

# Modified Krill Herd Optimization Algorithm using Focus Group Idea

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**Abstract:** Krill Herd algorithm is one of most recently developed nature-inspired optimization algorithms which is inspired by herding behavior of krill individuals. In order to improve the performance of this algorithm to deal more effectively with high dimensional numerical functions, we propose a new method, called Focus Group idea to modify the solutions found by searching agents in group cooperation. In order to evaluate the effect of the proposed method on the performance of the Krill Herd algorithm, we conducted experiments on a set standard benchmark functions. The obtained results demonstrate the ability of the proposed method to improve the performance of the Krill Herd optimization algorithm.

## 1 INTRODUCTION

Recently the world has been grappling with high dimensional and complex real world optimization problems. The urgency for solving these challenging problems has caused heated discussion among scientist in different areas (Yang and Press, 2010). It has been proven that due to the high dimension of these problems, the logical and classical methods cannot come in useful, as using them is so time consuming and in some cases inapplicable. Due to mentioned reasons, the meta-heuristic means which are inspired by physical or biological processes would be among the appropriate options. These evolutionary techniques have earned much popularity due to their success in dealing with hard to solve problems (Yang and Press, 2010). Although they do not guarantee finding the optimal solution, they have shown impressive performance in accessing acceptable solutions in very efficient time. Two main components of each meta-heuristic algorithm are exploration and exploitation. Each meta-heuristic algorithm uses a combination of these two components which is the main reason why the related searches are so powerful. Evolutionary techniques include Genetic algorithms (GAs) which was developed based on Darwinian theory (Goldberg and Holland, 1988; Mitchell, 1998), Ant Colony Optimization (ACO), inspired by collective foraging behavior of ants (Dorigo et al., 1996), PSO, inspired by bird flocking and fish

schooling (Kennedy, 2011), vector-based evolutionary algorithm proposed by Storn (Storn and Price, 1995), Artificial Bee Colony (ABC), a further development of ACO, proposed by Karaboga (Karaboga, 2005), Firefly Algorithm (FA), inspired by fireflies' behavior in emitting light in order to attract other fireflies (Yang, 2010a), Gravitational Search Algorithm (GSA), introduced based on the law of gravity and mass interactions (Yang, 2010a), Bat-Inspired algorithm (Rashedi et al., 2009) and Cuckoo Optimization Algorithm (COA), inspired by eggs laying and breeding characteristics of cuckoos (Yang, 2010b). Due to their ability to deal with hard optimization problems, these solving methods have been widely applied to different areas including pattern recognition, control objectives, image processing and filter modeling.

Krill Herd (KH) is a nature-inspired optimization algorithm which was developed based on herding behavior of krill in the nature (Bhandari et al., 2014). The minimum distances from the food and the highest density of the krill herds are considered as objectives of the KH algorithm. Although meta-heuristic algorithms are able to deal with different optimization problems, enhancing and improving their performance to deal with wider range of problems or become able to deal with specific applications is an open issue. Various ideas and methods such as chaotic sequences or fuzzy methods have been applied and combined with meta-heuristic algorithms to do so. In this regard, we introduce a new method called Focus

Group to enhance the performance of the Krill Herd algorithm when solving high dimensional numerical functions. Focus Group is a method which tries to modify the solutions found by group's members in a group discussion. Despite the other evolutionary algorithms which concentrate on the use of the best solutions, this method enables algorithms to consider all the solutions found to reach the optimal one. To evaluate the effect of this new method in improving the performance of Krill Herd optimization algorithm, we applied it to a set of standard benchmark functions. The obtained results demonstrate the ability of the proposed method in enhancing the performance of Krill Herd algorithm.

