Computer Modeling and Programming in Algebra

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Abstract: This paper introduces a novel approach to providing high school students with access to computer science experiences as part of an Algebra unit on linear functions. The approach is being developed and tested as part of a funded National Science Foundation study. The unit piloted in the study integrates computational thinking and computer modeling into a project-based Algebra unit on linear functions. Literature on computational thinking, access to computer science in secondary settings, modeling approaches, project-based learning, and design-based research is described to provide a rationale for the study design. The ultimate goal of the study is to develop a paradigm for integrating computer science experiences into algebra as a way to increase engagement in STEM and computing among students from all backgrounds.

1 INTRODUCTION

Whereas algebra has long been discussed as a "gatekeeper" to college-preparatory mathematics tracks (Spielhagen, 2006), we argue for repositioning algebra as a *gateway* both to college and to STEM and computing careers. This paper describes a novel approach for incorporating computer modeling and programming into a project-based exploration of algebra using engineering applications. The approach is currently being evaluated through a two-year National Science Foundation funded pilot study that proposes to increase student understanding of functions and integrate 21st-century skills into classroom experiences through the strategic infusion of computational thinking (CT).

Although computational thinking has been defined in various ways (Grover and Pea, 2013), in this pilot study, teachers and students develop an understanding of computational thinking as a way of creatively approaching tasks using fundamental concepts from computer science (Barr et al., 2011). This study leverages the power of computational thinking for 21st-century learning by piloting a manageable yet compelling integration of modeling and computer programming into a project-based exploration of linear functions using engineering applications.

2 CONCEPTUAL FRAMEWORK

The approach taken in the study's unit reflects the benefits of context-based experiences with mathematics (Bickmore-Brand, 1993) and follows a Guided Inquiry and Modeling Instructional Framework (EIMA) with the following progression: Engage, Investigate, Model, and Apply (Schwarz and Gwekwerere, 2006). Although based on earlier research on learning progressions and teaching cycles (Bybee, 1997), EIMA goes beyond discovery to position the creation, revision, and application of models as the focus of inquiry. Further, EIMA was explicitly developed to pave the way for student engagement with "computer models and simulations that are central in modern science and engineering" (Schwarz and Gwekwerere, 2006: 160).

EIMA effectively sets the stage for students' encounters with computer science. The project is developing an Interactive Computer Modeling (ICM) platform based on SAGE (Software for Algebra and Geometry Experimentation) and the Python computer language. SAGE, an open-source program by Stein (2008), can be used to generate models using data gathered from their observations. SAGE is a powerful tool for approaching mathematical tasks from a computational perspective. Its web-based platform (Notebook) allows users to enter equations and data using a

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command-line interface, and a graphical interface allows users to visualize and interact with data (Gray, 2008). The project incorporates into SAGE a set of user-friendly interfaces that will allow students to easily manipulate variable amounts. Consequently, when making predictions, they can quickly observe the link between algebraic and the graphical representation of functions. A robust body of research suggests that creating and manipulating dynamic models may enhance both student understanding of mathematical concepts (in this case, linear functions) and their ability to use modeling strategically in a mathematics context (Borba and Villareal, 2006; Zbiek and Conner, 2006).

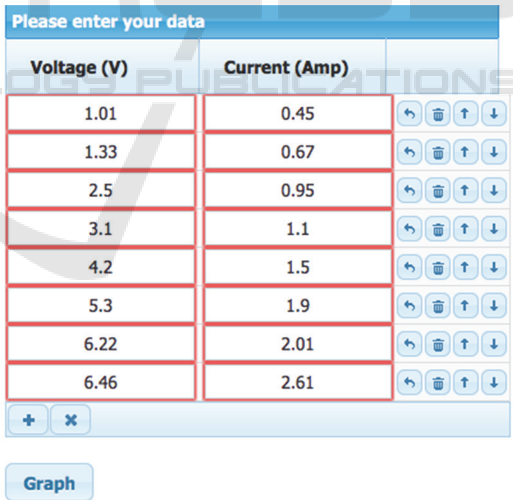
Historically, efforts to improve mathematical problem solving have been limited by an overemphasis on heuristic strategies at the expense of the metacognitive skills that are needed to manage the application of these strategies (Lester, 1983; Schoenfeld, 1983). By contrast, EIMA provides an ideal context for engaging computational practices (such as effective abstraction and iterative approaches to problem-solving) as well as computational perspectives that encompass the attitudes and dispositions of programmers, including confidence and persistence in the face of complex problems, tolerance for ambiguity, resourcefulness in the face of open-ended problems, and a capacity for cooperation with others in the pursuit of a common goal (Barr et al., 2011). The need for growth in this area is demonstrated by U.S. students' relatively weak performance on international assessments such as PISA where they are asked to model real-world situations in multi-step problems (Organization for Economic Co-operation and Development, 2012).

