

# Suremath

## *User Study and Related (Re-)Implementation of a Multitouch Application for Learning Math*

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**Abstract:** Learning of the relation between mathematical effects and the underlying formula is a huge step for learners, which are used to execute rather basic arithmetic calculations up to this point. This usually happens in high school and pupils are often overstrained by the amount of abstraction which is required. In order to help students to overcome this gap, we have developed an application for a multitouch table. The pupils are able to grasp and move function graphs and see how the parameters of the formula change immediately. We have claimed that this approach leads to a better and faster understanding of the facts to be learned. In order to evaluate our approach we have carried through several workshops with a focus group and a small user study with pupils at the relevant age. In this paper we will describe the findings and we will shortly sketch the resulting (re-) implementation of our system.

## 1 INTRODUCTION

Learning mathematical facts is a hard task for many students. Whereas basic mathematic knowledge more or less still reflects the daily life. More complex and abstract features appearing in the pre-calculus curriculum are hard to match with daily topics.

To help learners to get a better understanding of the more theoretical concepts we have developed a multitouch table application, where they can interact with the mathematical concepts in a natural way, nevertheless providing a link to the underlying concept. They can move function graphs using their fingers by touching and dragging on that table. In parallel they can observe the changes of the parameters of the belonging equations in real time (Blanke and Schneider 2011). Furthermore the application provides an integrated exercise module.

The system has two application scenarios. First, it shall add another view and another media to the regular class. There should be a change between regular teaching methods with pen and paper and the multitouch application. We also imagine that pupils should have time to experiment with the application on their own for a certain period of time in order to verify their concepts or to discover correlations. Second, the pupils shall work independently with the

application at home using the built-in exercise mode for exam training.

The evaluation of our application in a real world setting in order to figure out if we can prove that the application leads to better results in learning these specific topics was another goal. Therefore we have carried out several workshops with a focus group and a small user study with pupils and their teacher. Having only a relatively small group of participants, we have focussed on finding hints for improving our system and its acceptance among learners, which we wanted to integrate afterwards.

In the following we will describe the study and the workshop we have carried through as well as the results.

Finally we have deduced different factors to support the aspects identified in the studies and incorporated the results into our application. This has led to a complete reimplementing of the system since these features could not be incorporated into the current system.

Accordingly, we will describe our new approach at the end of this paper.

## 2 RELATED WORK

First of all, we have to ask the question if at all, there is a hint that computer supported learning leads to better results. (Means et. Al. 2009) conducted a meta-analysis of 46 studies in the area of online learning, incorporating different learning techniques, like blended learning. One of the main effects they discovered was, that classes with online learning (including blended learning) produce stronger student learning outcomes than classes with solely face-to-face instruction.

Novel user interfaces based on multitouch tables in the field of mathematics are presented in (Zeleznik et al 2010). The application supports the manipulation of formulas using gestures on the device. A small prototype evaluation with students of their university indicated acceptance of the prototype.

Similar applications like our system are "GeoGebra" (Hohenwarter, Fuchs, 2004) and "Cinderella" (Kortenkamp, Richter-Gebert, 2002). However the interaction is more complicated compared to our system and we strongly believe that integrated exercises are a very important feature for learners for self-assessment.

(Iijima 2012) presents a math application based on HTML5, which hence runs on many output devices. Therefore popular devices for students as tablets can be supported as well.

Even though the systems above reflect different aspects of the field of interactive multitouch math learning applications there is not yet a clear picture, which features must be integrated in order to build a successful application.

## 3 THE APPLICATION

Our multitouch application consists of a display area, where the function graph is displayed (see fig. 1 right part), an equation area, where the equation is displayed (see fig. 1 left upper part) and an exercise area, where the exercises are displayed (see fig. 1 left lower corner).

The students can create mathematical objects by drawing the shape of the object with their fingers on the table. When the object is recognized, the belonging equation is displayed as well. Afterwards they can move the objects by touching and dragging and observe how the parameters in the equation change in real time. Doing so, they can explore the topics and try to detect the interrelation on their own.

The exercises serve as a means for the students to check on their own if the concepts have been understood. The use of a traffic light symbol tells them if the exercise has been completely, partially or not at all completed correctly.

