Augmented Postprocessing of the FTLS Vectorization Algorithm
Approaching to the Globally Optimal Vectorization of the Sorted Point Clouds

Ales Jelinek and Ludek Zalud
Department of Control and Instrumentation, Brno University of Technology, Technicka 12, Brno, Czech Republic

Keywords: Vectorization, Point Cloud, Linear Regression, Least Squares Fitting, Mobile Robotics.

Abstract: Vectorization is a widely used technique in many areas, mainly in robotics and image processing. Applications in these domains frequently require both speed (for real-time operation) and accuracy (for maximal information gain). This paper proposes an optimization for the high speed vectorization methods, which leads to nearly optimal results. The FTLS algorithm uses the total least squares method for fitting the lines into the point cloud and the presented augmentation for the refinement of the results, is based on a modified Nelder-Mead method. As shown on several experiments, this approach leads to better utilization of the information contained in the point cloud. As a result, the quality of approximation grows steadily with the number of points being vectorized, which was not achieved before. Performance costs are still comparable to the original algorithm, so the real-time operation is not endangered.

1 INTRODUCTION

Vectorization is a process, which produces some kind of geometrical primitive (line segment, arc, polynomial curve) which describes a general shape of a given set of data points. This procedure has several advantages.

At first, vectorization converts the discrete measurements into their continuous abstraction. This effect is used in mobile robotics, where laser scanners are used for sensing of the environment around the robot. After the measurement, the point cloud is vectorized and utilized during the map building process (Nguyen et al., 2007). This is an important task of various mobile robots, especially of those dedicated to exploration and reconnaissance (Zalud et al., 2008). Very similar problem is the 3D surface or infrastructure reconstruction (Xu et al., 2015) from the airborne LiDAR used in cartography. Computer vision and image processing applications also take advantage of the vectorization in various applications, such as shape detection (Mathibela et al., 2013) and photogrammetry (Liu et al., 2011). Another contextual domain of application is trajectory tracking (Werner et al., 2014) and infrastructure networks mapping, such as roads in (Xiangyun Hu et al., 2014).

Second, the vectorization process has a generalization ability, which reduces noise and negligible details in the measurement. Noise reduction is very useful in precise laser scanning applications, requiring minimal error (Heinz et al., 2001). Filtration of the negligible details is widely used in cartography, where different levels of details are obtained with varying settings of the vectorizing algorithm (Shi and Cheung, 2006).

The third important feature of a vectorized data set is the reduction of the amount of the data stored. In a typical application with thousands of measured points converted into units or tens of line segments, the memory savings of two orders of magnitude can be easily observed. This fact is mainly utilized in cartography, where large maps have to be stored efficiently (Kandal and Karschti, 2014).

The examples summarized in the previous paragraphs clearly show, that the vectorization has a wide usage across a variety of scientific and technical domains. There are some applications, which are not time critical, but in general, the faster and more accurate the vectorization algorithm is, the higher is its suitability for practical applications. The motivation of the presented research is to devise a method, which would be fast enough for the real-time applications in robotics and image processing and which would provide results with the maximal accuracy. At the cost of slightly reduced speed of the original algorithm, the further described method fulfils this objective.

Jelinek, A. and Zalud, L.
Augmented Postprocessing of the FTLS Vectorization Algorithm - Approaching to the Globally Optimal Vectorization of the Sorted Point Clouds.
DOI: 10.5220/0005962902160223
In *Proceedings of the 13th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2016) - Volume 2*, pages 216-223
ISBN: 978-989-758-198-4

2 STATE OF THE ART

There are plenty of vectorization methods described in the literature. Various approaches are devised for different situations, but the two main categories, which can be distinguished, differ in the nature of the input data. The general algorithms accept any arbitrary set of data points, but this advantage is redeemed by a larger computational complexity. The second category of the algorithms works only on the sorted data, but usually performs much faster.

In the following text, we are going to speak about the "optimality" of vectorization and the "globally optimal" results. Since every vectorization algorithm employs some error function to enumerate quality of the approximation, optimal result is achieved, when this function reaches its minimum. Global optimality is used to emphasize minimization of the error function for the whole data set, not just a single line segment.

