

# Model Predictive Control for Y-source Boost DC-DC Converter

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Abstract: Recently a new topology called Y-source impedance network has been proposed to enhance the performance of boost dc-dc converters. The Y-source boost dc-dc converter has shown its ability to offer high gain voltage with small duty ratio. This paper presents an algorithm based on Model Predictive Control (MPC) to control the Y-source boost DC-DC converter. An analytical MPC algorithm reducing the computation time is proposed. Using this technique a fast response and steady state output can be achieved. Besides, the proposed controller controls directly the switch position, so Pulse-Width Modulation (PWM) is not required in this technique. The proposed algorithm offer optimal solution in reasonable time and it is not considered as a computation burden, thus real-time implementation is possible; overcoming the inherent drawback of classical MPC controller. Simulation results, demonstrating the controller capabilities to produce the required high gain voltage, are presented.

## 1 INTRODUCTION

Recently many of researchers focused on the development of boost dc-dc converter with high gain voltage. Several impedance networks have been proposed to enhance the power conversion with high voltage gain. Introducing coupled magnetic has been lately proposed to improve the impedance network while using a shorter duty ratio. In this direction several techniques has been presented in literature like the T-Source (Strzelecki et al., 2009), Z-source (Qian et al., 2011), TZ-source (Nguyen et al., 2013),  $\Gamma$ -source (Loh et al., 2013) and Y-source (Siwakoti et al., 2014). The obtained gain of Y-source is presently not matched by other networks operated at the same duty ratio; a mathematical derivation and experimental results have proven this capability (Siwakoti et al., 2014). The Y-source has been considered as generic network, from which the other networks can be derived (Siwakoti et al., 2015).

On the other hand, Model Predictive Control (MPC) appears to be an efficient strategy to control many applications in numerous industries. It can efficiently control a great variety of processes, including systems with long delay times, non-minimum phase systems, unstable systems, multivariable systems, constrained systems and hybrid systems (Camacho and Bordons, 1999), (Maciejowski, 2002), (Thomas et al., 2004), as well

as systems with discrete inputs only (Thomas, 2012). MPC has become an accepted standard for constrained multivariable systems (Mayne *et al.*, 2000).

Many applications of MPC controller in power electronics area have been presented in literature, for example in (Wang, 2012) and (Vazquez et al., 2014), including controls of traditional boost converters (Beccuti et al., 2007) and (Murali et al., 2010). The main contribution of this paper is to develop a MPC algorithm to control the output voltage of the Y-source boost dc-dc converter. The proposed controller based on analytical computation of the cost function for both of On and Off states of the single switch. The proposed MPC algorithm controls directly the switch position to obtain the required gain voltage.

The rest of the paper is organized as following; section 2 briefly presents the Y-source boost dc-dc converter, while the concepts of MPC has been presented in section 3. Section 4 presents the proposed MPC algorithm for Y-source dc-dc converter. Results are demonstrated in section 5. Finally conclusion and some remarks are given in section 6.

## 2 THE MATHEMATICAL MODEL OF Y-SOURCE BOOST DC-DC CONVERTER

Recently, the Y-shaped impedance network that can offer a high-voltage gain converter while using a small duty ration is proposed (Siwakoti et al., 2014). It uses a tightly coupled transformer with three windings. An application of Y-shaped impedance network with a single switch dc-dc converter (Figure 1) has shown its ability to offer more degrees of freedom for varying its gain. The voltage gain of Y-source boost dc-dc converter is given by (1); the mathematical derivation is given in (Siwakoti et al., 2014):

$$V_{out} = V_{in} / [1 - Kd_{st}] \quad (1)$$

where  $K = \frac{N_3 + N_1}{N_3 - N_2}$ ,  $N_1, N_2, N_3$  are the winding turns of the three windings and  $d_{st}$  is the duty ratio. The gain of (1) and its related turns ratio are summarized in Table 1 (Siwakoti et al., 2014).

Table 1: Gain of Y-source boost dc-dc converter with different winding factor K and Turns Ratio ( $N_1:N_2:N_3$ ).

