

# MILP-based Approach for Optimal Implementation of Reconfigurable Real-time Systems

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**Abstract:** This paper deals with the design and implementation of reconfigurable uniprocessor real-time embedded systems. A reconfiguration is a run-time operation allowing the addition-removal of real-time tasks or the update of their parameters. The system is implemented then by different sets of tasks such that only one is executed at a particular time after a corresponding reconfiguration scenario according to user requirements. The problem is to optimize the system code while meeting all related real-time constraints and avoiding any redundancy between the implementation sets. Based on the Linear Programming (MILP), we propose a multi-objective optimization technique allowing the minimization of the number of tasks and their response times. An optimal reconfigurable POSIX-based code of the system is manually generated as an output of this technique. We apply the paper's contribution to the study of the performance evaluation.

## 1 INTRODUCTION

A real-time system is any system which has to respond to externally generated input stimuli within a finite and specified delay (Burns and Wellings, 2009). The development of real-time systems is not a trivial task because a failure can be critical for the safety of human beings (Cottet and Grolleau, 2005). The researchers are moving today toward proposing techniques for programming concurrent reconfigurable real-time systems.

To provide design-time guarantees on timing constraints, different scheduling methodologies can be used, such as Rate Monotonic (RM) which is a scheduling algorithm which was defined by Liu and Layland (Liu and Layland, 1973) where the priority of tasks is inversely proportional to their periods. The authors of (Bouaziz et al., 2015) provide a method to drive the designer by producing a set of design solutions based on RM scheduling algorithm. In (Mehiaoui et al., 2013; Woźniak, ), the authors are interested in the optimization of deployment techniques from functional and platform models of real-time systems by using mixed integer linear program-

ming (MILP). An MILP formulation is easily extensible, re-targetable to a different optimization metric and can easily accommodate additional constraints or legacy components (Mehiaoui et al., 2013). There are many programming languages designed for the development of real-time systems such as POSIX (Portable Operating System Interface) (Lewine, 1991). The POSIX standard promotes portability of applications across different operating system platforms. The authors (Obenland, 2000) use POSIX in the development of software for real-time and embedded systems.

The synthesis of a valid and optimal implementation model consists in building the set of tasks implementing the applicative functions while meeting all related real-time constraints. The reconfiguration at the implementation level consists in adding/removing tasks or modifying their timing parameters to go from one implementation to another, which may require an additional time for reconfiguration. So that, the resulting implementation model should avoid redundancy between the different implementations to minimize the possible overhead.

In this paper, we present an approach toward an op-

timal implementation of reconfigurable uniprocessor real-time systems. The proposed approach aims to automatically produce a valid and optimal task model from a given specification. The task model consists of a set of tasks implementing the applicative functions that we assume independent and periodic. We assume also that assigning priorities to tasks is performed using rate monotonic algorithm RM (Klein et al., 1993).

The paper is organized as follows. Section 2 gives an overview on related works. Section 3 provides the formalisation of approach. Section 4 explains in details the proposed approach to obtain a valid and optimal implementation model from the user specification. Section 5 illustrates the approach on the chosen case study and evaluates its efficiency. Finally, we summarize our work and discuss the future work in Section 6.

## 2 RELATED WORKS

In this section, we present the related works that deal with real-time systems and reconfigurable architectures.

In the literature, many approaches have been carried out in the area of schedulability analysis for meeting real-time requirements. Pillai and Shin (Pillai and Shin, 2001) propose an optimal algorithm for computing the minimal speed that can make a task set schedulable. Concerning the system model and the optimization objectives, the proposed approach is closely related to the research works in (Bertout et al., 2014; Bouaziz et al., 2015; Mehiaoui et al., 2013). In (Bertout et al., 2014), the authors propose a technique to minimize the number of tasks in a real-time system while satisfying timing constraints. The approach in (Bouaziz et al., 2015) aims both to reduce the number of preemptions for minimizing timing overheads and to maximize the laxity of tasks in order to improve the schedulability of the design model. There are also related approaches which generate complete real-time systems. In (Pillai and Shin, 2012), the authors deliver an approach called TASTE to enable the generation of a complete real-time distributed system. The authors in (Pagetti et al., 2011) provide a framework that allows designers to automatically generate by the Prelude language, a set of real-time tasks that can be executed on a uniprocessor architecture.

Nowadays, many research works have been proposed to develop reconfigurable systems. The authors in (Gharsellaoui et al., 2012) propose an approach that deals with reconfigurable systems to be implemented with different tasks under deadline constraints according to user requirements. In (Krichen et al., 2010),

the authors aim to provide an automated development process from modeling to implementation for the dynamic software part of reconfigurable systems.