2.1 General Algorithms

General algorithms are able to produce globally optimal results, or resemble optimality at least. For example, Hough transformation (Hough, 1962) in its classical implementation relies on the systematic search through the whole state space of possible solutions. This process can be enhanced (Norouzi et al., 2009) to focus on the most promising areas, which might result in globally optimal vectorization. Another popular method is RANSAC (Fischler and Bolles, 1981), which is based on random sampling of the data set. With enough iterations and luck, it converges to globally optimal solution as well. Another examples are optimization techniques, suited for highly complex problems, such as genetic algorithms (Mirmehdi et al., 1997).

The paper (Nguyen et al., 2007) proves this kind of algorithms to be significantly slower than the methods, that we are going to focus on in the rest of this paper, but for the sake of completeness, since general algorithms can produce globally optimal results, we have decided not to omit them.

2.2 Algorithms for the Sorted Data Sets

Sorted data points preserve the information about the order, in which they come after each other. The known succession means, that only the consecutive points can form a single line to be extracted, which can be exploited to improve performance.

Sometimes the sorted data cannot be obtained, but for practical situations it is not as constraining, as it might seem. For example a border in cartography is by a polyline, where each point has exactly defined neighbours. A similar situation arises in the laser scanning case as well: the laser usually rotates around and the distinct measurements are obtained int the short intervals one after another, so the order is known as well. In the image processing we can find the edge extracting algorithms, which produce their results in a form of a chain of the adjacent pixels. All of these situations are the examples of naturally ordered data sets, which makes the algorithms focused on this type of input very useful.

The simplest and the fastest (Nguyen et al., 2007) family of algorithms in this category are the point eliminating (PE) algorithms. A large number of these methods is described in the literature (Shi and Cheung, 2006). The main idea behind them is the elimination of points, which do not carry a crucial information about the general shape of the point set. After application of such algorithm, only the most important points remain, forming a polyline approximating the whole set. Due to loss of information from the eliminated points, these methods cannot provide very accurate results. In the experimental part of the paper, we have decided to examine the Reumann-Witkam (RW) algorithm (Reumann and Witkam, 1974), which has $O(n)$ computational complexity and is probably the fastest vectorization technique in practical usage. The second PE method, which will be tested, is the Douglas-Peucker (DP) algorithm (Douglas and Peucker, 1973), which is $O(n \log n)$ complex in average and represents a widely accepted standard in real-time vectorization, being more accurate than the RW method (Shi and Cheung, 2006), but still very fast (Nguyen et al., 2007).

The second group of the algorithms for vectorization of the sorted data is based on the total least squares (TLS) fitting of the regression line. The main advantage over the PE methods consists in utilization of all of the information contained in the data set. This leads to higher accuracy, which can effectively suppress the noise. Classical implementation of the TLS vectorization is the Incremental (INC) algorithm (Nguyen et al., 2007). The optimized version of the INC algorithm is the Fast Total Least Squares (FTLS) method (Jelinek et al., 2016). It produces the same output as the INC algorithm, but significantly faster, especially when a large set of points is being processed. Both of these methods will be examined in the experimental section, because INC algorithm is widely known and FTLS is its recent faster alternative. Asymptotic complexity of both algorithms is $O(n)$. FTLS is also a base for the optimization proposed in this paper.

2.3 The Problem in the Threshold based Vectorization

Every vectorization algorithm has an error function. The general algorithms usually search each line segment in the data set separately, which leads to minimization of the error function in each case. Conversely, the algorithms for the sorted data, exploit the continuity of the given point cloud, traverses through the data set and set up the break points of the approximating polyline. This approach is prone to give the nonoptimal results as shows the following example.

Fig. 1 displays two point clouds: one precise (black) and one noised (red). Both point clouds are ordered, as if they were obtained by a laser scanner. Fig. 2 shows the characteristics of the error function generated by a TLS method for a given number of points, counted from the beginning of the cloud. As expected, the first half of the precise point cloud is vectorized with a zero error, since the points lie exactly in a line. After the midpoint, the error starts to grow monotonically. Though the whole shape is quite similar, the first half of the red characteristics is corrugated, contains local minima and the break point, where the error starts to grow rapidly, is not defined as well as in the previous case.

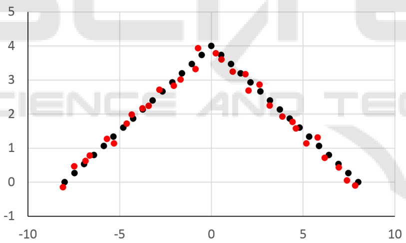


Figure 1: An example of two simple point clouds. The black one is precisely generated to form a sharp corner, while the red one has added a realistic noise.