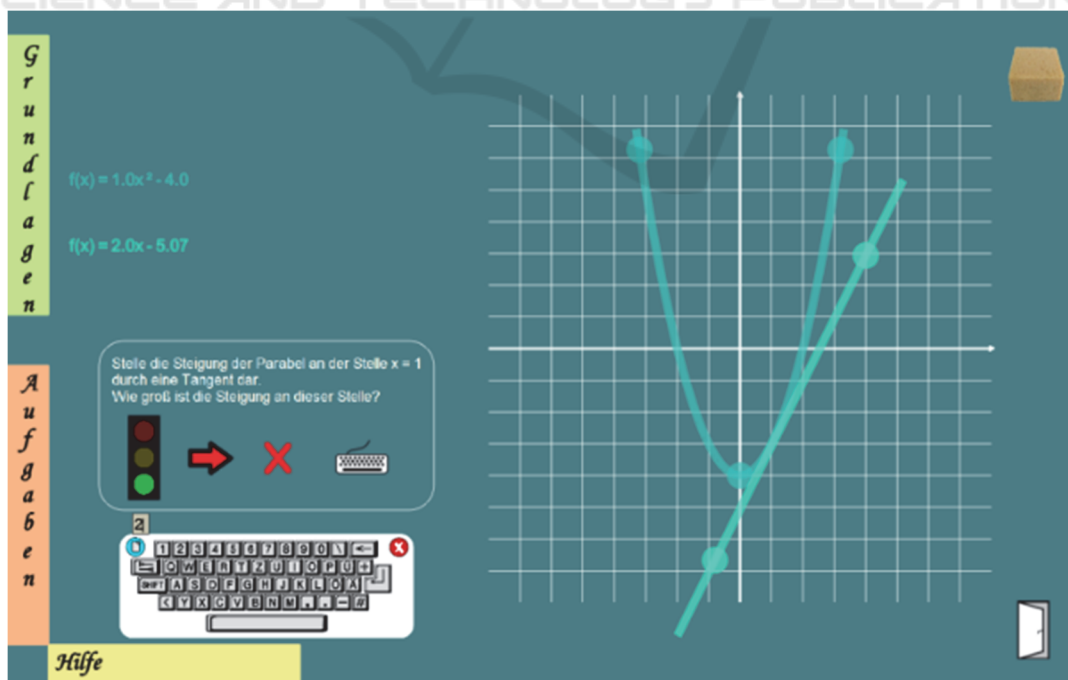


Figure 1: The user interface of the multitouch application.

## 4 STUDY DESIGN

In this section we will quickly sketch the design of our user study and the workshops with a focus group we have carried through.

### 4.1 Study Method and Participants

For the study design we have selected 9 pupils from a local high school for girls at the age between 13 and 14 years. The topics we have selected for the study were about quadratic equations and parabolas. The topics have not been taught at school yet. Therefore, all pupils have were new to the topics that were part of the study.

Our hypotheses were that:

- (A) Pupils will be able to solve the exercises faster with the multitouch application compared to pen and paper
- (B) Pupils gain a better understanding of the topic so that they perform better at the pen and paper exercise compared to the group that did not work with our application before the test.

The study started with a short introduction to the field, explained by one of their teachers.

Then the group has been split up into two groups, A and B. Both groups had to perform similar tasks. However in the first round, group A started to work on an exercise concerning moving a parabola along the x-axis and determine the effects on the parameters in the equation and vice versa, using pencil and paper, whereas group B used the multitouch table application for this task.

Then the groups switched roles and continued to work on the same topic. The table group used pencil and paper and the pencil and paper group used the application. For the second round, we have focussed on the movement along the y-axis. This time, group A started with pencil and paper, whereas group B used the table. Then they switched their roles again as in the first round. Each round consisted of 6 exercises, 3 exercises which had to be performed with pencil and paper and 3 exercises using the multitouch application.

Exercises looked like this:

The quadratic function  $f$  has the form

$$f(x) = x^2 + r \tag{1}$$

Chose a value for  $r$  in a way that  $f$  has

$$\text{Two x-intercepts: } r= \tag{2}$$

$$\text{Only one x-intercept: } r= \tag{3}$$

$$\text{No x-intercepts: } r= \tag{4}$$

The groups had 20 minutes for the exercises, independent of the working method.