## 2 KRILL HERD ALGORITHM

Krill Herd (KH) is a recently proposed meta-heuristic optimization algorithm which was inspired by Krill's herding behavior to solve global optimization problems (Gandomi and Alavi, 2012). The fitness function of each krill individual is defined as its distances from food and highest density of the swarm. The time-dependent position of an individual krill is influenced by the following three main factors: the movement induced by other krill individuals, the foraging activities and the random diffusion. When hunters attack krill, they remove krill individuals and this lead to diminish their density. Formation of the krill herd depends on many parameters. Increasing krill density and reaching food are considered as two main objectives of herding behavior after reducing krill density which these two lead the krill individuals to herd around the global optima. In this process, an individual krill moves toward the fittest individual that has found the best solution when it searches for the highest density and food. So the closer the distance to the high density and food, the better value is produced by the objective function. Predators hunt individuals and cause reduction of the average krill density, and distances the krill swarm from the food location. In the Krill Herd algorithm presented in Figure 1, the fitness of each individual is evaluated considering the distance of that individual from the food and from the highest density of the krill swarm.

For  $n$  dimensional decision space, the Krill Herd algorithm conforms to the following Lagrangian model:

$$\frac{dX_i}{dt} = N_i + F_i + D_i \quad (1)$$

where  $N_i$  is the motion induced by other krill individuals in the herd,  $F_i$  is the foraging motion and  $D_i$  is the physical diffusion of the  $i$ th krill individuals. The

```

Krill Herd Optimization Algorithm
begin
  Initial parameter setting
  Pre-calculation
  Initial krill positions
  For each iteration
    Population evaluation
    For each krill
      Calculation of Best krill effect
      Calculation of Neighbors Krill effect
      Movement Induced
      Calculation of food attraction
      Calculation of best position attraction
      Foraging motion
      Physical diffusion
      Motion process
      Crossover and mutation
      Update krill's position
    end
  Update the current best
end

```

Figure 1: The pseudo-code of Krill Herd optimization algorithm (Gandomi and Alavi, 2012).

movement of  $i$ -th krill can be considered as shown in (2) in motion induced by other krill individuals.

$$N_i^{new} = N^{max} \alpha_i + w_n N_i^{old} \quad (2)$$

where

$$\alpha_i = \alpha_i^{local} + \alpha_i^{target} \quad (3)$$

where  $N^{max}$  is the maximum induced speed,  $w_n$  is the inertia weight of the motion induced,  $N_i^{old}$  is the last motion induced and  $\alpha$  is direction of movement.  $\alpha_i^{local}$  is the local effect caused by neighbors and  $\alpha_i^{target}$  is target direction effect caused by the best individual. The effect of the neighbors can be assumed as an attractive/repulsive tendency between the individuals for a local search which is formulated as follow:

$$\alpha_i^{local} = \sum_{j=1}^{NN} \hat{K}_{i,j} \hat{X}_{i,j} \quad (4)$$

where

$$\hat{X}_{ij} = \frac{X_j - X_i}{\|X_j - X_i\| + \epsilon} \quad (5)$$

And also

$$\alpha_i^{target} = C^{best} \hat{K}_{i,best} X_{i,best} \quad (6)$$

where  $C^{best}$  is the effective coefficient of the krill individual with the best fitness to the  $i$ -th krill individual and defined as follow:

$$C^{best} = 2 \left( rand + \frac{I}{I_{max}} \right) \quad (7)$$

where  $rand$  is a random values between 0 and 1 and it is for enhancing exploration,  $I$  is the actual iteration number and  $I_{max}$  is the maximum number of iterations. The foraging motion is proportional to two main parameters, food location and previous experience about food location. This motion can be formulated as follow:

$$F_i = V_f \beta_i + \omega_f F_i^{old} \quad (8)$$

where

$$\beta_i = \beta_i^{food} + \beta_i^{best} \quad (9)$$

$V_f$  is the foraging speed,  $\omega_f$  is the inertia weight of the foraging motion,  $F_i^{old}$  is the last foraging motion and  $\beta_i^{food}$  is food attractive and  $\beta_i^{best}$  is the effect of the best fitness of the  $i$ -th krill so far. Physical diffusion of krill can be considered as (10):

$$D_i = D^{max} \left(1 - \frac{I}{I_{max}}\right) \delta \quad (10)$$

where  $D^{max}$  is the maximum diffusion speed,  $\delta$  is the random directional vector,  $I$  and  $I_{max}$  are  $i$ -th and maximum iteration number. The location vector of krill during the interval  $t$  to  $t+1$  is given by (11):

$$X_i(t + \Delta t) = X_i(t) + \Delta t \frac{dX_i}{dt} \quad (11)$$

where  $\Delta t$  is an important constant that should be carefully set based on the optimization problem. After Motion calculation, to improve the performance of KH algorithm, crossover and mutation operators are added to algorithm.

