The Compression Algorithm of the S-Transform and Its Application in MFCC

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Abstract: The S-Transform which is used in many fields, is better in the methods of time-frequency analysis. Although the S-Transform improves the time-frequency analysis, it also increases the algorithm complexity, resulting in rapidly increasing the computing time and needing more memory. Therefore, the S-Transform is difficult to be applied for real-time procession of signal. Based on the characteristics that human being is insensitive to the voice frequency resolution, in this paper, we propose a compressing algorithm of the S-transform. Through re-sampling frequency, the algorithm decreases data size, the computing time and computer memory usage in the S-transform. The application of the algorithm in MFCC analysis shows the results is reliable under the condition of re-sampling frequency resolution $\Delta f < 82Hz$.

1 INTRODUCTION

The beginning of research on speech features was in the 1930s (Cheng et al., 1996). Lendbergh and his colleagues first conducted the research on the personal speech features. After decade, many kinds of coefficients about speech features were proposed, including the most important one, Mel Frequency Cepstral Coefficients (MFCC) (Davis et al., 1980). The MFCC was put up based on the ability of auditory feeling in 1980 by Davis and Mermelstein, is better in speech recognition and has been widely applied in the speech research. Since 1995, computation of MFCC has been improved based on time-frequency analysis and Neural Network (Abdalla et al., 2013).

In 1996, the S-Transform(ST) is proposed (Stockwell et al. 1996), aiming at processing the seismic wave in earthquake exploration. ST is the expanding of Short Time Fourier Transform (STFT) (Liu et al., 2000) and Wavelet Transform (WT) (Qian, et al., 2008). ST is better than other methods (Chen et al., 2006, Lin et al., 2013) in time-frequency resolution. According to the previous research (Lin et al., 2013), ST has a more straightforward relationship with Fourier Transform comparing with WT and a clear physical significance which transform coefficients are invariant frequency (Vidakovic et al.,1995). The well performance of ST leads to its extensive use (Assous et al., 2006). However, ST increases algorithm complexity (Brown et al., 2010), needs more computing time and computer memory, and that is less practical in MFCC while quick response needed.

For reducing computation cost of ST, some efficient algorithm of ST are put forward. Through eliminating redundant data, Brown et al. (2010) suggest a fast ST that improves ST efficiency, but its frequency is different from general ST, so how to get real frequency is a problem (Zhang, 2013). Depending on the features of Power Quality Disturbances, Yi et al. (2009) proposed a incomplete S-transform(IST) that also is efficient than general ST, but IST only treats main frequency, so it can only be applied in special situation.

Basing on the features that speech is wide frequency band and most people can recognize limited frequency band, we propose an efficient algorithm of ST that replaces a small section in frequency domain with a point in the section, the method decreases computation cost of ST in MFCC, and was called Compressing S transform (CST).

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2 S TRANSFORMATION AND ITS IMPROVEMENT

2.1 S Transformation

ST of continuous time signal $h(t)$ can be represented as (Stockwell et al., 1996)

$$S(\tau,f)=\int_{-\infty}^{\infty}h(t)\left\{\frac{|f|}{\sqrt{2\pi}}\exp\left(-\frac{f^2(\tau-t)^2}{2}\right)\exp(-2\pi ift)\right\}dt \quad (1)$$

In this formula, t and τ is the same parameters about time, and f is the frequency.

Obviously the width of the window (the product of $|f|/\sqrt{2\pi}$ and the real exponential) will decrease with increasing frequency. Because the narrowed integral window for the different frequencies, it will expose different resolution. It means that ST can do multi-scale analysis. For (1), its Fourier transformation is

$$S(\tau,f)=\int_{-\infty}^{\infty}H(\alpha+f)\left\{w(\alpha+f)\exp(-2\pi i\alpha\tau)\right\}d\alpha \quad (2)$$

$$H(f)=\int_{-\infty}^{\infty}h(t)\exp(-2\pi ift)\,dt \quad (3)$$

$$W(\alpha,f)=\int_{-\infty}^{\infty}w(t,f)\exp(-2\pi i\alpha t)\,dt \quad (4)$$

Here, α is the frequency after convolution.

