Individual and Global Assessments with Signed Distance Defuzzification, and Characteristics of the Output Distributions based on an Empirical Analysis

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Keywords: Individual Assessment, Finanzplatz Zürich: Umfrage 2010, Fuzzy Logic, Fuzzy Statistics, Global Assessment, Linguistic Questionnaire, Normality, Output Distribution, Sampling Weights, Signed Distance Defuzzification Method.

Abstract: Considering a particular kind of questionnaires called linguistic questionnaires, we apply fuzzy logic to provide individual and global weighted evaluations in the case of the signed distance defuzzification method. We test our method on real data coming from a survey of the financial place of Zurich (Switzerland). Furthermore, we have been enable to give a look at the output distributions, and put into evidence their statistical properties. In particular, normality of distributions draws our attention. One of our main findings is that the individual evaluations calculated with the signed distance defuzzification method tend to be normally distributed.

1 INTRODUCTION AND MOTIVATION

A linguistic questionnaire is a set of items called "questions", coded as categorical variables. Indeed, the modalities of the variables are defined as linguistics. This type of questionnaires has been used frequently in many fields such as in satisfaction studies or in different economic studies, etc. Due to this particular kind of questions, the answers are deeply market by imprecision and vagueness, for the human response often weaving between different answers. Fuzzy logic was precisely introduced to deal with such imprecision.

The treatment of the data provided by the survey via the questionnaires can be done in different ways. We support below the notion of assessment or evaluation of questionnaires. This approach aims to bring out more information about the overall questionnaires compared to classical methods of treatment, often giving partial information. Fuzzy logic is a great tool to produce quantitative outcomes — the evaluation — from a given linguistic questionnaire. For instance, (Lin and Lee, 2009) and (Lin and Lee, 2010) presented the global evaluation of a linguistic questionnaire, and in the same way, (Berkachy and Donzé, 2015) and (Berkachy and Donzé, 2016a) displayed the individual ones.

The third part of a fuzzy process is the defuzzification, i.e. the way of producing quantitative crisp outcomes from the given fuzzy sets, (Yager, 1996). Among several known defuzzification methods seen in (Runkler, 1997), one has especially retained our attention: the signed distance method defended initially by (Yao and Wu, 2000). We have already explored some of the nice features of the signed distance method in (Berkachy and Donzé, 2015) and (Berkachy and Donzé, 2016a). In addition, (Berkachy and Donzé, 2016b) presented a comparison between the signed distance and other defuzzification methods in order to highlight some statistical characteristics of distributions obtained by applying this method.

We intend in the following to present the measures for the individual and global evaluations of questionnaires. The aim is to adapt the method described in (Berkachy and Donzé, 2015) and (Berkachy and Donzé, 2016a) by introducing the sampling weights in the calculations. In addition, we emphasize the relations between these two assessments' methods. The empirical part of the study, based on data collected during a regional survey concerning the financial place of Zurich (Switzerland), shows how one can implement these measures. Furthermore, the estimation gives us the possibility to analyse the properties of the distributions produced by the defuzzification process, and in particular if the resulting distributions

Berkachy, R. and Donzé, L.
Individual and Global Assessments with Signed Distance Defuzzification, and Characteristics of the Output Distributions based on an Empirical Analysis.
DOI: 10.5220/0006036500750082
In *Proceedings of the 8th International Joint Conference on Computational Intelligence (IJCCI 2016) - Volume 2: FCTA*, pages 75-82
ISBN: 978-989-758-201-1

are normal.

The section 2 is devoted to a short presentation of the signed distance measure, the formalisation of a linguistic questionnaire in fuzzy sense, as well as the global and individual evaluations. In Section 3, we describe the survey "Finanzplatz Zurich: Umfrage 2010", the database and the set up of the evaluations. In addition, this section summarises some analysis on an individual assessment. We finally give some arguments about the use of evaluations through fuzzy logic and classical methods of data analysis.

2 GLOBAL AND INDIVIDUAL EVALUATIONS AND THE SIGNED DISTANCE MEASURE

2.1 Definitions

Let us start by defining the signed distance measure for a fuzzy set as defended by (Yao and Wu, 2000) and (Lin and Lee, 2010).

We consider a family F of fuzzy numbers on $R = (-\infty, \infty)$. Let $\tilde{D} \in F$ be a fuzzy set on R, such as $\tilde{D} = \{(x, \mu_{\tilde{D}}(x)) | x \in R\}$ where $\mu_{\tilde{D}}(x)$ is the membership function of x in $\tilde{D}$ which maps R to the closed interval [0,1].