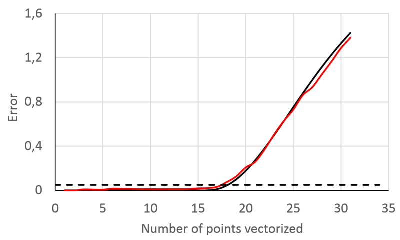


Figure 2: The error functions generated for the various number of points from example point clouds in Fig. 1, obtained by the TLS vectorization algorithm. The black characteristics belongs to the precise data set and the red chart to the noised one. The dashed line illustrates a defensive choice of a threshold level for a robust vectorization.

All of the fast vectorization methods (whether PE or TLS) employ a threshold, which is compared to the error function in each iteration and when exceeded, the break point is set up and a new approximation is started. Due to the fact, that the error characteristics is not ideal, the threshold must be set with appropriate reserve, to ensure robustness and immunity to the noise. Such a defensive choice of the threshold leads to higher error than necessary for every line segment in the approximating polyline, except the last one, as can be seen in Fig. 3.

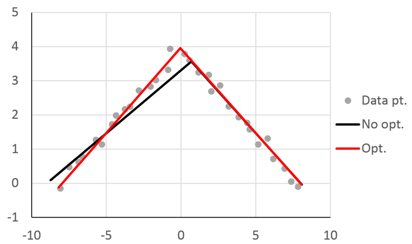


Figure 3: Optimal (red) and suboptimal (black) vectorization of the noised point cloud (gray). The red and black line segments approximating the second half of the point cloud overlap each other.

The negative influence of the defensively chosen threshold was slightly magnified for the illustrational purposes in this case, but even without that, it is a serious drawback of all of the fast vectorization algorithms. This effect manifests regardless the algorithms traverse the point cloud incrementally (INC and RW), or perform some sort of an adapted binary search (FTLS and DP). It is also impossible to suppress it by calibration (Kocmanova et al., 2013) or identification (Zalud et al., 2015) of the laser scanner, because the described problem is an inherent feature of the TLS and PE algorithms, not a flaw of the input data. The rest of the paper is focused on minimization of this unnecessary error.

3 AUGMENTATION OF THE FTLS ALGORITHM

Globally optimal vectorization of a continuous point cloud is a really challenging task. There is no prior information either on the number of line segments in the approximating polyline, or the position of the break points. This high degree of uncertainty leads to employment of the traditional vectorization algorithms to get an initial guess of the approximation and than use it for further optimization.

As the primary algorithm, we have chosen the FTLS algorithm (Jelinek et al., 2016), which is based o the TLS method and is reasonably fast. The algorithm has three stages of the data processing. The first one produces precomputed sums for every data point in the cloud. The data vector belonging to each point is as follows:

$$\boldsymbol{a}_i = \left[x_i, y_i, \sum_{k=1}^{i} x_k, \sum_{k=1}^{i} y_k, \sum_{k=1}^{i} x_k^2, \sum_{k=1}^{i} y_k^2, \sum_{k=1}^{i} x_k y_k \right], \tag{1}$$

for $i = 1 \ldots N$, where N is the number of points being vectorized. For our purposes, we define an additional vector $\boldsymbol{a}_0 = [0, 0, 0, 0, 0, 0, 0]$ and form an augmented data matrix:

$$\mathbf{A} = \begin{bmatrix} \boldsymbol{a}_0 & \boldsymbol{a}_1 & \ldots & \boldsymbol{a}_N \end{bmatrix}^T. \tag{2}$$

We also define a submatrix of $\mathbf{A}$ as:

$$\mathbf{A}[p,q] = \begin{bmatrix} \boldsymbol{a}_p & \ldots & \boldsymbol{a}_q \end{bmatrix}^T, \tag{3}$$

determined by the row indices p and q, which follow the condition $0 \leq p < q \leq N$.

The second stage of the FTLS algorithm is dedicated to line extraction. The output is an ordered set of extracted lines, which approximate the whole point cloud. Every extracted line has an exactly defined interval of points, which belong to it. This information is used to build a break point vector, where the index of the last point of every line is stored:

$$\boldsymbol{b} = [b_0, b_1, \ldots, b_L]. \tag{4}$$

The $b_0 = 0$ is a dummy element representing a virtual point before the beginning of the point cloud. The last break point always refers to the end of the point cloud, therefore $b_L = N$. All the other break points satisfy the condition: $b_{i-1} < b_i$ for $i = 1 \ldots L$.