$N_1:N_2:N_3$	$K = \frac{N_3 + N_1}{N_3 - N_2}$	Gain
(1:1:3),(2:1:4),(1:2:5),(3:1:5),(4:1:6), (1:3:7)	2	$(1-2d_{st})^{-1}$
(1:1:2),(3:1:3),(2:2:4),(1:3:5), (4:2:5)	3	$(1-3d_{st})^{-1}$
(2:1:2),(1:2:3),(5:1:3),(4:2:4), (8:1:4)	4	$(1-4d_{st})^{-1}$
(3:1:2),(2:2:3),(1:3:4),(7:1:3), (6:2:4)	5	$(1-5d_{st})^{-1}$
(4:1:2),(3:2:3),(2:3:4),(1:4:5), (9:1:3)	6	$(1-6d_{st})^{-1}$
(5:1:2),(4:2:3),(3:3:4),(2:4:5)	7	$(1-7d_{st})^{-1}$
(6:1:2),(5:2:3),(4:3:4),(3:4:5), (2:5:6)	8	$(1-8d_{st})^{-1}$
(7:1:2),(6:2:3),(5:3:4),(4:4:5)	9	$(1-9d_{st})^{-1}$
(8:1:2),(7:2:3),(6:3:4)	10	$(1-10d_{st})^{-1}$

In the following the mathematical models of the Y-source converter for both of the ON and OFF positions of the switch SW are delivered.

When SW is turned ON,  $D_1$  and  $D_2$  are reverse-biased (i.e. turn off), causing  $C_1$  to charge the magnetizing inductance of the transformer. At the same time,  $C_2$  discharges to power the load. the equivalent circuit in this case is shown in Figure 2, and the circuit analysis is as follows:

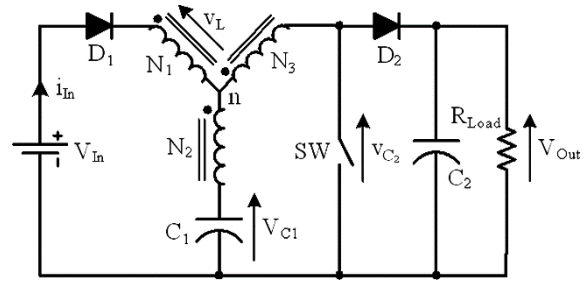


Figure 1: Single-switch Y-source dc-dc boost converter.

$$V_{c1} + V_L / n_{12} - V_L / n_{13} = 0 \quad (2)$$

where  $n_{12} = N_1 / N_2$  and  $n_{13} = N_1 / N_3$ , then:

$$i'_L(t) = \frac{n_{12}n_{13}}{L(n_{12} - n_{13})} V_{c1}(t) \quad (3)$$

$$V'_{C1}(t) = \frac{-i_{C1}}{C_1} = -\frac{n_{12}i_L(t)}{C_1} \quad (4)$$

$$V'_{C2}(t) = \frac{-i_o}{C_2} = -\frac{V_{c2}(t)}{C_2 r_o} \quad (5)$$

Using the Euler discretization with a sampling time  $T_s$ :

$$i_L(k+1) = i_L(k) + T_s \frac{n_{12}n_{13}}{L(n_{12} - n_{13})} V_{c1}(k) \quad (6)$$

$$V_{C1}(k+1) = V_{C1}(k) - T_s \frac{n_{12}}{C_1} i_L(k) \quad (7)$$

$$V_{C2}(k+1) = \left(1 - T_s \frac{1}{C_2 r_o}\right) V_{C2}(k) \quad (8)$$

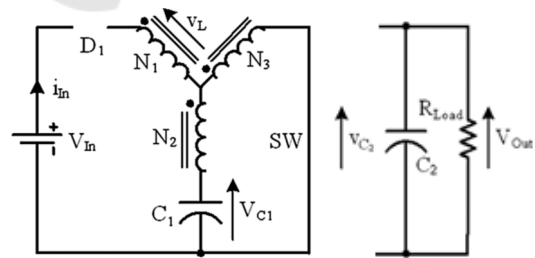


Figure 2: The equivalent circuit during SW ON.

When the SW is turned OFF,  $D_1$  and  $D_2$  are reverse conducting, causing  $V_{in}$  to recharge  $C_1$ . Energy from  $V_{in}$  and the transformer will also flow to the load. The equivalent circuit is shown in Figure 3, and the circuit analysis is as follows:

$$V_{in} - V_L - V_L / n_{13} - V_{c2} = 0 \quad (9)$$

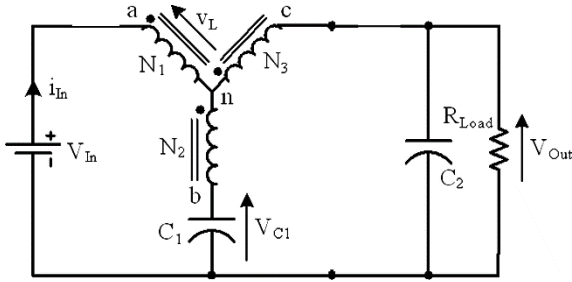


Figure 3: The equivalent circuit during SW OFF.