We denote by $D_L(\alpha)$ and $D_R(\alpha)$ the left and right α-cuts of the fuzzy set $\tilde{D}$, where $0 \leq \alpha \leq 1$. We assume that $D_L(\alpha)$ and $D_R(\alpha)$ exist and are integrable for $\alpha \in [0,1]$. Moreover, the fuzzy set $\tilde{D}$ can be represented using its α-cuts as

$$\tilde{D} = \bigcup_{0 \leq \alpha \leq 1} [D_L(\alpha), D_R(\alpha); \alpha].$$

We are now able to define the signed distance of the fuzzy set $\tilde{D}$.

Definition 2.1. *The signed distance of $\tilde{D}$ measured from the fuzzy origin $\tilde{0}$ is:*

$$d(\tilde{D}, \tilde{0}) = \frac{1}{2} \int_0^1 [D_L(\alpha) + D_R(\alpha)] \mathrm{d}\alpha. \tag{1}$$

In the case of a triangular fuzzy number $\tilde{D} = (p, q, r)$, we can easily find the corresponding signed distance measure. For the left and right α-cuts $D_L(\alpha)$ and $D_R(\alpha)$, where $D_L(\alpha) = p + (q - p)\alpha$ and $D_R(\alpha) = r - (r - q)\alpha$, the signed distance of $\tilde{D}$ measured from $\tilde{0}$ is given by

$$d(\tilde{D}, \tilde{0}) = \frac{1}{4}(p + 2q + r). \tag{2}$$

We will apply this particular signed distance later in our calculations.

2.2 Notation and Conditions

Inspired by (Lin and Lee, 2010), we decompose a linguistic questionnaire into r weighted main-items B_j, $j = 1, \ldots, r$, with weight b_j, such as $0 \leq b_j \leq 1$ and $\sum_{j=1}^{r} b_j = 1$, and the corresponding m_j weighted sub-items B_{jk}, $k = 1, \ldots, m_j$, with weight b_{jk} such as $0 \leq b_{jk} \leq 1$ and $\sum_{k=1}^{m_j} b_{jk} = 1$. Each sub-item counts m linguistic terms with the corresponding series of fuzzy numbers $\tilde{L_1}, \tilde{L_2}, \ldots, \tilde{L_q}, \ldots, \tilde{L_m}$, where $q = 1, 2, ..., m$, as seen in Table 1.

In addition, we required some other conditions:

- The fuzzy numbers are linearly ordered as well as their signed distance measure i.e. $d(\tilde{L_1}, \tilde{0}) < d(\tilde{L_2}, \tilde{0}) < \ldots < d(\tilde{L_m}, \tilde{0})$,
- only one answer is possible by sub-item,
- missing values are not allowed,
- the answers are weighted,
- each unit sample i has a sampling weight α_i, $i = 1, \ldots, N$.

We denote by δ_{jkqi} an indicator of an answer at a linguistic term L_q:

$$\delta_{jkqi} = \begin{cases} 1 & \text{if the observation i has an answer} \\ & \text{for the linguistic } L_q \text{ ;} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Let α_i be the sampling weight. Then $n_{jkq\bullet}$ is the total number of weighted answers at the linguistic term L_q of the sub-item B_{jk} where

$$n_{jkq\bullet} = \sum_{i=1}^{N} \alpha_i \delta_{jkqi}. \tag{4}$$

Furthermore, under the condition that missing values are not allowed, we have

$$\begin{aligned} n_{jk\bullet\bullet} &= \sum_{i=1}^{N} \sum_{q=1}^{m} \alpha_i \delta_{jkqi} \\ &= \sum_{i=1}^{N} \alpha_i, \end{aligned} \tag{5}$$

where $j = 1, ..., r$ and $k = 1, ..., m_j$.

2.3 Evaluation Measures

In their paper, (Lin and Lee, 2010) showed a so-called global assessment in linguistic questionnaires, while

Table 1: Decomposition of a weighted main-item B_j into m_j sub-items and the signed distance measures corresponding to their m fuzzy numbers.