### 3 FOCUS GROUP

The Focus Group (FG) idea is inspired by the behavior of the members of a group in sharing, correcting and improving their ideas on a specific topic until reaching the best one. More precisely, FG consists of some people discussing a given subject and sharing ideas in order to reach the best solution to a given problem. The main characteristic of the FG is its ability of producing and improving ideas or data based on cooperation of its members. This approach could be considered as an optimization operator, used for optimization purposes. In a FG, each member's idea will be taken into consideration by all members and will affect their idea. After repetitions of this procedure, the best and the most conceivable choice will be reached. It is worth mentioning that another characteristic of a FG is that, at each time just one member is allowed to talk and this is mainly due to importance of hearing all ideas with care in order to take them into

account. While different, all these ideas should be valued. Better ideas possess higher values and have greater impact on the other members' ideas.

Referring to KH optimization algorithm proposed by Gandomi (Gandomi and Alavi, 2012), when predators such as seabirds, penguins and seals attack krill, they remove krill individuals which results in reduction in the krill density and distancing them from food. So reformation of the krill's herd after reduction of krill density depends on many factors. This process (herding of the krill individuals) is a multi-objective process which follows two main goals: increasing krill density and reaching food. To achieve the first goal, krill individuals make their ultimate effort and try to increase and maintain the herd density and move due to their mutual effects. The movement direction of each krill individual,  $\alpha_i$ , is influenced by local swarm density as local effect, target swarm density as target effect and repulsive swarm density known as repulsive effect. This movement is as follow:

$$N_i^{new} = N^{max} \alpha_i + w_n N_i^{old} \quad (12)$$

where

$$\alpha_i = \alpha_i^{local} + \alpha_i^{target} \quad (13)$$

where  $\alpha_i^{local}$  is the effect caused by individual neighbors and  $\alpha_i^{target}$  is the effect caused by the best krill individual. This means that the movement direction of krill individuals is influenced by their neighbors and the best individual. According to the focus group idea in a group, all members have their own impact on the other members' ideas in a way that the best krill individual has greatest impact and the worst krill has the least impact on the other members' ideas. So contrary to the Gandomi's idea in the Krill Herd optimization algorithm about the movement of the krill individuals, the focus group idea points to the fact that the movement of each krill individual should be influenced by all krill individuals. However, individuals with higher fitness have higher effects and individuals with lower fitness have lower effects. So, fitting the focus group description, equation (13) should be reformulated as follow:

$$\alpha_i = \sum_{j=1}^N (C_j \hat{K}_{i,jbest} \hat{X}_{i,jbest}) \quad (14)$$

where

$$\hat{X}_{i,jbest} = \left( \frac{X_{j,ibest} - X_i}{\|X_{j,ibest} - X_i\| + \epsilon} \right) \quad (15)$$

and

$$\hat{K}_{i,jbest} = \frac{K_i - K_{j,ibest}}{K_{worst} - K_{best}} \quad (16)$$

where  $N$  is the population size,  $X_i$  is the current location of  $i$ -th individual,  $X_{j,ibest}$  is the best location

visited by  $j$ -th individual in all iterations and  $C_j$  is the coefficient effect of  $i$ -th krill individual which is the exponentially distributed random number. The larger random numbers are allocated to individuals with higher fitness and the smaller ones are allocated to the individuals with lower fitness. Also  $K_{j,ibest}$  equals to best fitness of  $j$ -th individual in all iterations and  $K_i$  equals to fitness of  $i$ -th individual,  $K_{best}$  and  $K_{worst}$  are the best and worst fitness achieved by all krill individuals.