$$i'_L(t) = -\frac{n_{13}}{L(n_{13}+1)}V_{c2}(t) + \frac{n_{13}}{L(n_{13}+1)}V_{in} \quad (10)$$

$$V'_{C1}(t) = \frac{i_{C1}}{C_1} = \frac{n_{12}i_L(t)}{C_1} \quad (11)$$

$$V'_{C2}(t) = \frac{i_{C2}}{C_2} = \frac{i_{L3} - i_o}{C_2} = \frac{n_{13}i_L(t)}{C_2} - \frac{V_{c2}(t)}{C_2 r_o} \quad (12)$$

Using the Euler discretization:

$$i_L(k+1) = i_L(k) - T_s \frac{n_{13}}{L(n_{13}+1)}V_{c2}(k) + T_s \frac{n_{13}}{L(n_{13}+1)}V_{in} \quad (13)$$

$$V_{C1}(k+1) = V_{C1}(k) + T_s \frac{n_{12}}{C_1}i_L(k) \quad (14)$$

$$V_{C2}(k+1) = T_s \frac{n_{13}}{C_2}i_L(k) + \left(1 - T_s \frac{1}{C_2 r_o}\right)V_{c2}(k) \quad (15)$$

These developed equations will be used by the MPC controller to predict the future outputs of the Y-source boost dc-dc converter in both ON and OFF states; (6)-(8) and (13)-(15) respectively.

### 3 MODEL PREDICTIVE CONTROL

Predictive control was first developed at the end of 1970s, and was published by Richalet *et al.*, (1978). In the 1980s, many methods based on the same concepts are developed. Those types of controls are now grouped under the name Model Predictive Control (MPC) (Camacho and Bordons, 1999). MPC has proved to efficiently control a wide range of applications in various industries.

The main idea of predictive control is to use a model of the plant to predict future outputs of the

system. Based on this prediction, at each sampling period, a sequence of future control values is developed through an on-line optimization process, which maximizes the tracking performance while satisfying constraints. Only the first value of this optimal sequence is applied to the plant. The whole procedure is repeated again at the next sampling period according to the 'receding' horizon strategy (Maciejowski, 2002). The objective is to lessen the future output error to zero with minimum input effort. The cost function to be minimized is generally a weighted sum of square predicted errors and square future control values, e.g., in Generalized Predictive Control (Clarke et al., 1987):

$$J(N, N_u) = \sum_{j=1}^N \beta [\hat{y}(k+j|k) - w(k+j)]^2 + \sum_{j=1}^{N_u} \lambda [u(k+j-1)]^2 \quad (16)$$

where  $\hat{y}, u$  are the predicted output and the control signal respectively.  $N, N_u$  are the prediction horizons and the control horizon, respectively.  $\beta, \lambda$  are weighting factors. The control horizon permits a decrease in the number of the calculated future control assuming  $\Delta u(k+j) = 0$  for  $j \geq N_u$ .  $w(k+j)$  is the reference trajectory.

Constraints over the control signal, the outputs and the control signal changing, can be added to the cost function:

$$\begin{aligned} u_{\min} &\leq u(k) \leq u_{\max} \\ \Delta u_{\min} &\leq \Delta u(k) \leq \Delta u_{\max} \\ y_{\min} &\leq y(k) \leq y_{\max} \end{aligned} \quad (17)$$

The solution of (16) gives the optimal sequence of the control signal over the horizon  $N_u$  while respecting the given constraints of (17).

### 4 MPC FOR Y-SOURCE DC-DC CONVERTER

As the switch of the inverter has only two different positions; ON and OFF, an analytical computation of the tracking performance, for the two possible position combinations can be performed. Then the position of the switch, which is the manipulated variable, which maximizes the tracking performance is selected.

The objective function that captures the tracking performance includes the error between the actual output voltage  $V_{out}$  and the reference trajectory of the output voltage. To minimize the inverter switching frequency a penalty term on the control variations is included in the objective function. The considered objective function is:

$$J = \sum_{j=1}^N Q(\hat{V}_{out}(k+j|k) - V_r(k+j))^2 + \sum_{j=0}^{N_u-1} P_j (u(k+j) - u(k+j-1))^2 \tag{18}$$

where  $\hat{V}_{out}$  is the predicted future output voltage,  $V_r$  is the output voltage reference,  $u$  is the ON/OFF control signal, and where  $Q$  and  $P_j$  the weighting matrixes are positive constants. The second term penalizes the switch position variation. The objective function (18) is minimized subject to constraints that describe the discretized dynamics in (6)-(8), and (13)-(15).