There are many programming languages designed for the development of real-time systems. Among the most used real-time languages, we cite real-time java (RT-java) which aims to support the programming of real-time codes from different directions used by other software development platforms. POSIX (Portable Operating System Interface) is a standard written in terms of the C programming language. (Lewine, 1991). This standard facilitates the application portability that is why we adopt it as a target language to implement a reconfigurable real-time system in the current paper.

The main contributions of this paper are four-fold. The first part consists in ensuring the respect of timing properties before the effective implementation of the real-time system (i.e. at the design level). Second, we are interested in the reconfiguration of real-time systems where the addition and removal of tasks are applied at run-time. Third, we propose a multi-optimization metric. Indeed, the proposed approach aims to minimize the reconfiguration time by avoiding a redundancy between the different implementations from one side. From the other side, it aims to minimize the response times of the real-time tasks in order to maintain the performance of the system. The fourth part, this work automatically generates a complete reconfigurable real-time system from the specification level by using the programming language POSIX. None of the existing works is solving all the four problems together.

## 3 SYSTEM FORMALIZATION

In this section, we present a formal description of a reconfigurable uniprocessor system. We present in addition real-time prerequisites required to introduce the paper's contribution.

It is assumed in this work that a reconfigurable real time system  $Sys$  is defined as a set of implementations:  $Sys = \{imp_1, imp_2 \dots imp_m\}$ . We denote by  $Sys(t)$  the implementation defining the system at a particular time  $t$  (i.e.  $Sys(t) = imp_i$ ). An implementation  $imp_i$  is composed of  $n$  tasks that we assume *independent* and *periodic* (i.e.  $imp_i = \{\tau_1, \tau_2, \tau_3 \dots \tau_n\}$ ). Each task  $\tau_i$  executes a set of applicative functions  $\tau_i = \{F_1, F_2, F_3 \dots F_k\}$ . A function  $F_i$  is characterized by static parameters  $F_i = (T_{fi}, C_{fi})$  where  $T_{fi}$  is the activation period of the function  $F_i$  and  $C_{fi}$  is an estimation of its worst case execution time WCET. Note that these parameters are considered as inputs

to the proposed approach and must be specified by the user. Each task  $\tau_i$  is characterized by a set of real-time parameters  $(r_i, T_i, C_i, D_i, P_i)$ : its release time  $r_i$ , we assume that  $r_i = 0$ , its activation period  $T_i$  which is deducted from the activation periods of the functions implemented by this task, its capacity or worst case execution time  $C_i$  which is equal to the sum of the WCETs of the functions executed by this task, its deadline  $D_i$  we assume that  $D_i = T_i$ , the priority  $P_i$ , we assume that  $P_i = 1/T_i$  since we adopt the Rate Monotonic priority assignment(RM).

Let  $U$  be the processor utilization factor defined by:  $U = \sum_{i=1}^n \frac{C_i}{T_i}$ . We perform Rate-Monotonic (RM) response time analysis based on the computation of an upper bound of the response time  $Rep_i$  of the different tasks constituting the task model. This analysis aims to verify whether these tasks complete their computations within the time limit specified by the real-time application i.e. the deadline ( $Rep_i \leq D_i$ )(Klein et al., 1993). The reconfiguration scenario corresponds to adding/removing tasks or modifying timing parameters. Thus, we introduce the reconfiguration time  $T_{reconf}$  which refers to the time required to jump from one implementation to another according to user requirements (i.e. reconfiguration conditions). This parameter is defined as follow:

$$T_{reconf} = A * T_{delete} + B * T_{creat}$$

Where  $A$  is the number of deleted tasks,  $B$  is the number of created tasks,  $T_{delete}$  is the spent time to delete a task and  $T_{creat}$  is the spent time to create a task. We assume that the blanking time  $T_{delete}$  and creation time  $T_{creat}$  of all the tasks are equals for a considered platform (i.e.  $T_{delete} = T_{creat}$ ). We denote by  $T_{cost}$  the spent time to create a task or to delete it (i.e.  $T_{delete} = T_{creat} = T_{cost}$ ). Thus, the reconfiguration time is given as follow:

$$T_{reconf} = (A + B) * T_{cost}$$

## 4 PROPOSED APPROACH

In this section, we present an overview on our approach and detail the structure of different modules involved in this work. We deliver an approach which automatically converts a high-level specification of a reconfigurable real-time system into an executable running on POSIX platform. Figure 1 shows the process of the proposed approach. As entry, the designer provides the specification model which defines the reconfiguration conditions, the applicative functions that must be executed under a considered condition and the temporal parameters of each function. This model presents the input of the task generator step which aims to produce an initial task model. Then,