## 4 EXPERIMENTATION

In order to evaluate the performance of the proposed algorithms, we applied it to a set of standard benchmark functions listed in Table 1. These benchmarks include high dimensional functions which are difficult to solve due their dimension (Fister et al., 2013) and are being used most frequently by the researchers to examine the performance of the different optimization algorithms. If we exclude Sphere and DixonPrice which are unimodal functions, the rest are multimodal functions. In spite of unimodal functions which have one local optimum, multimodal functions have many local minimum points with a risk of being trapped in them. In Table 1,  $n$  is the dimension of the functions, Search Space is the problem space which is a subset of  $R^n$ . The Global Minimum is the minimum value of the functions which are zero for all functions except for Michalewicz function (Its minimum point is -9.66). The dimension for F1 to F9 functions is considered 20 and for F10 is considered 10. Below is the description of the benchmark functions we have used in our experiments. More details can be found in (Ali et al., 2005).

- F1.** Ackley function: This is a popular test problem for evaluating the performance of the optimization methods. Its many local optimum solutions challenge the performance of the optimization methods by posing a risk on them, to be trapped in one of local optimum solutions and this is specially the case for the hill climbing methods. Ackley is continuous, differentiable, non-separable, scalable and multimodal. The global minimum of the function is  $f(x^*) = 0$  with corresponding solution  $x^* = (0, \dots, 0)$ . The test domain is  $32.768 \leq x_i \leq 32.768$ .
- F2.** Griewank function: This function has many local optimum solutions that are regularly distributed in the problem space. This is also a non-separable, scalable and a differentiable function. Its global optimum solution is  $f(x^*) = 0$  with correspond-

ing solution  $x^* = (0, \dots, 0)$ . The domain of test is  $600 \leq x_i \leq 600$ .

- F3.** Levy function: Levy is a continuous optimization problem with several local optimum solution distributed in the problem space. It has global optimum solution  $f(x^*) = 0$  which is located at  $x^* = (1, \dots, 1)$ . This problem is subject to  $10 \leq x_i \leq 10$ .
- F4.** Rastrigin function: Rastrigin is a continuous multimodal function with many local optimum solution distributed in the search space. It is a difficult problem to solve due to its large search space and large number of local optimum solutions. Its global optimum solution is  $f(x^*) = 0$  with corresponding zero vector  $x^* = (0, \dots, 0)$ . The test domain is  $5.12 \leq x_i \leq 5.12$ .
- F5.** Schwefel function: Schwefel belongs to continuous multimodal class of test functions. It is also differ-entiable, separable and scalable functions. Its many local optimum solutions make it generally difficult solution to solve. Its global minimum is  $f(x^*) = 0$  which is located at  $x^* = (0, \dots, 0)$ . The problem constraint is  $500 \leq x_i \leq 500$ .
- F6.** Dixon Price function: This function is continuous, differentiable, non-separable and unimodal function. It has global minimum  $f(x^*) = 0$  which is located at  $x^* = (2(\frac{2^i - 2}{2^i}))$  for  $i = 1 \dots n$ , where  $n$  is the dimension of the problem. The test domain is  $10 \leq x_i \leq 10$ .
- F7.** Rosenbrock function: Rosenbrock is a popular function for gradient-based optimization algorithms. It is continuous, differentiable, non-separable and unimodal function. It has global minimum  $f(x^*) = 0$  which is located in narrow valley. The corresponding solution is  $x^* = (1, \dots, 1)$  and the problem constraint is  $5 \leq x_i \leq 10$ .
- F8.** Sphere function: Sphere is a popular test function which is used most frequently by the researchers for examining the performance of the optimization methods. This function is continuous, differentiable, separable and unimodal test function. Its global optimum solution is  $f(x^*) = 0$  with corresponding solution  $x^* = (0, \dots, 0)$ , where  $5.12 \leq x_i \leq 5.12$ .
- F9.** Powell function: Powell function is continuous, differentiable, separable and unimodal function. It has global optimum  $f(x^*) = 0$  which located at  $x^* = (3, 1, 0, 1 \dots, 3, 1, 0, 1)$  where  $4 \leq x_i \leq 5$ .
- F10.** Michalewicz function: This function is continuous multimodal function. It has global minimum  $f(x^*) = 9.66$  for 10 dimension version ( $n = 10$ ). This problem is subject to  $0 \leq x_i \leq \pi$ .