2.2 The Compressing S Transform

Using the discrete version on (2) , we have

$$\mathrm{S}[\tau,k]=\sum_{\alpha=0}^{\frac{N}{2}-1}H[\alpha+k]\left\{W[\alpha,k]\exp(2\pi i\alpha\tau)\right\} \quad (5)$$

$$H[k]=\sum_{t=0}^{N}h[t]\exp(-2\pi ikt) \quad (6)$$

$$W[\alpha,k]=\sum_{t=0}^{N}w[t,k]\exp(-2\pi\alpha t) \quad (7)$$

From (5), the algorithm complexity of ST is $o(n^2\log n)$ larger than FFT $o(n\log n)$. The memory need for ST is $o(n^2)$. Here $n=F_s\cdot T$, T is duration of speech signal and F_s is the sample frequency.

Table 1 is the running time and memory of ST for actual speech signal that sample frequency is 14 KHz. The table shows that run time and needed memory will increase sharply with increasing duration of speech. For example, 60 seconds speech needs nearly 5000 multiple run time and 3800 multiple memory needed with 1 second speech signal.

Table 1: Computation Cost of ST.

Sample Length（second）	Runs (*10^9)	Similar memory(GB)
1s	1.87	0.73
2s	8.03	2.92
10s	232.3	73.01
60s	9625.2	2628

Generally, the frequency range of speech is 20 Hz to 20 kHz, and the human hearing is far more sensitive to sound between 100Hz and 500Hz, and the frequency resolution of the human hearing is about $\Delta f\approx 1.8Hz$. It means we are unable to receive the information a speech that frequency is too high or too low. Depending on the features of speech and human hearing, this article re-samples frequency points of ST to compress the amount of data in ST operation within an acceptable range of error, and improves the operation efficiency of ST. we call the ST with compressing data size as the compressing algorithm of ST or compressing ST, Abbreviated as CST. Through re-sampling, CST does not need to process all time-frequency data.

For a small enough section, the algorithm only picks up the intermediate point of section to participate in operation of ST. Setting parameter C as the compression rate, Fs is the sample rate, N is the data size before compression and T is signal duration, for section $[k_C-(C-1)/2, k_C+(C-1)/2]$, k_C point will be picked up, then after resample, data size of ST will change from N to Nc, and frequency resolution will be

$$\Delta f_c=\mathrm{C}\Delta f=C\frac{Fs}{N}=C\frac{Fs}{T*Fs}=\frac{C}{T} \quad (8)$$

When replaced section is small, approximately we have

$$\mathrm{S}[\tau,k_C]\approx\frac{\sum_{k=k_C-\frac{C-1}{2}}^{k_C+\frac{C-1}{2}}\mathrm{S}[\tau,k]}{C} \quad (9)$$

Then, we have

$$\mathrm{S}[\tau,k]=\sum_{\alpha_C=0}^{\frac{N}{2C}-1}H[C\alpha_C,k]\left\{W[C\alpha_C,k]\exp(2\pi iC\alpha_C\tau)\right\} \quad (10)$$

$$W[C\alpha_C,k]=\sum_{t=0}^{\mathrm{N}}w[t,k]\exp(-2\pi iC\alpha_C t) \quad (11)$$

here α_c is the re-sample value of α , and compression rate C is

$$\mathrm{C} = \frac{N}{N_c} = \frac{TF_s}{N_c} \tag{12}$$

From (10), the data size of ST will decrease C times that will lower computation cost.

3 THE APPLICATION OF THE COMPRESSED ST IN MFCC

MFCC is an analysis method on hearing system of human beings. For a time-domain speech signal, procedure of MFCC includes: pre-emphasis, framing, windowing, then for each frame, its Fourier amplitude spectrum will be filtered with Mel filter group, after that, all the filter output will be done with logarithm, Discrete Cosine Transform (DCT) and so on. Following is the Mel filter

$$H_m(k) = \begin{cases} 0 & k < f(m-1) \\ \dfrac{2(k - f(m-1))}{(f(m+1) - f(m-1))(f(m) - f(m-1))} & f(m-1) < k \le f(m) \\ \dfrac{2(f(m+1) - k)}{(f(m+1) - f(m-1))(f(m) - f(m-1))} & f(m) < k \le f(m+1) \\ 0 & k > f(m+1) \end{cases} \tag{13}$$

Here, $H_m(k)$ is the Mel filter group, m is the mth Mel filter, $f(m)$ is the centre frequency of mth Mel filter, k is frequency.