Every extracted line has also attached an error value, which describes, how well the line approximates the given data. The TLS methods use the variance of perpendicular distances of points to the line. The paper (Jelinek et al., 2016) define it in the following manner:

$$\sigma^2 = \frac{1}{n} \left(a^2 \sum_{i=1}^{n} x_i^2 + 2ab \sum_{i=1}^{n} x_i y_i + b^2 \sum_{i=1}^{n} y_i^2 \right) - c^2. \tag{5}$$

n is the number of points belonging to the line and a, b and c are the coefficients of a line in a neutral format: $ax + by + c = 0$ in XY coordinate system. If a half-open interval of points $(p, q]$ is given, the sums can be easily obtained by a simple subtraction of vectors $\boldsymbol{a}_q$ and $\boldsymbol{a}_p$. Since the coefficients a, b and c can be expressed as a function of $\mathbf{A}[p,q]$ as well, we can denote the variance: $\sigma^2(\mathbf{A}[p,q])$.

This adjustment of the notation allows us to express the error E of the approximation of the whole point cloud. It is defined as follows:

$$E(\boldsymbol{b}) = \sum_{i=1}^{L} \sigma^2(\mathbf{A}[b_{i-1}, b_i]) \frac{b_i - b_{i-1}}{N}. \tag{6}$$

The original FTLS algorithm does not evaluate any global error metrics, its third stage consists only in turning the extracted lines into the polyline, which describes the general shape of the point cloud. This approximation is necessarily suboptimal, due to the effect described in the Subsection 2.3. To get the globally optimal results analytically, the following set of the equations would have to be solved:

$$\frac{\partial E(\boldsymbol{b})}{\partial b_1} = 0 \quad \ldots \quad \frac{\partial E(\boldsymbol{b})}{\partial b_{L-1}} = 0. \tag{7}$$

There is only $L-1$ of equations for L extracted lines, because b_0 and b_L coefficients are defined to be constant, when building the break point vector $\boldsymbol{b}$ according to the equation (4). Unfortunately, the analytic solution of the system (7) is extremely complicated or even impossible, because the error function $\sigma^2(\mathbf{A}[p,q])$ is highly non-linear. It also does not grow monotonically, which possibly leads to an unpredictable number of local minima.

To overcome this issue, we have decided to employ the Nelder-Mead (NM) method (Nelder and Mead, 1965). It is a non-gradient optimization technique, which only relies on enumeration of the optimized function for various input arguments. This is a key feature, because thanks to the precomputed sums of the FTLS algorithm, the evaluation of the error function is very fast. The NM method is designed for localization of extremes of the non-linear functions with arbitrary number of arguments. The method is based on a simplex composed of $D+1$ vertices in a D-dimensional space of possible arguments of the optimized function. Each vertex has its function value attached and every iteration the least suitable one is replaced by a newly computed successor. The function value of the new vertex is usually better than the previous one, so as the iterations proceed, the simplex shrinks and closes to the set of arguments giving the optimal value.

In the original NM method, the simplex is described by a set of $D+1$ vectors with D elements. In our case of the search for the optimal break point positions, the situation is slightly more complex, because every argument vector for the optimized error function (6) has constants at the beginning and at the end. At the beginning of the optimization process, there is only one initial guess on position of the break points produced by the second stage of the FTLS algorithm, therefore $L-1$ new vertices have to be generated. The resulting simplex has the structure:

$$\mathbf{S} = \begin{bmatrix} \boldsymbol{s}_0 & \boldsymbol{s}_1 & \ldots & \boldsymbol{s}_{L-1} \end{bmatrix}^T, \tag{8}$$

generated by the following rules:

$$\begin{aligned} &\boldsymbol{s}_0 = \boldsymbol{b}, \\ &s_{i,j} = b_j \qquad i \neq j, \\ &s_{i,j} = b_j - \tau \quad i = j, \end{aligned} \tag{9}$$

for $i = 1\dots(L-1)$ and $j = 0\dots L$. As the first vertex, the initial guess is adopted without any changes. The remaining $L-1$ vertices are generated by subtracting a user defined value τ from a single element of the vector $\boldsymbol{b}$ (different element for every new vertex). The τ parameter is an integral variable and should fulfil the condition $\tau < b_0$ to ensure, that all the elements of the vertex will satisfy the constraints discussed already beside the equation (4). τ should correspond to an expected number of extra points assigned to the line segment, but the NM method is robust enough to handle even very rough estimates.