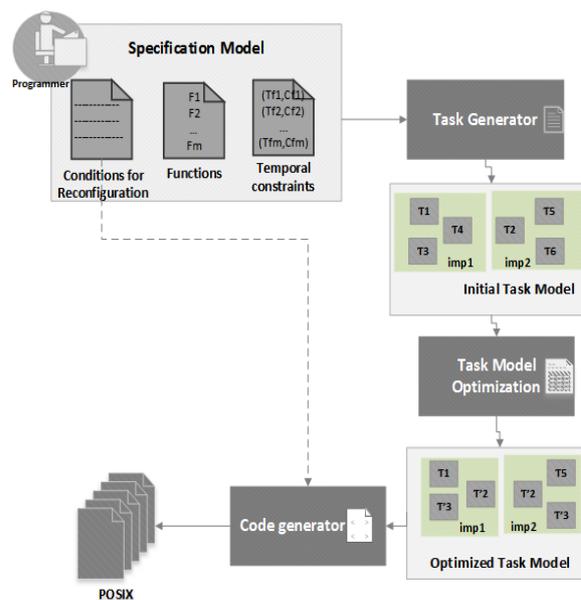


Figure 1: Process overview.

the optimization step receives the generated model and proposes a valid and optimal task model. This model is finally converted into an executable program running under POSIX.

### 4.1 Task Generator

The first step consists in generating the initial task model. This stage considers the specification model as an input and aims to generate the initial task model which defines a possible implementation of the considered system. For each reconfiguration condition, this step generates an implementation and associates its appropriate functions. Then, for each generated implementation, it regroups the functions having the same period  $T_{fi}$  to be executed by one task  $\tau_i$ . Since we assume that the release time  $r_i = 0$  and  $P_i = 1/T_i$ , the task  $\tau_i$  is characterized only by  $(T_i, C_i, D_i)$  where the period  $T_i$  corresponds to the period of the grouped functions,  $C_i$  is the sum of WCETs of the grouped functions and the deadline  $D_i$  of each task is equal to the corresponding period  $T_i$ . Let us note that for the generation of this model, the optimization and real-time feasibility concerns are not considered.

### 4.2 Task Model Optimization

This phase aims to produce a feasible and optimal implementation of the reconfigurable real-time system from the initial task model.

In order to avoid redundancy between the sets of implementation and reduce the number of tasks, this

phase aims to merge the tasks belonging to different implementations but implementing the same functions and/or having the same periods. To implement properly the problem by taking into consideration the different constraints, we propose a MILP formulation of our problem. So we should define the objective function and the required constraints for parameters and variables.

**Definitions.** Let  $m$  be the number of tasks in the initial model, let  $N$  be the number of tasks in the new task model, let  $s$  be the starting time which corresponds to effective starting time of each task. We denote by *InitTask* the initial task model which is a three column matrix where the first column presents the period  $T_i$  of task, the second one presents their WCETs  $C_i$  and the third column is their deadline  $D_i$ . *NewTask* is the resulting task model after merging the different tasks (i.e. optimized task model).

### Objective Function.

$$\text{Maximize } \sum_{i,j \in \{1,m\}} \text{Merge}_{ij} - \sum_{i,j \in \{1,m\}} \text{Rep}_{ij} \quad (1)$$

This expression defines the objective function of our problem. *Merge* denotes a boolean variable used to mention whether two tasks  $\tau_i$  and  $\tau_j$  are merged. More in detail,  $\text{Merge}_{ij}$  is equal to 1 if task  $t_i \in \text{imp}_k$  and task  $t_j \in \text{imp}_l$  are merged. The expression(1) aims to maximize the number of merged tasks and minimize the sum of response times of the different tasks constituting the task model. In order to limit non meaningful merging situations, we define in addition the following constraints:

### Merging Situation Constraints.

The constraints (2) and (3) introduce the merging condition such as tow the tasks  $\tau_i \in \text{imp}_k$  and  $\tau_j \in \text{imp}_l$  will be merged if they have the same period.

$$\forall i, j \in \{1 \dots m\} \text{ et } i \neq j, \\ \text{if}(\text{InitTask}[i, 1] - \text{InitTask}[k, 1]) = 0 \text{ then } \text{Merge}_{ij} = 1 \quad (2)$$

$$\forall i, j \in \{1 \dots m\} \text{ et } i \neq j, \\ \text{if}(\text{InitTask}[i, 1] - \text{InitTask}[k, 1]) \neq 0 \text{ then } \text{Merge}_{ij} = 0 \quad (3)$$