3.1 MFCC with CST

For general MFCC, how to select the frame length of speech signal and window are two problems that influence results of MFCC. The frame length must be short enough to meet short-time steady state, but is not too short to ensure sufficient frequency resolution for FT. It is difficult what the window function should be selected for the different window function will result in different MFCC. The application of ST in MFCC can simultaneously resolve two problems of general MFCC.

For speech time-history $y(t)$, after pre-emphasis, its time-frequency signal $Y(t, f)$ can be got with ST. Then faming $Y(t, f)$ can be done in time domain according to (15) and need not to do FT for every fame.

$$Y[i,k] = \frac{\sum_t^N Y[t,k]}{N} \tag{15}$$

Here, $N = T_f \cdot F_s$ is the point number. Therein, $Y[i,k]$ is the discrete $Y(t, f)$. For CST, replace N with N/C.

Every frame energy of $y(t)$ is

$$E[i,k] = \left[Y[i,k]\right]^2 \tag{16}$$

The frame energy through Mel filter group is

$$\Omega(i,m) = \sum_{k=0}^{N-1} E(i,k) H_m(k) \tag{17}$$

For $\Omega(i,m)$, do logarithm, then compute the cepstrum of DCT transform, finally we have

$$mfcc(i,n) = \sqrt{\frac{2}{M}} \sum_{m=0}^{M-1} \log\left[Y(i,m)\right] \cos\left[\frac{\pi n(2m-1)}{2M}\right] \tag{18}$$

Here i is the i^{th} frame, n is the spectral line, $mfcc(i,n)$ is N-dimension characteristic vectors of MFCC.

3.2 Error Analysis

The application of CST in MFCC must cause the distortion of MFCC feature vector. Simply, the distortion can be expressed with the mean square error (MSE) as

$$D(X,Y) = \|X - Y\|_{2,1} = \frac{1}{N_T \cdot N} \sum_{i=0}^{N_T} \sqrt{\sum_{n=1}^{N} \left(x_{i,n} - y_{i,n}\right)^2} \tag{19}$$

Here, $D(X,Y)$ is MSE about MFCC feature vector Y, X with or without CST.

4 RESULT VERIFICATION OF COMPRESSING ALGORITHM

4.1 Operation Efficiency

From (10), the computing time-consuming of CST is N/2C times of Inverse FFT with the algorithm complexity $o(n \log n)$. In FFT, split-radix FFT (Johnson, et al., 2007) is a better method with computational complexity ($4N \log N - 6N + 8$). For CST, its time complexity is $o(n^2 \log n / C)$, space complexity is $o(n^2 / C)$, and has $N / \mathrm{C} \times N$ time-frequency points，then its computational complexity is $(4N \log N - 6N + 8) / \mathrm{C}$. So it can reduce run time and memory as C times in ST.

Under the condition of Inter Core I7-4790 processor, DDR3 1866 16GB memory, 64-bit MATLAB software, we processed some speech signals with duration from 1s to 4s at interval of 0.2s. For CST, its compression rates are set from 1 to 13 at interval of 2. For each speech time-history, six tests are performed, and the average of memory consumption and run time is computed.

The figure 1 illustrates the relationship among run time, compression rates and signal duration. The figure 2 illustrates the relationship among memory, compression rates and signal duration.

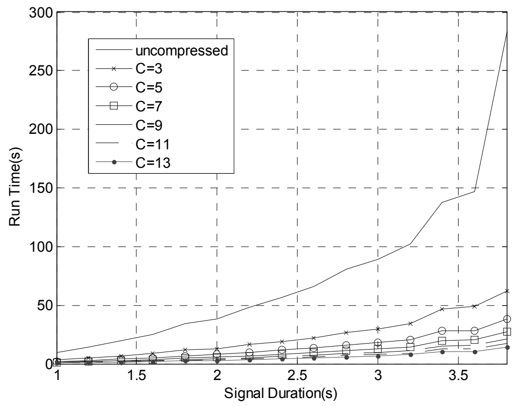


Figure 1: Relationship of Compression rate, sample time and run-time (C is Compression rate that change from 1,3,5,7,9,11 to 13).