The vertex manipulating operations, which compose the core of every iteration of the NM method, were implemented in our algorithm according to (Lagarias et al., 1998). The structure of the iteration is well described, therefore we are not going to reproduce the details. The error function (6) and the simplex (8) are enough to start the optimization process.

There are two major differences to the original approach. The first is the introduction of constraints to otherwise unconstrained method. In general, adding constraints to the NM method can be very complicated (Le Floc'h, 2012), but in our case, adding limits cannot lead to worse results, than the initial guess, neither can cause a collapse of the method. Because the break points must keep the order determined by the points in the source point cloud, the algorithm does not search through the whole $(L-1)$-dimensional space, but only through its part, limited by the inequality $s_{i,j-1} < s_{i,j}$ for $i = 0\dots(L-1)$ and $j = 1\dots L$. This condition is checked every time after a new vertex is generated and proceeds from the end to the beginning of the particular vector. Any break points out of the range are trimmed to fit into the boundaries. A new error function value is computed after this check.

The second difference to the original approach is the discreteness of the arguments of the optimized function. While the original method is designed to be used with real numbers, the break points can be only integers. In combination with the constraints discussed above, this means, that there is a finite number of possible vertices. At the end of the iteration process, as the solution becomes more precise, the vertices inevitably start to overlap and the simplex collapses. If that happens, or if the vertices end up in one hyperplane, the algorithm is stopped and the best solution is taken as the output of the optimization.

The concept described above ensures, that no worse result than the initial guess can appear and in the vast majority of cases a significant improvement can be observed. The postprocessing, after the modified NM optimization is over, is the same as in the original FTLS algorithm.

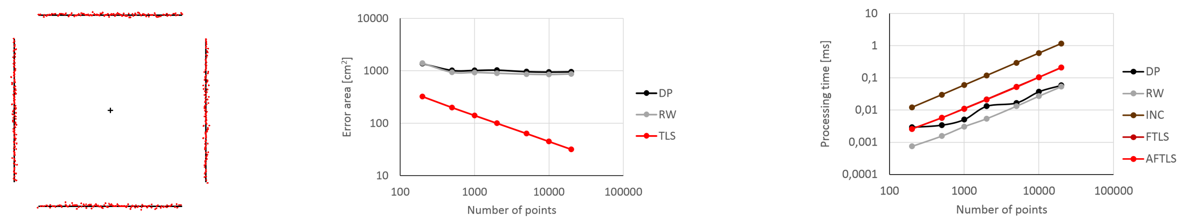
4 EXPERIMENTAL VALIDATION OF THE METHOD

This section presents several experiments for evaluation of the precision gains and the performance costs of the augmented FTLS method. For the convenience, we are going to use the AFTLS abbreviation. Because the AFTLS method is derived directly from the FTLS method, we are going to follow the experimental methodology presented in (Jelinek et al., 2016) and repeat some benchmarks with the new augmented vectorization technique. The datasets for the experiments in Fig. 5-8 are directly adopted from (Jelinek et al., 2016) and so are the illustrational pictures (the (a) subfigure in all of the specified cases). All of the measurements are performed again to unify the test conditions.

All of the algorithms and benchmarks were implemented in the C++ programming language and compiled with Microsoft Visual Studio 2012, using the O2 optimization setting. No other optimizations were made to keep the tests as general as possible. An interested user of the algorithms may benefit from multithreaded implementation, SIMD instructions, loop unrolling and many other techniques, which can improve the performance significantly, but are highly platform dependant. The computer used for running the benchmarks had a six-core, 64-bit AMD FX-6350 CPU, which runs on 3.9 GHz. Processor cache memory is large enough to hold all of the data processed in one iteration.

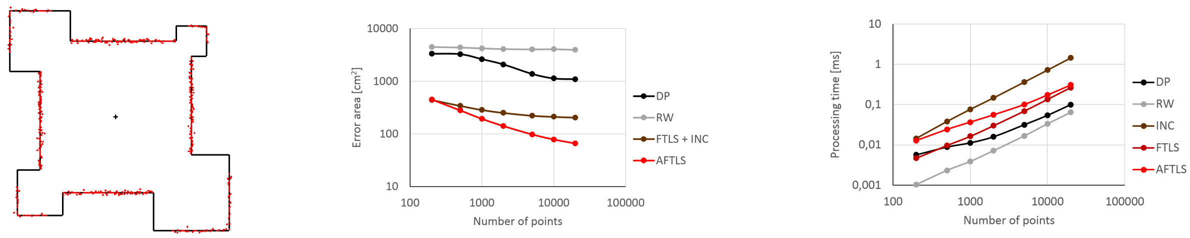
The metrics used for precision evaluation of the examined methods is the same as in (Jelinek et al., 2016), which is based on computation of the error area demarcated by the true edge and the extracted polyline approximation. Neither the error metrics for a single line (5), nor the error function for the whole continuous point cloud (6), used during the vectorization process, are suitable, because both of them apparently cannot take into account the real edges.