After completing all exercises, we had handed out a questionnaire asking their opinion about the application.

At the end, we had a short discussion with the whole group.

### 4.2 Focus Group Workshops

The workshops consisted of a focus group of pupils from another high school. They all were part of a STEM special interest course. The number of pupils varied during the several meetings between 6 and 11. Their age was between 15 and 17, both male and female. All of them were familiar with the mathematical background.

The kick-off of the workshops has been an introduction into our system. Afterwards the pupils have met during about 9 months for open discussions. Altogether, there were approximately 6 general meetings with all participants. The goal of the workshops has been to suggest further features, that the students would find useful for an interactive math application, having in mind their recent learning experience.

## 5 RESULTS OF THE STUDY

Whereas the user study tried to find a hint to support our hypotheses. Especially we were keen to know if the pupils perform better, when they use an interactive application compared to a group with pencil and paper.

The focus group workshop was targeted at improving the interactive math application in general in order to improve its acceptance among pupils in the classroom.

### 5.1 Findings of the User Study

As it concerns the results of the exercises, we could verify our first hypothesis (A): The multitouch group was already finished with their exercises mostly in less than half of the available time, whereas the pen and paper group needed almost the whole time slot.

Concerning hypothesis (B) there were no clear findings if the pencil and paper group performed better or worse than the multitouch group. The pen and paper groups even performed slightly better in our study. However, the small number of participants

and the small number of exercises is a weakness of this study. Hence, we do not see a clear hint that one of the methods would be better for learning the concept than the other.

We made several interesting discoveries, when we met after the exercise to discuss the application.

The teacher made a comment that she could introduce the concept of the quadratic equation and the displacement of the function graph in half an hour compared to a week (about 4 teaching hours) using the traditional way. This comment points toward a high potential of the application for the use in a classroom from the teacher's point of view.

Afterwards she encouraged the pupils to use the application for further experimentation and asked more in depth question. Although the pupils have performed very well in both exercises, they had not yet profoundly acquired the underlying concepts. For several questions, they selected wrong parameters (especially the wrong sign). Experimenting with the application, they quickly discovered their mistake and corrected the parameters. This observation together with the fact that the pupils executed the exercises much quicker using the application led us to the belief that the use of a careful didactic instruction and especially careful selection of exercises is needed. Learners may be entrapped by the application to make faster but not always carefully elaborated decisions.

Further results have been discovered by evaluating the questionnaires. On a 7 point likert scale the students had to answer several questions, where "1" was "strongly agree" and "7" was "strongly disagree". In the following, we highlight the most interesting results. In general, there was a great acceptance of the multitouch application and the worst results for individual questions has been an average of 2.44.

We had asked questions concerning the topics ease of use, work effort, maturity of the hard- and software, usage scenarios (basic introduction, repetition, in-depth studies) and general opinion.

In the following, we mention only some selected answers that we found the most useful.

The question "The interaction with the table was obvious" has been rated 1.0. Asking if "The application motivates to deepen the topics autonomous" has been rated 2.11. "The critical examination of the topics is stimulated" scores 2.33 and the question "The examples of the multitouch application improve my interest in the topic" scores with 1.66. Most students were convinced that the application helps them to determine their individual learning pace (1.55). The pupils wanted to use the application to get an overview of the related topic

(1.67), acquire basic knowledge (1.62), evaluate theoretical knowledge (1.56) and repeat and deepen (1.67) the content. In general the pupils felt well prepared for the math course at school using the application (1.72). Summing up they gave a grade of 1.67 for the application and they would recommend the application to their fellow students with a value of 1.22.

There was an additional possibility to write free-form text concerning advantages and drawbacks of the application. As advantages the students mentioned at first "Having more fun to learn the topics", which has been mentioned 5 times. Working with the multitouch application means spending less amount of work for the same result has been mentioned 5 times as well. Three positive comments relate to the integrated exercises. Two utterances relate to the positive experience with experimental learning.

On the other site the pupils mentioned as drawbacks that the output device is not well suited for their learning environment (too expensive, too hard to transport, limited number of users at the same time). 4 comments related to this point. One student pointed out, that the tool does not replace the explanation of a teacher.