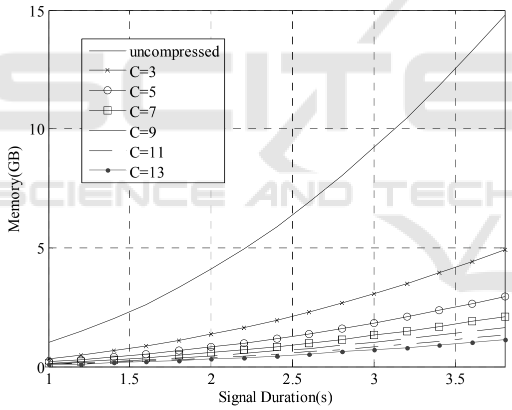
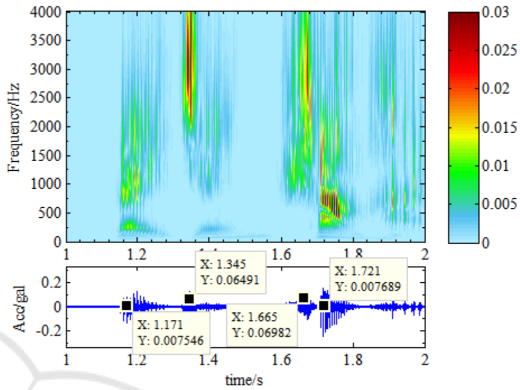


Figure 2: Relationship of Compression rate, sample time and memory (C is Compression rate that change from 1,3,5,7,9,11 to 13).

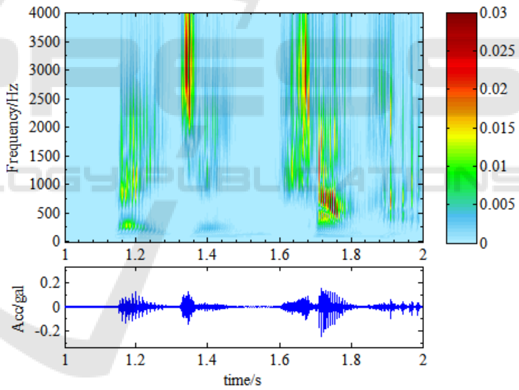
When signal duration is longer than 4 seconds, the memory is not enough to store the data and some data must store in hard disk, the computation efficiency drops rapidly and the run time increases to 1514s. With the increasing compression, the needed memory and the run time decrease significantly. For instance, when the duration of speech is 3.6s, ST need 147s and about 13GB memory, CST with triple compression needs about 49s and 4.4GB memory which reduced 66% run time, and with thirteen compression needs about 10.3s and 1GB which reduced 93% run time.

4.2 The Influencing of Compressing

Figure 3 is the time-frequency diagram of speech with 44.1KHz sampling frequency. In the figure 3, (a) is uncompressing, (b) is 20 times compression. The bottom of 3(a) and 3(b) is the time-history of speech, the top is the time-frequency diagram, time-frequency energy distribution in the figure is corresponding with the time-history diagram. There is no significant difference between two time-frequency diagrams.



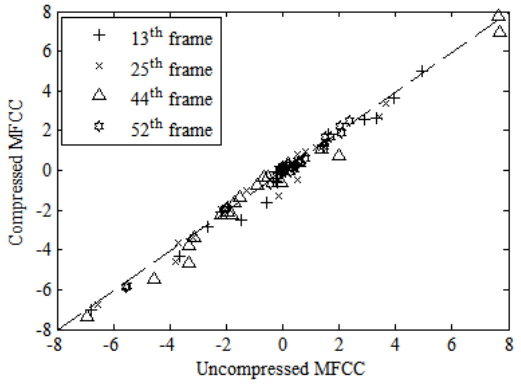
(a) uncompressed time-frequency signal



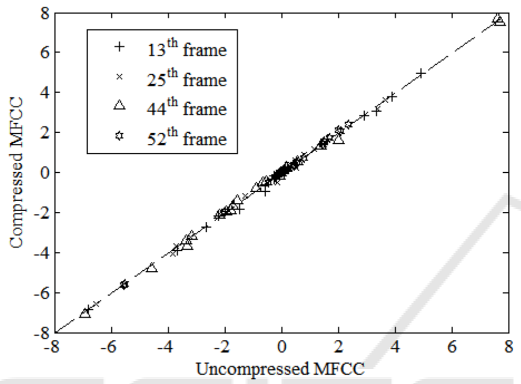
(b) 20 times compression time-frequency signal

Figure 3: ST Time-Frequency analysis diagram.