The first tested case is shown in Fig. 4a. The continuous parts of the point cloud can be approximated by a single line, which is a perfect situation for comparison of the PE and the TLS based methods. From the precision benchmark in Fig. 4b it is clear, that the PE methods are not capable to utilize the information



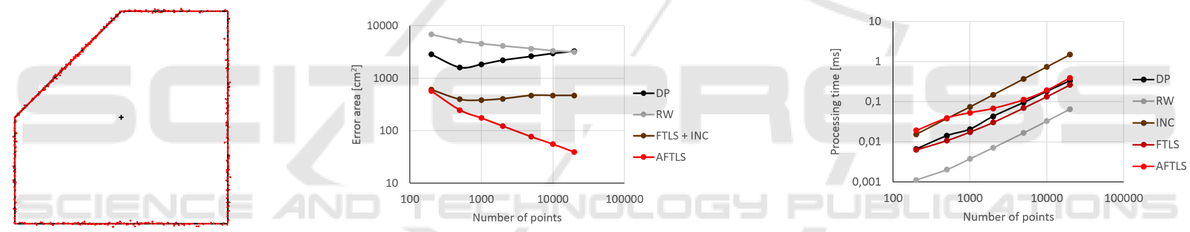
(a) The point cloud. (b) The precision benchmark. (c) The speed benchmark.

Figure 4: An example of a simple point cloud where no break points are present. All of the TLS based methods provide the same precision of vectorization and the FTLS and AFTLS algorithms exhibit the same speed.



(a) The point cloud. (b) The precision benchmark. (c) The speed benchmark.

Figure 5: This example consists of eight continuous parts, four of which contain one break point and four is straight without bending. The AFTLS algorithm provides the most accurate results.



(a) The point cloud. (b) The precision benchmark. (c) The speed benchmark.

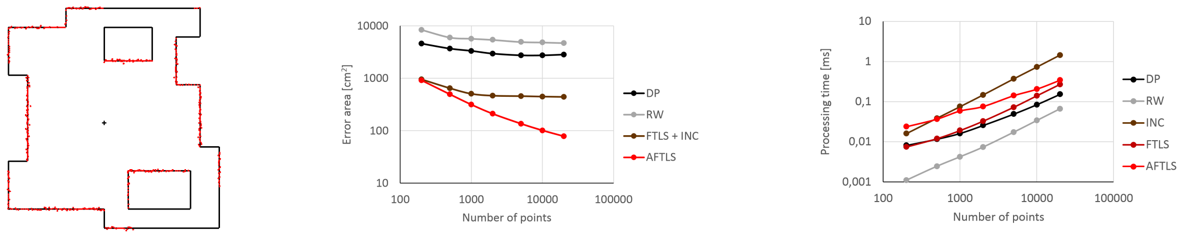
Figure 6: The continuous point cloud examined in this experiment has four bendings. As the number of break points in the cloud increases, the advantage of the AFTLS method becomes even more evident.

from all of the processed points, while the TLS methods use it to gradually refine the results as the number of points grows. The speed benchmark in Fig. 4c proves, that there is no performance penalty of the AFTLS algorithm, if no break points are present.

The second, example in Fig. 5a contains straight lines and polylines with one break point. The FTLS and INC algorithms, although TLS based and significantly more accurate than the PE methods, cannot lower the error under certain threshold, because of the problem discussed in Section 2.3. The AFTLS algorithm effectively solves this issue and provides the better results, the higher is the amount of the input points (see Fig. 5b). Computational costs are evident from the speed benchmark in Fig. 5c. The relative performance loss decreases with the number of points being processed.