Main-item	Weight	Sub-items	Weights	Linguistic terms	Fuzzy numbers	Signed Distance measures
B_j	b_j	B_{j1}	b_{j1}	L_1	$\tilde{L}_1$	$d(\tilde{L}_1,\tilde{0})$
				$\vdots$	$\vdots$	$\vdots$
				L_m	$\tilde{L}_m$	$d(\tilde{L}_m,\tilde{0})$
		$\vdots$				
		B_{jk}	b_{jk}	L_1	$\tilde{L}_1$	$d(\tilde{L}_1,\tilde{0})$
				$\vdots$	$\vdots$	$\vdots$
				L_q	$\tilde{L}_q$	$d(\tilde{L}_q,\tilde{0})$
				$\vdots$	$\vdots$	$\vdots$
				L_m	$\tilde{L}_m$	$d(\tilde{L}_m,\tilde{0})$
		$\vdots$				
		B_{jm_j}	b_{jm_j}	L_1	$\tilde{L}_1$	$d(\tilde{L}_1,\tilde{0})$
				$\vdots$	$\vdots$	$\vdots$
				L_m	$\tilde{L}_m$	$d(\tilde{L}_m,\tilde{0})$

(Berkachy and Donzé, 2015) and (Berkachy and Donzé, 2016a) showed an individual one. Nevertheless, we present in this part a revised global evaluation measure taking into account the sampling weight of each sample unit. We also display the relation between the evaluations on both levels.

First, let us express the evaluation of a whole questionnaire in terms of global or aggregative evaluation. A weighted fuzzy number is $\frac{n_{jkq\bullet}}{n_{jk\bullet\bullet}}\tilde{L}_q$, and the signed distance d_{jkq} of this quantity can be written as

$$d_{jkq} = d((n_{jkq\bullet}/n_{jk\bullet\bullet})\tilde{L}_q,\tilde{0}) = (n_{jkq\bullet}/n_{jk\bullet\bullet})d(\tilde{L}_q,\tilde{0}).$$

Using (4) and (5), we can derive the aggregative evaluation P of the linguistic questionnaire in the following way:

$$\begin{aligned} P &= \sum_{j=1}^{r} b_j \sum_{k=1}^{m_j} b_{jk} \sum_{q=1}^{m} d_{jkq} \\ &= \sum_{j=1}^{r} b_j \sum_{k=1}^{m_j} b_{jk} \sum_{q=1}^{m} \frac{\sum_{i=1}^{N} \alpha_i \delta_{jkqi}}{\sum_{i=1}^{N} \alpha_i} d(\tilde{L}_q,\tilde{0}). \quad (6) \end{aligned}$$

On an individual level, we define the individual evaluation P_i of the questionnaire for an individual i as:

$$P_i = \sum_{j=1}^{r} b_j \sum_{k=1}^{m_j} b_{jk} \sum_{q=1}^{m} \delta_{jkqi} d(\tilde{L}_q,\tilde{0}). \quad (7)$$

Proposition 2.1. *The weighted mean of the individual evaluations $P_i, i = 1,\dots,N$, is equal to the global evaluation P.*

Proof 2.1. The weighted mean of the individual evaluations is equal to

$$\frac{\sum_{i=1}^{N} \alpha_i P_i}{\sum_{i=1}^{N} \alpha_i}.$$

Then, from (7), we have

$$\frac{1}{\sum_{i=1}^{N} \alpha_i} \sum_{i=1}^{N} \alpha_i \sum_{j=1}^{r} b_j \sum_{k=1}^{m_j} b_{jk} \sum_{q=1}^{m} \delta_{jkqi} d(\tilde{L}_q,\tilde{0}) = \sum_{j=1}^{r} b_j \sum_{k=1}^{m_j} b_{jk} \sum_{q=1}^{m} \frac{\sum_{i=1}^{N} \alpha_i \delta_{jkqi}}{\sum_{i=1}^{N} \alpha_i} d(\tilde{L}_q,\tilde{0}) = P.$$

As a corollary, if all sample units have the same weight $\alpha_1 = \dots = \alpha_i = \dots = \alpha_N = \alpha$, then the arithmetic mean of the individual evaluations $P_i, i = 1,\dots,N$, is equal to the global evaluation P.

3 EMPIRICAL APPLICATION

3.1 Description of the Questionnaire

We apply the individual and global assessments presented previously to the survey "Finanzplatz Zürich: Umfrage 2010" (BAK, 2010), which was conducted by the Office of Economy of the canton of Zurich (Switzerland) in 2010. The survey was intended to understand the situation on the financial market of the canton of Zurich. A sample of actors of the place (mostly financial enterprises, e.g. banks or insurances) were asked through a written questionnaire about their expectations of demand, income and employment for a foreseeable future. The questionnaire relative to this survey consisted principally of 21 questions divided into 4 groups. We note that 19 of these questions are linguistic ones with five possible answers going from 1 (bad) to 5 (good) while only 2 are quantitative.