Figure 4 illustrates correlation of the MFCC results computed from un-compression and compression methods through linear regression analysis. Two diagram show the results with 50 times compression give the larger deviation that means that error of results will increase with increasing compression.



(a) 50 times compression



(b) 20 times compression

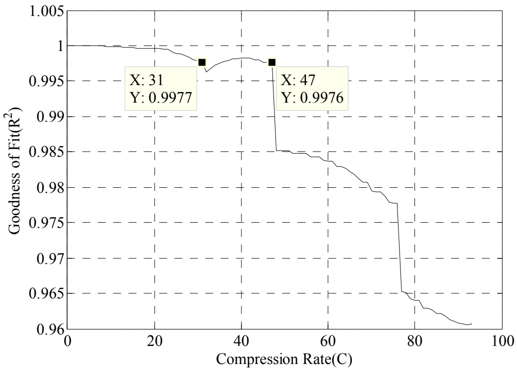
Figure 4: Comparing the MFCC results with different compression and uncompressing (horizontal ordinate is uncompressed, vertical ordinate is compression).

Goodness of fit reflects level of similarity between two vectors, defined as

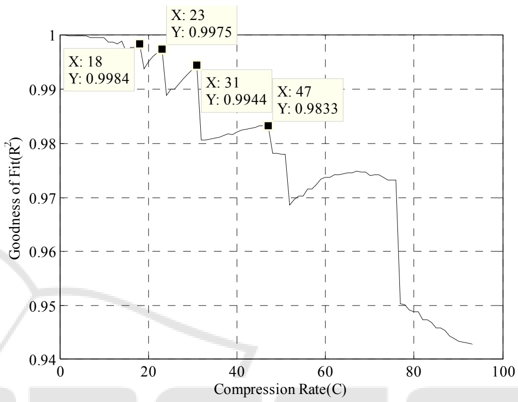
$$R^2 = 1 - \frac{\sum (y-x)^2}{\sum (y-\overline{y})^2} \tag{20}$$

Here, $\overline{y}$ is the average of y, and figure 5 show that the goodness of fit change with compression rate. In figure 5, (a) and (b) are the results respectively with male voice and female voice.

For figure 5, the signal duration is 0.38s that eliminates zero energy points in speech. From diagrams, the goodness of fit will decrease with the compression rates increase. In the diagram, we can find some important points, for male, these points are C=31, 47, and for female, these points are C=18, 23, 31, 47. These points maybe reflect some features of speech. From figure 5, we can get conclusion that the larger compression can be used for male in the same goodness of fit.



(a) The result with male voice



(b) The result with female voice

Figure 5: The Change of Goodness of Fit with Compression ratio (in figure, horizontal ordinate is compression rate, vertical ordinate is goodness of fit).

In speech recognition, a small difference of MFCC will result in large recognition error, so in order to ensure the reliability of the recognition results, we limit that the goodness of fit is not smaller than 0.99. Then for figure 5, we can find a key point where the compression C=31, as long as the compression is not larger than 31, either male or female, the goodness of fit will is larger than 0.99. For this key point, from (8), we have

$$\frac{C}{T} = \Delta f_c < 82Hz \tag{21}$$

That means, as long as re-sampling frequency Δf_c is less than 82Hz, the results will be reliable.

5 CONCLUSIONS

ST is excellent in time-frequency analysis for high resolution, energy concentration, and without cross terms. The application of ST can resolve two key problems and improve accuracy in MFCC analysis, but also be hindered due to the enormous operational consumption.

Based on the features of human hearing and speech is insensitive with speech signal resolution, the CST re-sampling the frequency points in ST frequency space to reduce data size in ST operation, effectively reduces the run time and memory consumption of ST. Applying CST in MFCC, the actual speech signal analysis proves while re-sampling interval satisfies $\Delta f < 82Hz$, the results of MFCC is reliable.

In this article, we set that the goodness of fit is not smaller than 0.99, although it satisfies the reliability of MFCC, it is at the expense of efficiency. So, whether or not to adopt a smaller goodness value of fit, when goodness of fit reduces, what phenomenon will produce in recognition of speech based on MFCC, and what a smallest goodness value of fit is that can ensure in recognition of speech. Resolving these problem will contribute to the effective application of CST in MFCC.

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