## 5.2 Results of the Workshops

The participants of the focus group analysed the multitouch application and elaborated several topics in their meetings concerning different parts of the application and the learning process with the application in general. The following sections sketch their proposals.

### 5.2.1 Support of the Introduction Phase

There should be an introduction into the specific topics, which is integrated in the application in a way that the topics are self-contained and do not need an external explanation.

### 5.2.2 Different Learning Modi

There should be different ways for working with the exercises. A learning mode could be used to focus on specific subjects only.

Assigning difficulty levels to exercises should help students to deepen their knowledge in a way that only exercises related to a specific level will be selected.

An exam training mode should present different questions of the topic field, possibly also varying in

the difficulty.

A self- assessment mode should only go from one topic to another if a certain number of exercises has been solved correctly.

Furthermore the participants suggested to incorporate some kind of gamification e.g. using high scores so that pupils of a class could compete against each other.

### 5.2.3 Misconception Detection

The application should be able to discover a misconception and to actively help the learner to overcome the error. E.g. if a student drags the parabola in the wrong direction, the application should detect this (common) mistake and remind the user to rethink her approach and possibly give her relevant information that is needed in this situation.

## 6 OPERATIONALIZATION OF THE RESULTS

The results of the study and the workshops require to integrate certain concepts that are not part of the system yet.

As it concerns the exercises, right now they are “hard-wired” into our application. In order to guarantee a high amount of exercises to make the application attractive to use, it is necessary to provide an authoring tool which can be easily used by teachers and pupils. Hence there must be a possibility to load the exercises into the system at run-time.

We have decided to integrate a web based authoring tool and to store the exercises on a server in order to build-up a pool of exercises that everybody can use.

A further requirement was the dynamic compilation of exercises.

Therefore each exercise can be associated with metadata, which will be used from the system to select exercises at run-time. Accordingly different selection criteria have to be integrated into the application for supporting the different learning modi (Section 5.2.2).

Concerning the remarks about the poor availability of the system in schools, we have decided to port the implementation to a platform that supports several clients, which are more commonly available at schools, like whiteboards, PCs or tablets (Section 5.1).

We are about to make it possible to display text, sound and animation. This enables possibilities for the introduction phase (Section 5.2.1), like explaining

mathematical concepts and instructions how to work with the system. Especially concerning the animations, we will offer the possibility to control the display of graphs, formula text and sound using a simple scripting language.

Finally, the detection of mistakes (Section 5.2.3) is supported by the system through attaching events to the mathematical objects. If a user has to move a parabola to the right in order to correctly solve the exercise, it is possible to attach an event to the parabola so that moving the parabola to the left will trigger an event displaying hints or pointing again to the theoretical foundations.

## 7 SYSTEM DESIGN

The program is implemented in the C# language, using the Monogame framework (Monogame 2016). This framework evolved from Microsofts XNA Game Studio (Microsoft 2016) and can support multiple operating systems like Microsoft Windows, Linux, OS X, Android, iOS and more. For implementing applications for Android and iOS, the Mono implementation from Xamarin (Xamarin 2016) is necessary. The programming is mostly done loop based like a video game rather than event based. This allows easier programming and smoother handling like moving the function graphs on the drawing area. Each frame, the input is read and the program reacts on changes.

### 7.1 Input

Suremath has input handling for mouse, touch and pixelsense (SUR40 (Samsung 2016)). Input from most interactive whiteboards is possible as mouse input simulation. As long as the input is not the mouse device (or its emulation) the program can handle multitouch.

### 7.2 Gesture Recognition

The recognition of the mathematical functions drawn on the grid is realized using the Protractor algorithm (Li 2010). This algorithm is a geometric template matcher that is based on the angular distance between the template and the drawn input. This allows very easy and fast recognition, but some additional work on the templates is needed. For example a normal parabola (5) and one with a negative leading coefficient (6) have different angular sums, and hence need two separate templates.