3 PROJECT GOAL

The research project seeks to construct a learning environment that effectively integrates computer modeling and programming into a project-based algebra unit on linear functions. To accomplish this goal, we are developing a project-based algebra unit that uses computer modeling and programming to explore engineering applications involving linear functions. Next, we are designing a 10-day summer STEM+C Institute to support the unit's implementation by math educators from secondary schools. Researchers and graduate research assistants will document teachers' engagement in the summer institutes, gather data on the implementation of the unit, and assess teacher and student outcomes using pre- and post-tests, interviews, and other data sources. Following the pilot implementation of the unit, participating teachers will engage in a 5-day STEM+C Institute II to explore data from the study and examine student work. Researchers will assess the effect of the modeling and programming unit on teachers' and students' understanding of functions, problem-solving practices, persistence, and computational thinking.

4 THE UNIT DESIGN

The proposed unit opens with an engagement activity that allows the students to discover the needed components of a circuit by attempting to use a battery and wires to light a bulb. This pre-activity focuses the students on the observation of a phenomenon in the world before they consider a mathematical representation (Sullivan, 1997). Once students determine how a circuit must be physically connected, they can be introduced to electrical meters that will allow them to generate a table of findings focusing on voltage and amperage. The students will be prompted to enter their data into the project's newly ICM platform (see figure 1).


Please enter your data

Voltage (V)	Current (Amp)
1.01	0.45
1.33	0.67
2.5	0.95
3.1	1.1
4.2	1.5
5.3	1.9
6.22	2.01
6.46	2.61

Graph

Figure 1: Data Entry Table.

The platform will produce a graph of the situation based on the data collected (see figure 2). The students will be asked to use these findings to model their observations by developing a function that will allow them to be predictive of what is happening in the circuit. When prompted to model their observations by developing linear equations (Ohm's law), some students will produce equations that keep voltage constant while varying 1/resistance (slope). Others will produce equations where resistance is the constant and voltage varies (slope). Students then enter the equations into ICM platform to generate interactive graphical representations that allow them to modify slope or resistance using sliders to further develop their understanding of the relationship between observed phenomena, algebraic and graphical representations of these phenomena (see Figure 2).

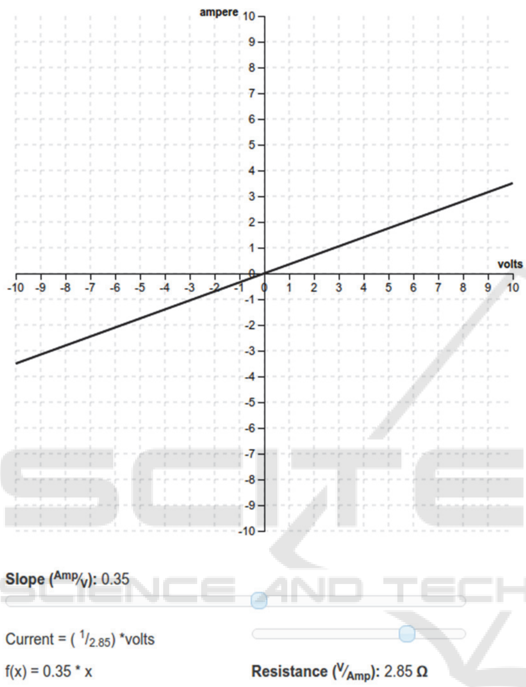


Figure 2: Graph representing the data.

The final stage of the opening experience is for students to discuss and make observations about *how* the ICM platform produces these graphs by examining the “backside” code in Python that makes them possible. The unit will scaffold the students in production of coding sequences starting with iterative loops that will allow the students to solve real-world engineering application problems that would be tedious to solve by hand. This will give the students the opportunity to experience coding in an unintimidating environment using a platform that is used for science and engineering applications.