The constraint (4) means that we have to maximize the number of merged tasks and thus minimize the number of tasks used in the task model. Indeed, this equation serves as a bound for the objective function (i.e. the number of merging operations).

$$N = m - \left( \sum_{i,j \in \{1 \dots m\}} \text{Merge}_{ij} \right) / 2 \quad (4)$$

### Real-time Constraints.

*NewTask* is a three column matrix where the first column presents the periods of the new tasks computed by the constraint (5). The second column presents the WCETs of the tasks computed by the constraint (6) and the last column is the deadline presented by the constraint (7)

$$\forall k \in \{1 \dots N\}, \forall i \in \{1 \dots N\} : \text{NewTask}[k, 1] = \text{InitTask}[i, 1] \quad (5)$$

$$\text{NewTask}[i, 2] = (\text{InitTask}[i, 2] + \text{InitTask}[j, 2])\text{Merge}[i, j] + (1 - \text{Merge}[i, j])\text{InitTask}[i, 2] \quad (6)$$

$$\forall i \in \{1 \dots N\}, \text{NewTask}[i, 3] = \text{NewTask}[i, 1] \quad (7)$$

The constraint (8) verifies whether the new model meets the timing constraints.

$$U = \sum_{i=1}^N \frac{\text{NewTask}[i, 2]}{\text{NewTask}[i, 1]} \leq \sum_{i=1}^N N(2^{\frac{1}{N}} - 1) \quad (8)$$

Constraint (9) ensures that the response times  $\text{Rep}_i$  of the different tasks in the optimized model are lower or equal than their deadlines:

$$\forall i \in \{1 \dots N\} \text{Rep}_i \leq \text{NewTask}[i, 3] \quad (9)$$

Constraint (10) gives the computation formula of the response time  $\text{Rep}_i$  of task  $\tau_i$ :

$$\text{Rep}_i = s[i] + \text{NewTask}[i, 2] \quad (10)$$

The response time  $\text{Rep}_i$  of a task  $\tau_i$  is defined as the sum of its start time and its execution time.

$$\forall i \in \{1 \dots N\} s[i] - s[j] \geq \text{NewTask}[j, 2] \quad (11)$$

$$\forall i \in \{1 \dots N\} s[j] - s[i] \geq \text{NewTask}[i, 2] \quad (12)$$

To ensure a single executed task at any time, we should have either

$$s[i] - s[j] - \text{NewTask}[j, 2] \geq 0 \text{ or } s[j] - s[i] - \text{NewTask}[i, 2] \geq 0, \text{ for every pair of tasks } t_i \text{ and } t_j$$

$$\forall i \in \{1 \dots N\} s[i] \leq r[i] \quad (13)$$

The task model generated by the linear program will be interpreted by the code generator in order to generate a running program in POSIX.

## 4.3 Code Generator

The last step of our approach consists in building the executable application from the optimized task model. We manually generate a POSIX code on the basis of transformation rules. For each task in the optimized task model, the code generator implements a POSIX thread by using pthread. In addition, this step produces the controller code of the reconfigurable real-time system, which allow moving from implementation to another, following well-defined conditions (i.e. user requirements).

## 5 CASE STUDY

In this section, we illustrate the proposed approach through a case study. The considered case study consists in a Global Positioning System (GPS). Firstly, we present the GPS specification. Then we apply the proposed approach to an automatic construction of a feasible and optimal implementation of a reconfigurable real-time system.

### 5.1 GPS Presentation

The global positioning system (GPS) is used to define the position of an object on a plan or a map using the information provided via radio signals by the associated satellites.

To show the applicability of our approach, we consider in this paper a simplified version of this system. For clarity, several features of the system (GPS) were omitted. Therefore, we only define two modes of operation:

#### Insecure Mode:

This mode is defined by seven functions:

- (i)  $F_1$ : *ControlBase* used to synchronize the satellite clock,
- (ii)  $F_2$ : *GpsSatellite* used to send radio signals (analogue) terminal,
- (iii)  $F_3$ : *Position* used to receive the satellite signal,
- (iv)  $F_4$ : *Receiver* used to convert the analogue signal into a digital signal,
- (v)  $F_5$ : *Decoder* used to decode the digital information and separate the information of calculating the distance from time information,
- (vi)  $F_6$ : *TreatmentUnit* used to calculate the distance of the satellite and determine position.
- (vii)  $F_7$ : *Encoder* used for encoding information of time and position.