Table 1: The standard benchmark functions used in our experiments.

ID	Name	Function	Domain	Global Minimum
F1	Ackley	$f(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}\right) - \exp\left(n^{-1} \sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e$	$x_i \in [-32.768, 32.768]$	0
F2	Griewank	$f(x) = 1 + \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right)$	$x_i \in [-600, 600]$	0
F3	Levy	$f(x) = \sin^2(\pi \omega_1) + \sum_{i=1}^{n-1} (\omega_i - 1)^2 * [1 + 10 \sin^2(\pi \omega_n)] + (\omega_n - 1)^2 [1 + \sin^2(2\pi \omega_n)]$ , where $\omega_i = 1 + \frac{x_i - 1}{4}$ , for all $i = 1, \dots, n$	$x_i \in [-10, 10]$	0
F4	Rastrigin	$f(x) = 10n + \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i))$	$x_i \in [-5.12, 5.12]$	0
F5	Schwefel	$f(x) = 418.9829n - \sum_{i=1}^n x_i \sin(\sqrt{ x_i })$	$x_i \in [-500, 500]$	0
F6	Dixon Price	$f(x) = (x_1 - 1)^2 + \sum_{i=2}^n i (2x_i^2 - x_{i-1})^2$	$x_i \in [-10, 10]$	0
F7	Rosenbrock	$f(x) = \sum_{i=1}^{n-1} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2)$	$x_i \in [-5, 10]$	0
F8	Sphere	$f(x) = \sum_{i=1}^n x_i^2$	$x_i \in [-5.12, 5.12]$	0
F9	Powell	$f(x) = \sum_{i=1}^{n/4} \left[ \begin{matrix} (x_{4i-3} + 10x_{4i-2})^2 + \\ 5(x_{4i-1} - x_{4i})^2 \\ + (x_{4i-3} + 2x_{4i-1})^4 \\ + 10(x_{4i-3} - x_{4i})^4 \end{matrix} \right]$	$x_i \in [-4, 5]$	0
F10	Michalewicz	$f(x) = -\sum_{i=1}^n \sin(x_i) \sin^{20}\left(\frac{ix_i^2}{\pi}\right)$	$x_i \in [0, \pi]$	-9.66

**F11. Colville function:** Colville function is continuous, differentiable, non-separable and multimodal function. It has several local minimum solutions which make it tricky and hard to find global minimum. Its global minimum is  $f(x^*) = 0$  which is located at  $x^* = (1, \dots, 1)$ . The problem domain is  $-10 \leq x_i \leq 10$ .

**F12. Shubert function:** Shubert function is continuous, differentiable, separable and multimodal function. It has 18 global minimum some of which are located at  $(-1.4251, -7.0835)$ ,  $(-7.0835, -7.7083)$ ,  $(5.4828, 4.8580)$ . Its global minimum is  $f(x^*) = -186.73$ . The test domain is  $-10 \leq x_i \leq 10$ .

**F13. Six-hump camel function:** This function is continuous, differentiable, non-separable and multimodal function. It has two global optimum  $f(x^*) = -1.0316$  which are located at  $(-0.0898, 0.7126)$ ,  $(0.0898, -0.7126)$ . This function is subject to  $-10 \leq x_i \leq 10$ .

**F14. Bohachevsky1 function:** This function is continuous, differentiable, separable and multimodal. Its global optimum is  $f(x^*) = 0$  is located at  $(0, 0)$ . The test domain is  $-100 \leq x_i \leq 100$ .

**F15. De Jong N.5 function:** This function is continuous multimodal function with many sharp drops on an almost flat surface. The global minimum of the function is  $f(x^*) = 0.99$  where  $-65.536 \leq x_i \leq 65.536$ .

**F16. Easom function:** Easom function is two-dimensional function in domain  $-100 \leq x_i \leq 100$ . This function belongs to continuous, differentiable, separable and multimodal function class. Its global minimum is  $f(x^*) = -1$ , located at  $(\pi, \pi)$ .