The next phase of the unit involves students in further explorations using the ICM platform and scaffolded programming in Python using application-oriented exploratory STEM exercises and tasks modified from *Python programming: an introduction to computer science* (Zelle, 2010). These tasks are framed within the context of engineering applications further exposing students to STEM careers though engineering based scenarios that drive the students towards solving the problem based scenario using mathematical modeling and computer programming. Materials for these exercises build context for exploration of programming environments by highlighting actual uses for Python in the real world. As the students explore the engineering application activities, they will need to make predictions based upon their mathematical models thus showcasing and developing elements of computational thinking. Ultimately, the unit task shows how decision making in math can be used in real world applications to make educated decisions.

5 METHODS

The project employs design-based research methods that deliberately intertwine the design of innovative learning environments (in this case, the programming-infused algebra classroom) and the development of a theory of learning to generate relevant implications for practitioners and other research designers (DBRC, 2003). In this exploratory study, we follow the approach of progressive refinement (sometimes called iterative design) to revise both the learning environment and the theory of learning through cycles of design, implementation, analysis, and revision (Cobb, 2001).

5.1 Instruments

As shown in Table 1 and Table 2, we will collect data from multiple sources, including videos of teachers during STEM+C Institutes, classroom videos, semi-structured interviews, pre- and post-tests with open-ended questions designed to provide insights into learners’ thinking, and Likert-scale surveys to assess perceptions of programming and STEM fields (Bannan, 2007). Triangulating findings among various data sources and conducting preliminary analysis after each implementation cycle will enhance the reliability and validity of the study’s findings (Cobb and Gravemeijer, 2008).

When possible, we are using already constructed and validated instruments. However, in several instances the research team is constructing and validating instruments for specific purposes. In the case of the unit for students, the team will be designing minimally worked problems to use during semi-structured interviews. The minimally worked problems are incomplete representations of real world engineering problems that also have incomplete Python programming representations. The students will talk aloud as they work though problems, allowing researchers to observe their computational thinking.

Table 1: Data Sources by Student Participants.

Data Sources	Focus of coding/analysis
Pre/Post content tests	Understanding of function and programming – yet to be developed
CT STEM Attitudinal Survey (Weintrop et al., 2014)	5-pont Likert scale survey focused on attitudes towards CT and STEM; confidence in these subjects; and interest in fields related to computation
Semi-Structured interviews	Focus on perceptions of math and computer programming and knowledge of linear functions, CT and modeling through the use of Minimally Worked Problems (MWPs)
Videos of student work groups	Discourse around functions, CT, and modeling

Teachers will take a series of assessments that not only focus on their understanding of the content being covered but also their self-efficacy towards mathematical modeling, computer programing and functions. There are few instruments that are already validated that suit the needs of the study. The project team is currently working on validating a 46-question Likert scale survey focusing on teacher understanding of what constitutes a mathematical modeling task and on teacher perceptions of obstacles and supports that either discourage or encourage teachers' use of mathematical modeling tasks within the classroom. The survey was based on work done by Schmidt (2011). The survey includes organizational, student-related, and teacher-related obstacles, which influence teachers' decision-making about incorporating mathematical modeling tasks in their lessons (Blum, 1996). The validation study of this instrument should be complete by early April 2016.

6 ANALYSIS

Preliminary analysis of these data sources will occur at each stage in the design process followed by a final retrospective analysis after all phases of the project are completed (Molina et al., 2007). Triangulating findings among various data sources and conducting preliminary analysis after each implementation cycle will enhance the reliability and validity of the study's findings (Cobb and Gravemeijer, 2008).

Table 2: Data Sources by Teacher Participants.

Data Sources	Focus of coding/analysis
Pre/Post content tests	Understanding of function and programming – yet to be developed
Semi-Structured interviews	Focus on pedagogical content knowledge in algebra, ideas about the nature of mathematics and computer science, beliefs regarding problem-solving and real world applications as part of their curriculum
Algebra Teachers' Self-Efficacy Instrument (ATSE) (Gupta et al., 2015)	Likert survey that focuses on PCK, modeling and functions
Modeling Survey – in the process of being validated.	Likert survey that focuses on teachers' motivation to use mathematical modeling tasks

7 CONCLUSIONS

This work-in-progress report frames how students explore computational thinking as a way of creatively approaching mathematics using fundamental concepts from computer science. The presentation will evaluate concrete strategies for incorporating computer modeling and programming into algebra and examine real-world applications that can be used when exploring linear functions with learners.

The potential value of computational thinking and computer modeling for learning in secondary mathematics will be explored. The draft lessons explored during the presentation will provide an up-close look at an approach that has the potential to increase equity in education and broaden access to STEM careers.

The validation study of the Modeling Survey will be explored as well as its potential as a tool for studying teacher beliefs.

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