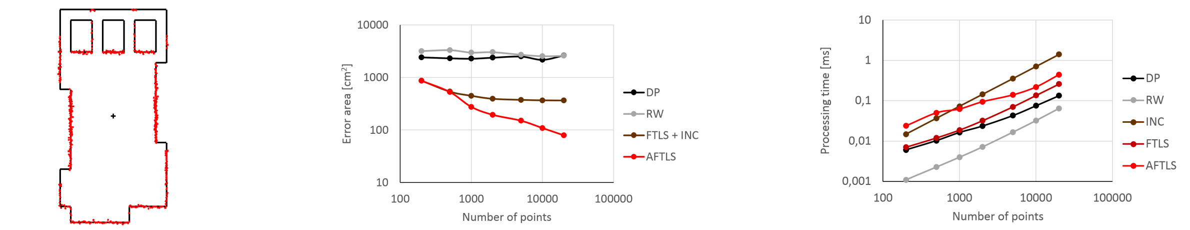
The shape of the point cloud depicted in Fig. 6a leads to even stronger manifestation of effects discussed in the previous paragraph. AFTLS keeps to refine the output polyline with the growing number of processed points as before (see Fig. 6b) and the speed sticks close to the FTLS characteristics as well (see Fig. 6c). We have not deeply examined the behaviour of the DP and the RW precision characteristics observable in Fig. 5b and Fig. 6b, because the paper is focused on the AFTLS algorithm. We believe, it originates from the hidden regularities in the point cloud generator, which are related to the given number of points. Nevertheless, the fact that the PE methods are not able to utilize the information from the eliminated points can still be considered valid.

The last two experiments shown in Fig. 7 and Fig. 8 are virtual simulations of an environment, which is commonly encountered by a mobile robots. Results are similar and correspond to the findings



(a) The point cloud. (b) The precision benchmark. (c) The speed benchmark.

Figure 7: The point cloud obtained in a virtually simulated environment. The AFTLS algorithm provides the most precise approximations, while the speed is comparable (yet lower) to the other algorithms.



(a) The point cloud. (b) The precision benchmark. (c) The speed benchmark.

Figure 8: Another example of a virtual environment similar to real world cases. The results correspond to the expectations about behaviour of the tested algorithms.

from the previous experiments. The AFTLS method consistently delivers the most accurate results. For point clouds with lower number of points, the precision gain is small and relative computational cost high in comparison with the FTLS method, but for the dense point clouds, it is definitely worthwhile.

Experimenting with the real laser scans was found to be very complicated. The first issue is a limited number of the output points of the laser scanners available these days. The Velodyne HDL 32, that we have used for testing, delivers ca. two thousands of points per scan. This is right enough to see the benefits of the AFTLS method, but the most significant results are expected in the future, with more output points. The second big issue is the precision of the reference measurement for the precision benchmarking. Every millimetre in the reference measurement taken in an empty room, had a serious impact on the error enumeration, which could easily cause the misleading results. We have tested only the speed on real data, which was proved to be in perfect agreement with simulations in Fig. 7a and 8a. The same we expect for accuracy.

5 CONCLUSIONS

This paper proposes a new approach to fast vectorization of the sorted point clouds. The presented method originates from the FTLS algorithm, which is based on the TLS line fitting, therefore it can utilize the information from every processed point in the cloud. It also incorporates an optimization mechanics based on a modified Nelder-Mead method, to overcome the general problem of the fast, threshold based, vectorization techniques. Since the optimization is based on the NM method and the function for estimation of the global error of the approximation is not guaranteed to have a single minimum, we cannot claim the method to be truly globally optimal. On the other hand, the algorithm with this augmentation is able to refine the vectorization results and approach the globally optimal solution very close. The results of several experiments have proved, that the augmented FTLS algorithm significantly enhances precision of the approximation of the point cloud, especially of the most dense ones containing many thousands of points. This fact is really promising, because the laser scanners with higher point densities can be expected in the near future.

Further research in this field is going to be focused on effective implementation of the algorithm on a special hardware architectures and the deployment to the practical applications, which could take advantage of a fast and highly precise vectorization, such as robotics and machine vision.

ACKNOWLEDGEMENTS

The grant No. FEKT-S-14-2429 - ”The research of new control methods, measurement procedures and intelligent instruments in automation" financially supported by the Internal science fund of BUT. The Department of Control and Instrumentation (BUT - FEEC). The project CEITEC (CZ.1.05/1.1.00/02.0068), financed from the European Regional Development Fund and by the TACR under the project TE01020197 - CAK3.

REFERENCES

Douglas, D. H. and Peucker, T. K. (1973). Algorithms for the Reduction of the Number of Points Required to Represent a Digitized Line or its Caricature. *Cartographica: The International Journal for Geographic Information and Geovisualization*, 10(2):112–122.