**F17. Matyas function:** Matyas function is continuous, differentiable, non-separable unimodal function. Its global minimum is  $f(x^*) = 0$  located at  $(0, 0)$ . The test domain is  $-10 \leq x_i \leq 10$ .

**F18. Beale function:** This is continuous, differen-

Table 2: The average of the normalized results of the proposed KHFG optimization algorithms and several famous meta-heuristic optimization algorithms in 50 trials for the benchmark functions.

ID	KHFG	KH	PSO	GA	ES	CS	ACO	ABC	TLBO
F1	0.97	0.94	0.89	1	0.77	0	0.99	1	1
F2	0.99	0.99	1	0.99	0	0.98	0.98	0.99	0.99
F3	0.98	0.36	0.68	0.36	1	0	0.44	0.86	0.37
F4	1	0.99	0.97	0.99	0	0.93	0.97	0.97	0.98
F5	0.21	0.3	0	1	0.99	0.33	0.99	0.24	0.25
F6	1	0.87	0.21	0.01	0	0.46	0	0.99	0.99
F7	0.72	0.58	0.9	1	0.29	0	0.99	0.99	1
F8	1	1	0.99	0.99	0	0.99	1	1	1
F9	1	0.92	0.24	0.06	0	0.9	0	0.99	1
F10	0.96	0.8	0.69	0	1	1	0.13	0.36	0.36
F11	0.99	0.98	0.99	0.97	0	1	0.98	0.99	0.99
F12	0.99	0.99	0.66	0	0.49	1	0	0.99	0.99
F13	1	1	1	0.77	0	1	0.93	0.99	0.99
F14	0.99	0.99	0	0.99	0.99	1	0.92	0.85	0.85
F15	0.99	1	0.97	0	0.99	0.62	0.01	0.75	0.75
F16	1	0.99	0.5	0.5	0	1	0.99	0.99	0.99
F17	1	0.99	0.54	0.53	0	0.99	0.96	1	1
F18	1	1	0	0.99	0.99	0.84	0.99	1	1
F19	0.57	1	1	0	1	0.99	1	1	1
F20	1	1	0.99	0.76	0	1	0.07	1	1
SUM	18.4	17.76	13.47	12.21	15.11	8.8	13.41	18.04	17.57
Rank	1	3	6	8	5	9	7	2	4

tiable, non-separable and unimodal test function. The global minimum of the function is  $f(x^*) = 0$  located at  $(3, 0.5)$ . This function is subject to  $-4.5 \leq x_i \leq 4.5$ .

**F19. Goldstein Price function:** Goldstein Price is continuous, differentiable, non-separable and multimodal function. Its global optimum is  $f(x^*) = 3$  which is located at  $(-1, 0)$  where  $-2 \leq x_i \leq 2$ .

**F20. Forrester's function:** This function is one-dimensional, continuous and multimodal function. It has a global optimum  $f(x^*) = -6.0207$  at  $(0.7572)$ . The test domain is  $0 \leq x_i \leq 1$ .

Several experiments with have been be carried out to obtain the real performance of an algorithm. The results are obtained on 50 trials with different initialization conditions. In all the experiments, the number of iterations and the number of krill individuals are set to 200 and 25 respectively. The results are normalized using (5).

$$X_{i,normalized} = 1 - \frac{(X_i - X_{min})}{(X_{max} - X_{min})} \quad (17)$$

where,  $X_{i,normalized}$  is the normalized value of solution  $i$ ,  $X_i$  is the fitness value of solution  $i$ ,  $X_{min}$  and  $X_{max}$  are the minimum and the maximum fitness value of the found solutions respectively.

KHFG is compared with the following eight well-known meta-heuristic optimization algorithms: Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Evolution Strategy (ES), Cuckoo Search (CS), Krill Herd (KH), Ant Colony Optimization (ACO), Artificial Bee Colony (ABC) and Teaching-Learning Based Optimization (TLBO). The obtained results of the proposed FG optimization algorithms together with the results of the mentioned meta-heuristic optimization algorithms are listed in Table 2. It should be mentioned that these obtained results are the average of the normalized results on 50 trials.