3.2 Database and Set up of the Evaluations

Our database is composed of 234 observations, with among others company size and branch variables, and the sampling weight. Considering our aim to transform linguistics into a more precise form, the 19 five-level Likert questions have been treated only. Finally, we have excluded the missing values and treated our database as a complete cases one. The consequence is that we will refer to 9 questions exclusively. The questionnaire was decomposed into 3 equally weighted main-items where 2 of them have 2 sub-items and the third 5 sub-items.

The individual and global evaluations in a linguistic questionnaire with the signed distance are implemented in *R* (R Core Team, 2015). Assuming triangular isosceles membership functions, the signed distance of the corresponding fuzzy number $\tilde{L_q} = (t_{q-1}, t_q, t_{q+1})$ is given by

$$d(\tilde{L_q}, \tilde{0}) = \frac{1}{4}(t_{q-1} + 2t_q + t_{q+1}),$$

and it follows that the expression of the individual assessment (7) is

$$P_i^{(j)} = \frac{1}{4}\sum_{k=1}^{m_j} b_{jk} \sum_{q=1}^{m} \delta_{jkqi}(t_{q-1} + 2t_q + t_{q+1}). \qquad (8)$$

We consider the five linguistic terms $L_1, \ldots, L_5$ with the triangular fuzzy numbers $\tilde{L_1}, \ldots, \tilde{L_5}$ where $\tilde{L_q} = ((q-1)(\frac{t_{m+1}}{m+1}), \frac{t_{m+1}}{m+1}, (q+1)(\frac{t_{m+1}}{m+1})), q = 1, \ldots, 5$. One can easily compute the individual evaluation of a main-item. In this case, we fix t_{m+1} to 30, which gives the following fuzzy numbers

$$\begin{aligned}
\tilde{L_1} &= (0,5,10) \\
\tilde{L_2} &= (5,10,15) \\
\tilde{L_3} &= (10,15,20) \\
\tilde{L_4} &= (15,20,25) \\
\tilde{L_5} &= (20,25,30).
\end{aligned}$$

Table 2 gives an example of the individual evaluation of an observation i from the treated database. We note that in our case, we can easily prove that changing the value of t_{m+1} do not affect the interpretation of the assessments. In fact, it is nothing but a translation to the new fuzzy numbers system.

3.3 Some Analysis on an Individual Level

3.3.1 Company Size

Our first analysis is made by company sizes: Small, Medium and Big companies. Some statistical measures appear in Table 3. Examining global assessments, we see that the evaluation given by small companies (18.09547) is higher than respectively the evaluations given by medium (17.58005) and big (16.89257) companies. This result indicates that small companies expect better business than the medium and the big ones. The boxplot Figure 1(a) displays this result as well.

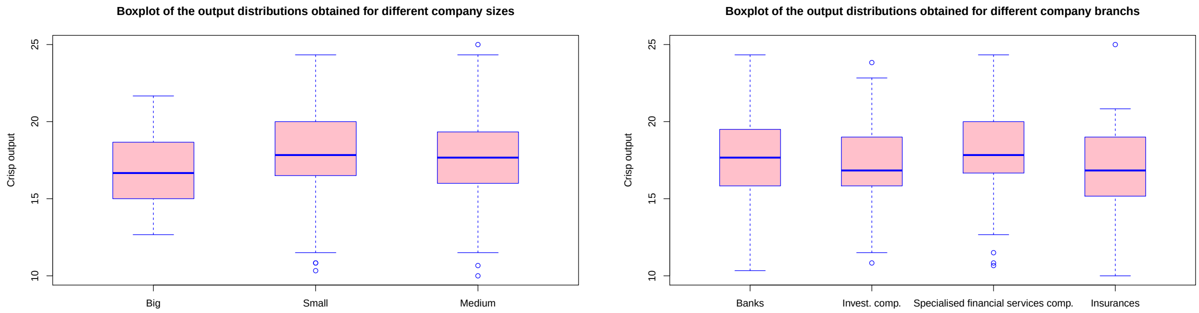
3.3.2 Company Branch

A second analysis can be made by company branches. Four branches were considered: Banks, Specialised financial services companies, Investment companies, Insurance companies. The global assessment of the Specialized finance services companies is the highest (18.27666) compared to the banks (17.43678), Investment companies (17.38439) and Insurances (16.98436), as seen in Table 4. Figure 1(b) shows boxplots of the 4 types of companies where the Specialized finance services companies seem to have a more positive vision to the future than other companies.