Fischler, M. A. and Bolles, R. C. (1981). Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography. *Communications of the ACM*, 24(6):381–395.

Heinz, I., Hartl, F., and Frohlich, C. (2001). Semi-automatic 3D CAD model generation of as-built conditions of real environments using a visual laser radar. In *Proceedings 10th IEEE International Workshop on Robot and Human Interactive Communication. ROMAN 2001 (Cat. No.01TH8591)*, pages 400–406. IEEE.

Hough, P. V. C. (1962). Method and Means for Recognizing Complex Patterns.

Jelinek, A., Zalud, L., and Jilek, T. (2016). Fast total least squares vectorization. *Journal of Real-Time Image Processing*.

Kandal, P. and Karschti, S. (2014). Method for simplified storage of data representing forms.

Kocmanova, P., Zalud, L., and Chromy, A. (2013). 3D proximity laser scanner calibration. In *2013 18th International Conference on Methods & Models in Automation & Robotics (MMAR)*, pages 742–747. IEEE.

Lagarias, J. C., Reeds, J. a., Wright, M. H., and Wright, P. E. (1998). Convergence Properties of the Nelder–Mead Simplex Method in Low Dimensions. *SIAM Journal on Optimization*, 9(1):112–147.

Le Floc'h, F. (2012). Issues of Nelder-Mead Simplex Optimisation with Constraints. *SSRN Electronic Journal*, pages 1–7.

Liu, Z., Zhang, B., Li, P., Guo, H., and Han, J. (2011). Automatic registration between remote sensing image and vector data based on line features. In *2011 19th International Conference on Geoinformatics*, number 2008, pages 1–5. IEEE.

Mathibela, B., Posner, I., and Newman, P. (2013). A roadwork scene signature based on the opponent colour model. In *2013 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 4394–4400. IEEE.

Mirmehdi, M., Palmer, P. L., and Kittler, J. (1997). Robust line segment extraction using genetic algorithms. In *Image Processing and Its Applications, 1997., Sixth International Conference on*, volume 1, pages 141 – 145 vol.1. IEEE.

Nelder, J. A. and Mead, R. (1965). A Simplex Method for Function Minimization. *The Computer Journal*, 7(4):308–313.

Nguyen, V., Gächter, S., Martinelli, A., Tomatis, N., and Siegwart, R. (2007). A comparison of line extraction algorithms using 2D range data for indoor mobile robotics. *Autonomous Robots*, 23(2):97–111.

Norouzi, M., Yaghobi, M., Siboni, M., and Jadaliha, M. (2009). Recursive line extraction algorithm from 2d laser scanner applied to navigation a mobile robot. In *2008 IEEE International Conference on Robotics and Biomimetics*, pages 2127–2132. IEEE.

Reumann, K. and Witkam, A. P. M. (1974). Optimizing Curve Segmentation in Computer Graphics. In *Proceedings of International Computing Symposium*, pages 467–472, Amsterdam. North-Holland Publishing Company.

Shi, W. and Cheung, C. (2006). Performance Evaluation of Line Simplification Algorithms for Vector Generalization. *The Cartographic Journal*, 43(1):27–44.

Werner, M., Schauer, L., and Scharf, A. (2014). Reliable trajectory classification using Wi-Fi signal strength in indoor scenarios. In *2014 IEEE/ION Position, Location and Navigation Symposium - PLANS 2014*, pages 663–670. IEEE.

Xiangyun Hu, Yijing Li, Jie Shan, Jianqing Zhang, and Yongjun Zhang (2014). Road Centerline Extraction in Complex Urban Scenes From LiDAR Data Based on Multiple Features. *IEEE Transactions on Geoscience and Remote Sensing*, 52(11):7448–7456.

Xu, C., Frechet, S., Laurendeau, D., and Miralles, F. (2015). Out-of-Core Surface Reconstruction from Large Point Sets for Infrastructure Inspection. In *2015 12th Conference on Computer and Robot Vision*, pages 313–319. IEEE.

Zalud, L., Kocmanova, P., Burian, F., Jilek, T., Kalvoda, P., and Kopecny, L. (2015). Calibration and Evaluation of Parameters in A 3D Proximity Rotating Scanner. *Elektronika ir Elektrotechnika*, 21(1):3–12.

Zalud, L., Kopecny, L., and Burian, F. (2008). Orpheus Reconnissance Robots. In *2008 IEEE International Workshop on Safety, Security and Rescue Robotics*, number October, pages 31–34. IEEE.