. This result suggests that the number of solutions respecting the constraint of minimal coverage is the bottleneck of solutions feasibility.

7 CONCLUSION

In this paper we addressed the police unit allocation model, presenting a hypercube queuing model to describe it and a VND heuristic approach to solve this model. Our objective was mainly show the suitability and viability of using the proposed model (MERLP-MECC) and the hypercube queuing approach to describe a realistic scenario of police unit allocation where mandatory closeness constraint are presented.

Considering this objective, we can state that this model is able to deal with a realistic scenario of locating police units. The presence of feasible solutions on many of situations tested and the low run time spent to find them are evidences sufficiently strong to confirm this statement.

Regarding to the tests of VND efficiency to solving the model, basically three measurements were considered: run-time, feasibility and objective function value. At the majority of the analysis, we focused the solutions feasibility and its influence on average objective function values.

Although we have found satisfactory values in both measures to some instances, few feasible solutions were found.

The problem with feasibility of solution, however, was already expected, once it was reported by (Ra-

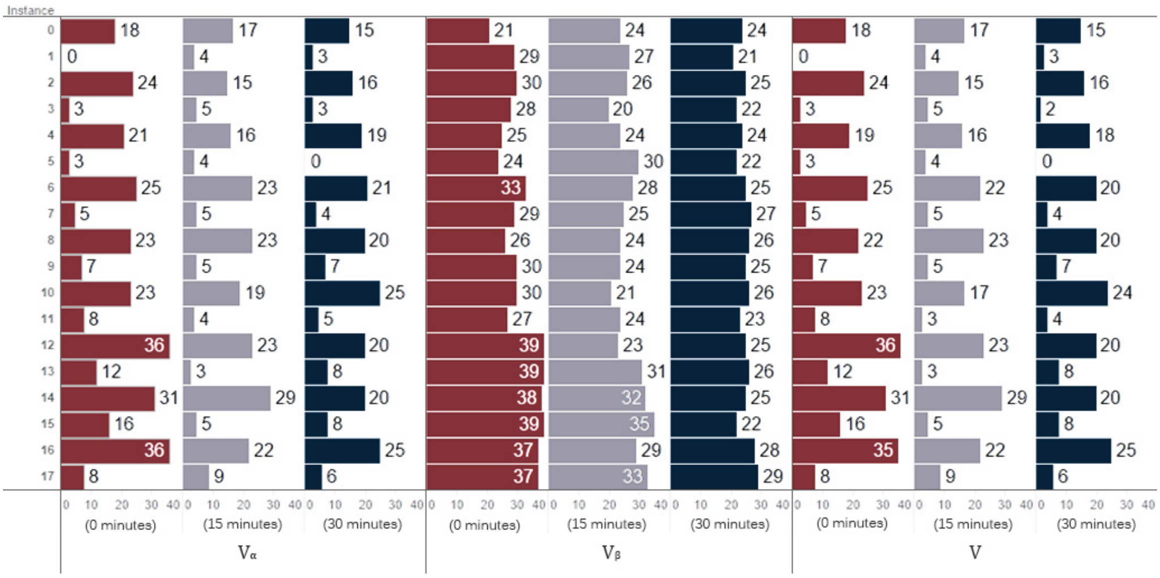

Figure 1: Number of feasible solutions got by VND with Tabu Search initialization using different services times.

jagopalan and Saydam, 2009) which introduced the MERLP, that was used as base to the MERLP-MECC. On our paper, whose model contains more constraints and the instances have tights action radius, this difficult would naturally appear. Beside this, as we have mentioned on previous section, the number of solutions satisfying the minimum coverage constraint can be a bottleneck for getting feasible solutions. Our intuition says that maybe through a strongest penalty when the minimum coverage constraint was not respected this problem could be solved. This hypothesis is a point to be tested on future works.

We highlight however that the feasibility, although desirable, is not always mandatory for our proposes. Almost feasible solutions could be satisfactory and some relaxations can be adopted on real situations, without a degradation of coverage quality, as for example, the lowering the α or β values, to adapt to some less probable scenarios. Some authors have used goal programming (Saladin, 1982) and fuzzy programming approaches (Araz et al., 2007) for dealing specifically with those situations.

We have seen also that the results obtained here were not so dependent of demand rates but, in another hand, the service time has a big impact on solutions quality. With the absence of these information, estimates may be done based on the literature, as we did, but if they were not near from real numbers, the efficiency of model to suggest a good allocation can be strongly affected.

Another question, that was not dealt on the text, but can be problematic is the imbalance of workload. The MERLP, as the MERLP-MECC, does not make any consideration about balancing the workload. This could be done with additional constraints on the model or a multi-objective approach, which can be a good source for future researches.

Another possibility is to built new models that not just states where police units should stay, but how they could patrol areas close to where they were put for optimizing some objective. Integrated models like this, that joins a coverage and a routing problem are still rare on the literature and can be explored starting from good deterministic or stochastic models.

Finally, a possibly more useful and hard to accomplish continuation to this research would be a study of case of implementation of this model in a city. This last suggestion is more challenging due to the ethical issues and bureaucracy involved.

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