3.4 Normality of the Crisp Output Distribution

(Berkachy and Donzé, 2016b) emphasized different statistical characteristics such as location, dispersion and asymmetry measures of distributions obtained using the signed distance method compared to different defuzzification methods. In the same way, since many statistical tests rely on the normality of a sample, it is advantageous to know whether the underlying distribution is symmetric or a fortiori tends to be normal. This can be done graphically via histograms and QQ-plots or by calculating the skewness and excess of kurtosis, and last but not least by using different statistical normality tests.

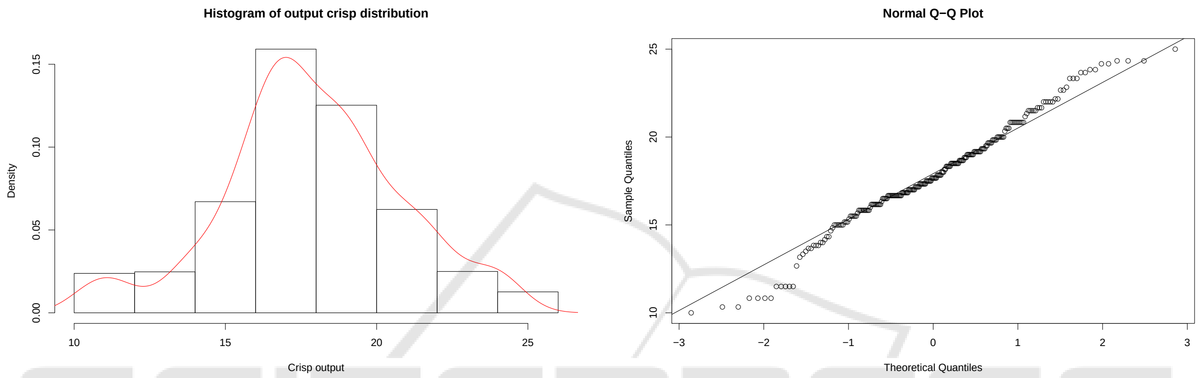
The histogram and QQ-plot corresponding to the output distribution (considering the whole dataset), seen in Figure 2, give us a hint about the normality hypothesis. We calculate as well the excess of kurtosis of this distribution and we get 0.14731 and a skewness of -0.09489 signalizing a symmetric distribution. At last, the normality tests used (Chi-square, Shapiro-Wilk, Anderson-Darling, Kolmogorov-Smirnov and



(a) Classification by company size

(b) Classification by company branch

Figure 1: The output distributions classified by: company size (a) and company branch (b).



(a) Histogram of crisp distribution

(b) QQ-plot of crisp distribution

Figure 2: The histogram (a) and qqplot (b) of the output distributions.

Lilliefors) couldn't reject the hypothesis H_0 of the normality of the distribution.

3.5 Analyses on Original Data vs. Evaluations through Fuzzy Logic

Statistical inference is described as the most substantial way of thinking regarding data analysis. Despite the fact that evaluations noted previously are quantitative and therefore should present less imprecision than the linguistic terms, some analyses can be done on the original data. Nevertheless, a fuzzy approach could be advantageous in several situation.

First of all, it has to be mentioned that evaluations through fuzzy logic offer the possibility to characterise units by their individual assessments and thus give another way for interpretations. We have done this kind of analyses for instance in section 3.3. Furthermore, we note that the resulting distribution of the outputs, as seen in section 3.4, tends to be normal, which is a nice property for statistical inferences.

Secondly, on a global level, we show that the mean of the means works as well as our aggregative evaluation. It is a useful property, especially when it comes to illustrate broadly information about the questionnaire. Last but not least, and it is a recurrent drawback when missing values occur, the mean of means applied on linguistic data cannot be "representative", especially if missingness is not ramdom and the estimates are biased. We present our measures under the assumption of no missing values. Indeed, it is not difficult to show that the global evaluation measure can be adapted in the case of missing values appearing in a dataset.

4 CONCLUSION

Due to the particularities of the variables — qualitative answers, the traditional treatment methods of linguistic questionnaires could encounter many difficulties. For instance the non-normality of the distribution could gravely impair the statistical inferences. Or, the results appear difficult to interpret. An approach with the fuzzy logic, and individual and global assessments could be an alternative way.