#### Secure Mode:

The secure mode is defined by eight functions. Compared with the insecure mode, we have replaced the function *Position* by *Position Secure* function noted  $F_3'$  and we added *AccessController* function  $F_8$  to ensure the secure reception of radio signals.

### 5.2 GPS Initial Task Model

The second step consists in generating the implementations and their tasks from the specification model by applying the Algorithm. We obtain two possible implementations of the GPS which refer respectively to the two execution modes already specified.

### 5.3 GPS Optimized Task Model

The third step corresponds to the generation of the optimized task model from the initial one. A tabular description of the task model generated by this phase is given in Table 1.

We can see from Tables 1, this model satisfies the

Table 1: Tabular description of the optimized task model of the GPS.

Execution mode	Task	$T_i$ (ms)	$C_i$ (ms)	$D_i$ (ms)	$Rep_i$ (ms)	Function
Insecure Mode	$\tau_1$	100	50	100	80	$F_1$
	$\tau_2$	200	40	200	200	$F_2$
	$\tau_3$	300	140	300	280	$F_3, F_3', F_4$
	$\tau_4$	400	100	400	360	$F_5, F_6$
	$\tau_5$	500	60	500	450	$F_7$
	$\tau_6$	100	60	100	50	$F_1$
	$\tau_7$	200	40	200	200	$F_2$
Secure Mode	$\tau_3$	300	140	300	280	$F_3', F_4$
	$\tau_4$	400	100	400	360	$F_5, F_6$
	$\tau_5$	500	60	500	450	$F_7$
	$\tau_8$	600	50	600	500	$F_8$
	$\tau_{11}$	600	50	600	500	$F_8$

timing constraints of the GPS system since the response times  $Rep_i$  of the different tasks are lower than their deadlines  $D_i$ .

### 5.4 Performance Evaluation

We denote by  $T_{reconf_{initial}}$  the reconfiguration time in the initial task model of the GPS and  $T_{reconf_{Current}}$  the reconfiguration time in the optimized task one. These parameters are given as follow:

$$T_{reconf_{initial}} = 5 * T_{delete} + 6 * T_{creat} = 11 * T_{cost}$$

$$T_{reconf_{Current}} = 1 * T_{delete} + 2 * T_{creat} = 3 * T_{cost}$$

We note that the proposed approach allows to reduce the reconfiguration time and thus improves the overall performance of the reconfigurable real-time system (GPS). It minimizes also the sum of the response times of the all considered tasks such as  $Rep_{initial} = 3010$  ms and after optimization it becomes  $Rep_{optimized} = 1920$  ms . In addition, we have randomly generated instances with 5 to 40 tasks. We compare the reconfiguration time and the sum of response times after optimization with the initial corresponding parameters. The numerical results are depicted in figures 2 and 3:

These figures show that the optimization of the reconfiguration and response times are clearer and more important for large scale reconfigurable real-time systems.

## 6 CONCLUSION

In this paper, we have proposed an approach to support the development of reconfigurable real-time systems.

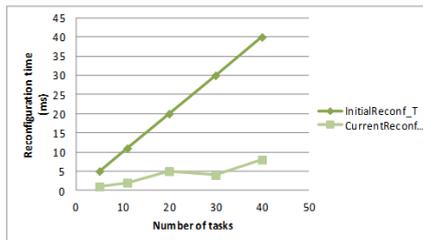


Figure 2: Comparison between Current reconfiguration time and initial one.

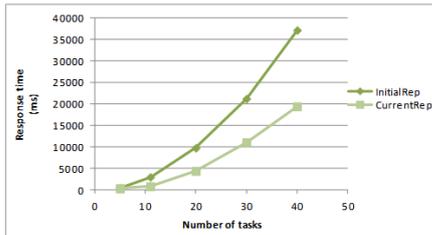


Figure 3: Comparison between Current response time and initial one.

tems. This approach consists in defining the specification such as reconfigurable conditions, functions and temporal constraints, then it generates an initial task model. After that, a MILP integral formulation is provided to compute the optimal solution for minimizing redundancy. Finally, the proposed approach generates a POSIX-based program describing the reconfigurable real-time system. We have evaluated the performance of the three-step approach. The numerical results show that the integer programming model allows to minimize the reconfiguration time and response times.

As a future work, we aim to extend our approach by considering multiprocessor systems and other optimization metrics. So that we expect to evaluate scalability of the proposed method with an industrial example.

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