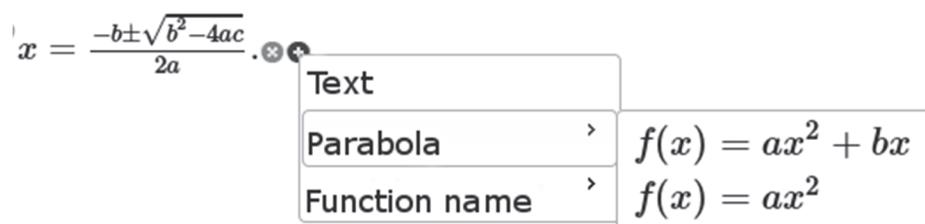


Figure 2: Formula creation using the authoring tool.

$$y = x^2 \tag{5}$$

$$y = -x^2 \tag{6}$$

However the algorithm is so flexible that new functions can be recognized by simply adding the corresponding templates. The templates are stored in an XML file. This guarantees a user friendly way to easily extend the set of gestures.

### 7.3 UI Elements

Since XNA/Monogame does not have any GUI elements included, all UI elements like windows, buttons, slider or textboxes have to be implemented separately. In our implementation the required images are generated programmatically on the fly.

### 7.4 Exercises

Our authoring tool allows to create exercises and to store them on a server. The exercises can be retrieved over the internet by entering a code in an input field on the exercises menu. A set of several exercises is stored in a ZIP file that contains a SQLite database file together with the metadata of the exercises and the images that contain the instructions. The use of images instead of text is necessary since complex formulas have to be displayed.

Exercises can have attached events, text or sound hints and animations. Giving an example, it is possible to add a text hint that is shown when a parabola is moved to the left instead of to the right. This allows the creation of self-explaining exercises.

Furthermore the integration of sound and animation can be used for an introduction into the subject. Using evented math objects it is even possible to integrate a mixed initiative introduction where the user is required to take some action before an animation continues.

### 7.5 Authoring Tool

The authoring tool is a web-based tool, which is integrated in a web page. It allows creating exercises

as a static and/or dynamic set of exercises. Static sets are a fixed list of exercises, created by the author. These sets are presented to the learner in the sequence as they have been created. Dynamic exercise sets can be compiled on the fly, based on metadata like difficulty, or topic.

The creation of the individual exercises in the web page is done using a very simple WYSIWIG LaTeX editor, based on tokens. Therefore it is possible to create professionally looking instructions for exercises without the need of knowing LaTeX or any other language to render equations.

Every token has two icons on the right hand side (see fig 2.).

A little “x” in a circle, used to delete this token.

A “+” in a circle adds a new token right to this token.

This tool uses conventional web technologies like Javascript and jQuery on the client side. For the user interface jQueryUI is used. The rendering of the equations is done with MathJax.

On the server-side asp.net and LaTeX are used. The application runs on a Linux system to render the text into images and create ZIP files and SQLite databases for deploying exercise-sets to the clients.

## 8 SUMMARY

In this paper, we have presented the evaluation of our multitouch application for learning math and the improved implementation based on the results we have gathered through our evaluation with pupils and workshops with a focus group.

First we have described the setup of our study and the approach for the workshops.

In the user study, we could show that learners can work faster with our application, compared to working with pen and paper.

Based on the findings of the comments from the user study and the remarks from the focus group, we have formulated the belonging features and changes of our application. These changes had to be integrated in order to support the learner and to improve the

acceptance and usability of our system. These were concepts like providing a kind of scripting to create an introduction to the domain. We have introduced meta data, e.g. in form of difficulty levels and different learning modes like competition, exam training etc.. Additionally we have introduced events to react flexibly on the user interaction.

Finally, we have shortly sketched our system (re) implementation, based on a video game based concept with an independent authoring tool, which is browser based.

In the future, we want to extend the use of our system. We have it already installed in one of the high schools running on a whiteboard for being used in the classroom. We also plan to make the Android version available in the app store soon. We hope to receive much more input from pupils and teachers over this channel. Additionally we want to carry through a more profound user study with our improved system.

As a further step, we plan to extend the functionality of our application to fields with similar challenges, e.g. trigonometric functions. More specifically, we want to extend the system to exercises of the type: "At a point 15 feet from the base of a church, the angle of elevation of the top of the church is  $43^\circ$ . Find the height of the church to the nearest foot." We imagine that moving objects back and forth and changing the angle of lines interactively while displaying the values of angles, lengths and trigonometric functions can help to grasp the concept behind relatively abstract functions like sine or cosine.

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