In order to make a fair comparison of the proposed KHFG and the mentioned optimization algorithms, the normalized results are summed and ranked. As we can easily see, KHFG optimization algorithm outperforms the other methods in 10 out of 20 functions.

In order to compare the performance of different algorithms different methods such as Chess Rating System and Wilcoxon Rank Sum Tests can be used. In this experiment Nonparametric Wilcoxon Rank Sum Tests is used. We conduct this experiments on the results achieved by the proposed algorithm and other mentioned meta-heuristic algorithms on presented benchmark functions and the results of this experiments is presented in Table 1. The results of this experiment are presented with  $P$ -value and  $h$

Table 3: Statistical Comparison between KHFG and the other five algorithms.

ID	KH		PSO		GA		ES		CS		ACO		ABC		TLBO	
	P-value	h	P-value	h	P-value	h	P-value	h	P-value	h	P-value	h	P-value	h	P-value	h
F1	4.85E-18	1+	3.68E-18	1+	5.12E-12	1-	3.68E-18	1+	4.59E-16	1+	1.14E-02	1-	2.14 E-02	1-	2.14 E-02	1-
F2	2.21E-14	1-	2.81E-15	1-	3.68E-18	1+	3.68E-18	1+	4.14E-11	1+	1.23E-02	1+	6.8 E-03	1-	5.8 E-03	1-
F3	1.92E-12	1+	4.19E-12	1+	3.68E-18	1+	2.51E-16	1-	3.68E-18	1+	5.44E-01	1+	1.215 E-01	1+	6.098 E-01	1+
F4	2.11E-04	1+	3.51E-05	1+	3.68E-18	1+	2.11E-19	1+	3.68E-18	1+	3E-02	1+	2.41 E-01	1+	1.11 E-02	1+
F5	4.73E-18	1-	3.68E-18	1+	2.48E-21	1-	5.26E-21	1-	8.27E-21	1-	7.77E-01	1-	3.28 E-01	1-	3.92 E-02	1-
F6	1.12E-06	1+	3.68E-18	1+	4.64E-08	1+	6.22E-14	1+	4.14E-06	1+	1	1+	4.00E-04	1+	4 E-04	1+
F7	2.87E-03	1+	4.58E-04	1-	6.98E-09	1-	8.40E-06	1+	1.32E-06	1+	2.73E-01	1-	2.82 E-01	1-	2.827 E-01	1-
F8	6.71E-01	0	1.41E-01	0	8.14E-12	1+	3.68E-18	1+	8.67E-02	1+	0	0	0	0	0	0
F9	2.37E-01	1+	4.46E-01	1+	3.68E-18	1+	3.68E-18	1+	3.68E-18	1+	9.946 E-01	1+	6 E-04	1+	5.4 E-03	1-
F10	7.96E-15	1+	3.26E-17	1+	3.68E-18	1+	2.21E-21	1-	3.51E-05	1-	8.309 E-01	1+	5.99 E-01	1+	5.995 E-02	1+
F11	9.26E-01	1+	7.68E-03	0	1.52E-05	1+	3.68E-18	1+	3.16E-01	0	1.65 E-02	1+	1.8 E-03	1+	1.8 E-03	1+
F12	1.45E-02	1-	1.29E-02	1+	1.38E-04	1+	1.64E-03	1+	3.62E-02	1-	9.999 E-01	1+	9 E-04	1+	8 E-04	1+
F13	8.47E-05	0	9.34E-04	0	2.32E-08	1+	3.23E-09	1+	5.12E-01	0	7.00E-02	1+	0	1+	0	1+
F14	2.60E-17	1+	3.68E-18	1+	3.68E-18	1+	1.12E-01	0	4.85E-21	1-	8.00E-02	1+	1.453 E-01	1+	1.404 E-01	1+
F15	5.97E-01	1-	3.68E-18	1+	4.31E-15	1+	1.18E-01	0	3.07E-21	1+	9.798 E-01	1+	2.333 E-01	1+	2.374 E-01	1+
F16	4.14E-04	1+	3.68E-18	1+	4.14E-04	1+	5.22E-11	1+	9.85E-01	0	1 E-02	1+	5 E-04	1+	5.5 E-03	1+
F17	1.51E-02	1+	3.68E-18	1+	3.68E-18	1+	3.68E-18	1+	8.18E-15	1+	4 E-02	1+	0	0	0	0
F18	2.07E-01	0	4.08E-02	1+	8.46E-03	1+	3.68E-18	1+	6.24E-10	1+	1 E-02	1+	0	0	0	0
F19	2.18E-02	1-	5.45E-01	1-	7.31E-03	1-	3.43E-19	1-	6.14E-19	1-	4.247 E-01	1-	4.247 E-02	1-	4.247 E-01	1-
F20	6.70E-01	0	3.68E-18	1+	3.68E-18	1+	3.68E-18	1+	1.45E-01	0	9.3 E-01	1+	0	0	0	0