The constants  $P_j$  should impose more penalties over the first time-steps than the later steps, to force the transition of the switch to occur as late as possible (Papafotiou et al., 2007). This is accomplished by the following constraints:

$$P_0 > P_1 > \dots > P_{N_u-1} \tag{19}$$

The objective function (18) is evaluated  $s = 2^{N_u}$  times at each time step, and the first control signal in the sequence  $u^{opt} = (u(k), \dots, u(k + N_u - 1))$  corresponding to the minimum objective function value is then selected and applied to the inverter switch.

Increasing the prediction horizon  $N$  will lead to more accurate choice of control signals. However, increasing the prediction horizon will increase the computational time. To account for that, we propose to use different discrete time models with different sampling times as described in (Thomas and Hansson, 2013). For the first sampling steps we use a model with the true sampling time, and then for later sampling steps we use another model with longer sampling time. This will increase the prediction interval with less number of prediction steps as compared to when using the same sampling time for all predictions.

To avoid examining all possible input switching over the control horizon  $N$  the following incremental algorithm is proposed to compute the

optimal control signal sequence. Here  $u^i$  is a candidate optimal control signal sequence that is an element in  $u \times u \times \dots \times u$ .

**Algorithm 1.**

- 1- Initializing with  $J_{opt} = \infty, J^i(k) = 0$
- 2- For  $u^i, i \in \{1, 2, \dots, s\}$  where  $s$  is the total number of possible input combinations over horizon  $N$
- 3- For  $j = 1 : N$ 
  - 4- Compute  $J^i(k+j)$  the cost function according to the control combination  $u^i$  for horizon  $j$  as following:  
 $J^i(k+j) = J^i(k+j-1) + f(\mathbf{x}(k+j), \mathbf{u}^i(k+j-1))$   
 where  $f(\mathbf{x}(k+j), \mathbf{u}^i(k+j-1))$  is the cost at instant  $(k+j)$  due to the control signal  $\mathbf{u}^i(k+j-1)$ .
  - 5- If  $J^i(k+j) > J_{opt}$   
     Break and go to step 2  
     end
  - 6- At  $j = N$   
     If  $J^i(k+N) < J_{opt} \rightarrow J_{opt} = J^i(k+N)$   
     end
- End
- 7-  $J^*_{opt} = J_{opt}$  the optimal solution

The incremental cost (in step 4 of Algorithm 1) is the predicted cost at time step  $k+j$  due to the control signal  $u^i(k+j-1)$ , and it is given by

$$f(\hat{v}(k+j|k), u^i(k+j-1)) = Q(\hat{V}_{out}(k+j|k) - w(k+j))^2 + P_j (u(k+j) - u(k+j-1))^2$$

Algorithm 1 stops the cost function calculations for the control sequence  $u^i$  prematurely if the cost function at prediction step  $j$ , where  $1 < j < N$ , is higher than the current upper bound  $J_{opt}$ . This saves computational time. The algorithm is similar to one of the pruning rules in the Branch and Bound (BB) algorithm for solving integer programs (Fletcher and Leyffer, 1995).

The proposed controller is faster than other standard techniques for solving integer programming

problems like for example BB, as the analytical computation of the objective function when the number of optimization variables is small, which is the case in the considered application, is much faster than solving a QP optimization problem. Moreover, the relaxed problem for the suggested MPC algorithm would not be a quadratic program, since we have introduced a penalty term on the number of switches so it could be expensive to solve with classical optimization technique.

The advantages of the proposed technique besides its simple design and implementation are that there is no complicated on-line optimization to be performed. Moreover there is no need to reformulate the system in the hybrid system framework, as done in (Beccuti et al., 2007).

The developed technique significantly reduces the computational time. Moreover, one extra dimension of freedom through the choice of the weights  $P_j$  has been added, which enables a trade-off between the average switching frequency and the voltage tracking performance. Note that reducing the ripple can only be achieved by increasing the switching frequency and vice versa.

#### 4.1 Constraints

Output signals and system states can be subject to constraints. This constraints could, for example, relate to safety or physical constraints. This constraints can be included in the proposed controller. by adding the following line to Algorithm 1:

$$if \quad x(y) > x_{max}(y_{max}) \rightarrow J^i(k+j) = \infty \quad (20)$$

Thus any control combination which will lead to violation of the output or state constraints will be avoided.