This study intends to present the individual and global assessments in linguistic questionnaires, an efficient method to manage uncertainty in this kind of questionnaires. Based on the signed distance defuzzification method, we estimated easily these evaluations in the fuzzy sense, considering furthermore the sampling weights in real databases. The signed distance calculations are made with triangular isosceles fuzzy numbers.

Moreover, in order to help using our method in empirical research, we applied these evaluation methods on a real dataset coming from a survey of the financial place of Zurich (Switzerland). This empirical part should give confidence that the individual and global assessments are easily implementable, and could provide interesting results.

From another side, the non-normality of distributions is one of the most appealing topics in statistical inference. Yet, we examined the normality of the output distributions and we found that the individual evaluations with the signed distance method tend to be normally distributed. This result can be profitmaking in terms of adequacy of data in further statistical analysis. Applying individual evaluation will enable us to assess easily topics in linguistic questionnaires per observation through fuzzy logic.

Finally, future work certainly encompasses the understanding of the effect of normality and symmetry of such distributions in different statistical modellings, investigating in particular databases with missing values.

ACKNOWLEDGEMENT

The authors would like to thank the Office of Economy of the canton of Zurich (Switzerland) which provided the permission to use the database.

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APPENDIX

Table 2: Application of the individual evaluation on Finanzplatz Zurich Umfrage 2010: Answers of an observation i for the 3 main-items equally weighted using the signed distance of triangular fuzzy numbers.

Main-items B_j	Weights b_j	Sub-items B_{jk}	Weights b_{jk}	Linguistics L_q	Answers δ_{jkqi}	Signed Distance	Individual Evaluation
1	$\frac{1}{3}$	The present state of business is	0.5	bad L_2 fair L_4 excellent	0 0 0 1 0	20	14.333
		The expected state of business in 12 months is	0.5	worst L_2 same L_4 better	0 0 1 0 0	15	
2	$\frac{1}{3}$	The demand for our services / products compared to the last 12 months is	0.5	smaller L_2 same L_4 bigger	0 1 0 0 0	10	
		The expected demand for our services / products in the next 12 months is	0.5	worst L_2 same L_4 better	0 0 1 0 0	15	
3	$\frac{1}{3}$	The gross profit compared to the last 12 months is	0.2	smaller L_2 same L_4 bigger	0 1 0 0 0	10	
		The employment compared to the last 12 months is	0.2	smaller L_2 same L_4 bigger	0 0 1 0 0	15	
		The expected gross profit in the next 12 months is	0.2	worst L_2 same L_4 better	0 1 0 0 0	10	
		The expected employment in the next 12 months is	0.2	worst L_2 same L_4 better	0 0 1 0 0	15	
		The expected use of technical and personal capacities in the next 12 months is	0.2	smaller L_2 same L_4 bigger	0 0 1 0 0	15	

Table 3: Statistical measures of the distribution obtained from individual evaluations: Classification by company size.

	Company size		
Statistical measure	**Small**	**Medium**	**Big**
Mean	18.09547	17.58005	16.89257
Median	17.83333	17.66667	16.66667
Variance	9.65144	7.21249	5.55479
Standard deviation	3.10667	2.68561	2.35686
Skewness	-0.21248	-0.05683	0.32202
Excess of Kurtosis	0.14485	0.45263	-0.72025
Quantiles:			
$P = 0$	10.33333	10	12.66667
$P = 0.25$	16.5	16	15
$P = 0.5$	17.83333	17.66667	16.66667
$P = 0.75$	20	19.33333	18.66667
$P = 1$	24.33333	25	21.66667

Table 4: Statistical measures of the distribution obtained from individual evaluations: Classification by company branch.

	Company branch			
Statistical measure	**Banks**	**Specialised financial services companies**	**Investment companies**	**Insurance companies**
Mean	17.43678	18.27666	17.38439	16.98436
Median	17.66667	17.83333	16.83333	16.83333
Variance	9.17247	8.20156	10.11543	7.02378
Standard deviation	3.02861	2.86384	3.18048	2.65024
Skewness	-0.43497	-0.03864	0.13030	0.31873
Excess of Kurtosis	-0.21930	0.38951	-0.33553	1.65608
Quantiles:				
$P = 0$	10.33333	10.66667	10.83333	10
$P = 0.25$	15.83333	16.66667	15.83333	15.16667
$P = 0.5$	17.66667	17.83333	16.83333	16.83333
$P = 0.75$	19.5	20	19	19
$P = 1$	24.33333	24.33333	23.83333	25