which help us to find out whether there is significant difference between the performances of two algorithms. In this experiment  $h$  can get three different values,  $1^+$ ,  $0$  and  $1^-$  and each value indicates different fact.  $h=1$  indicates that the performances of two compared algorithm is significantly different with 95% confidence.  $1^+$  shows that an algorithm has higher performance compared with another algorithm in the comparison and  $1^-$  vice versa. And  $0$  indicates that there is no statistical difference.

Table 3 shows the results of Null Hypothesis Significance Testing which has been done by Nonparametric Wilcoxon Rank Sum Tests on the proposed algorithm and other mentioned algorithms in MATLAB 2010. It also indicates that in most of the experiments the results achieved by KHFG are comparable with the results achieved by the other methods. Following are some discussion about the results presented in Table 3.

1. F1 is ackley function. The KHFG algorithm achieved the third best mean value compared with those of other algorithms and ranked second. There is statistically significant difference between KHFG algorithm and KH, PSO, CS and ES. GA, ABC, TLBO have significant performance over the KHFG algorithm.
2. F3 is Levy function. The KHFG algorithm achieved the second best mean value compared with those of other algorithms and ranked second. There is statistically significant difference

between KHFG algorithm and KH, PSO, CS, GA, ACO, ABC and TLBO. ES has significant performance over the KHFG algorithm

3. F8 is Sphere function. The KHFG algorithm achieved the best mean value compared with those of other algorithms. There is no statistically significant difference between KHFG algorithm and KH, PSO, ABC, ACO and TLBO algorithms. There is statistically significant difference between KHFG algorithm and GA, CS and ES algorithms.
4. F12 is Shubert function. The proposed method achieved the second best mean value in this experiment. The statistical results show that the proposed FGKH has significant importance over PSO, GA, ES, ACO, ABC and TLBO. There is no significant importance between the proposed method and KH algorithm. There is only CS algorithm that has significant importance over the FGKH algorithm.
5. F16 is Easom test function. The FGKH algorithm achieved the best results in this experiment among the mentioned evolutionary techniques. The proposed method has significant importance over almost all algorithms except ES algorithm. There is no significant importance between FGKH and ES algorithm.
6. F20 is Forrester's function. The proposed algorithm ranked first in this experiment. It has

also significant importance over PSO, GA, ES and ACO algorithm. There is no significant importance between FGKH and KH, CS, ABC and TLBO algorithms.

## 5 CONCLUSION

This paper introduces a new method called Focus Group Idea to improve the fitness of the KH algorithm by modifying the solutions found by the searching agents. This idea put the emphasis on utilizing all members' solutions with the focus on their fitness. According to the definition of the Focus Group Idea, each member can affect the other members' ideas considering the quality of its solution. In other words, the more quality solution it has the more impact it has on the other members' ideas (solutions). In order to evaluate the performance of the proposed method we experimentally compared KHFG to other well known evolutionary techniques for solving a set of standard benchmark functions. The results achieved by KHFG in comparison with those of the other methods, demonstrate the ability of proposed method in improving the performance of the KH algorithm.

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