### 5 RESULTS

The proposed control strategy is applied to the Y-source boost dc-dc converter shown in Figure 1, whose parameters are given in Table 2. After successive tuning iterations, the parameters of the MPC controller that give a good response are: control horizon  $N_u = N = 8$ , prediction interval  $= 20 \times T_s$ . The concept of multiple discrete models, as mentioned previously, is used to reduce the number of prediction steps; a model with sampling time  $T_s$  is used for the first four steps, and then a model with sampling time equal  $4T_s$  is used for the next 4 steps,

i.e. the prediction interval of in total  $20T_s$  is covered with 8 prediction steps. The weights in the objective function has been chosen as  $P_j = 100$ , and  $Q = 10000$ . A sampling time  $T_s$  of  $10\mu s$  is used.

Computer simulations have been carried out in order to validate the proposed scheme. The Y-source boost dc-dc converter is assumed to start at  $t=0$  with zero initial condition ( $i_l=0$ ;  $V_{c1}=0$ ;  $V_{c2}=0$  and  $V_{out}=0$ ; start-up) and it is required to support the load with a voltage  $V_{out}=200V$ , i.e. a gain of 4 is required. Figure 4 shows the output voltage with the proposed MPC controller, and also the inductance current  $i_l$ . It is obvious that the proposed MPC algorithm succeeded in providing the required output voltage.

Table 2: Parameters of the Y-source boost dc-dc converter.

Parameter/Description	Value
Input Voltage $V_{in}$	50V
Output Voltage $V_{out}$	200V
Capacitance $C_1$ & $C_2$	470 $\mu$ F
Turns Ratio: $N_1:N_2:N_3$	80:16:48=5:1:3
Winding Factor $K$	4
Inductance $L$	1mH
Load Resistance $r_o$	1K $\Omega$

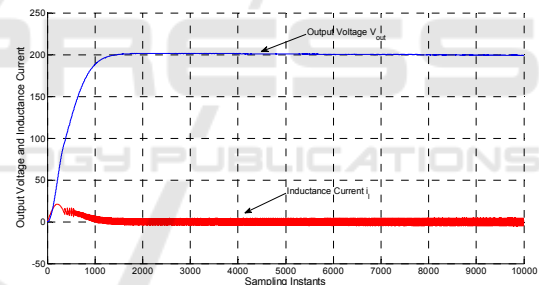


Figure 4: Output Voltage and Inductance current.

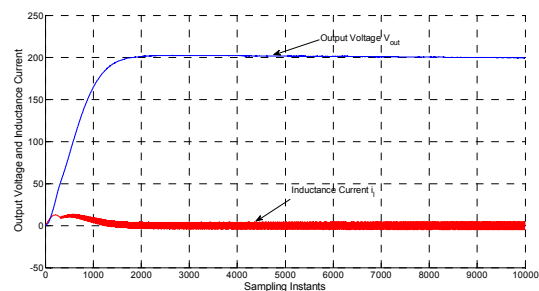


Figure 5: Output Voltage and Inductance current  $V_{in} = 30V$ .

Figure 5 shows the case when the input voltage source  $V_{in}$  drops to 30V, the controller succeeded in tracking the reference of output voltage, however it takes relatively more time to reach the steady state value, and the mode of ON was more selected by the

controller to compensate the voltage source drop.

Applying Algorithm 1 to reduce the number of cost function evaluation, for  $N=8$ , the average number of cost function evaluation was 95 times instead of  $2^8 = 258$  times, with reduction ratio of 62.9%. The technique presented here does not require average model of the switched system, moreover the proposed controller controls directly the switch, and hence the PWM inverter is not needed. This technique can be extended and applied to other types of converters possibly with multiple switches.

## 6 CONCLUSIONS

In this paper an algorithm based on model predictive control is used to control the Y-source boost dc-dc converter. The proposed algorithm computes analytically the cost function, a reduction technique to avoid evaluating the all possible cost function over the prediction horizon is used. The developed controller controls directly the inverter switches to track the output voltage trajectory. With this technique there is no need to use a PWM inverter, and moreover, it reduces significantly the computational time, which is an inherent drawback of classical MPC controllers. Thus real time implementation is possible. It is simple to construct, to implement and to tune.

Future work will include experimental works to validate this technique in practice. Finally, the same technique will be examined for other topologies with other types of converters possibly with multiple switches.

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