Integrated Production and Imperfect Preventive Maintenance Planning
An Effective MILP-based Relax-and-Fix/Fix-and-Optimize Method

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Abstract: This paper investigates the integrated production and imperfect preventive maintenance planning problem. The main objective is to determine an optimal combined production and maintenance strategy that concurrently minimizes production as well as maintenance costs during a given finite planning horizon. To enhance the quality of the solution and improve the computational time, we reconsider the reformulation of the problem proposed in (Aghezzaf et al., 2016) and then solved it with an effective MILP-based Relax-and-Fix/Fix-and-Optimize method (RFFO). The results of this Relax-and-Fix/Fix-and-Optimize technique were also compared to those obtained by a Dantzig-Wolfe Decomposition (DWD) technique applied to this same reformulation of the problem. The results of this analysis show that the RFFO technique provides quite good solutions to the test problems with a noticeable improvement in computational time. DWD on the other hand exhibits a good improvement in terms of computational times, however, the quality of the solution still requires some more improvements.

1 INTRODUCTION

Even though managed by two different departments in some factories, the production planning and maintenance planning are two closely interrelated functions. In the major modern factories effort is done to also carry these two planning functions in an integrated manner. Researchers have also proposed strategies and developed models to integrate the production and maintenance planning decisions both at the tactical as well as at the operational levels. Various mathematical models focusing on coordinating production and maintenance plans are proposed in (Lin et al., 1992; Gurevich et al., 1996; Agogino et al., 1997; Ben-Daya and Rahim, 2000; El-Amin et al., 2000; Kiyoshi et al., 2002; Chattopadhyay, 2004; Martorell et al., 2005; Aghezzaf et al., 2007; Dahal and Chakpitak, 2007; El-Ferik, 2008; Fitouhi and Nourelfath, 2012; Wang, 2013). A wide variety of solution techniques and algorithms including the whole spectrum of heuristic techniques, dynamic programming, tabu-search multi-objective optimization, expert systems and many other hybrid techniques are also proposed, see for example (Lin et al., 1992; Gurevich et al., 1996; Agogino et al., 1997; Ben-Daya and Rahim, 2000; Kiyoshi et al., 2002; Chattopadhyay, 2004; Martorell et al., 2005; Dahal and Chakpitak, 2007). Integrated production and imperfect preventive maintenance planning models were also proposed, see for example (Chung and Krajewski, 1984; Ben-Daya and Rahim, 2000; Sana and Chaudhuri, 2010; Fitouhi and Nourelfath, 2012; Aghezzaf et al., 2016). Imperfect preventive maintenance, when performed, brings the manufacturing system to an operating state that is between as bad as old and as good as new. The resulting mathematical models are naturally non-linear and involve many binary variables. In (Aghezzaf et al., 2016), the authors proposed a reformulation for the natural integrated production and imperfect preventive maintenance planning problem. The resulting optimization model is a mixed-integer linear programming problem which is solved using a MILP-based approximation method. The current paper proposes to adopt a Relax-and-Fix/Fix-and-Optimize approach and analyse its results.

This Relax-and-Fix/Fix-and-Optimize approach results in quite good solutions. However, it still requires a large amount of the computational time for medium and large scale instances of the problem. To deal with these large scale instances, some heuris-

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tics based on the Dantzig-Wolfe decomposition techniques and a new version of the Relax-and-Fix/Fix-and-Optimize method are investigated and developed. The main goal is to obtain good quality solutions within a reasonable amount of the computational time frame.

The remainder of this paper is organized as follows. In section 2, a slightly modified version of the mathematical model, for integrated production planning and imperfect preventive maintenance planning, propose in (Aghezzaf et al., 2016) is presented. In section 3, the developed Relax-and-Fix/Fix-and-Optimize techniques (RFFO) is introduced and presented in details. Section 4 presents the Dantzig-Wolfe (DWD) decomposition to solve the reformulated production and maintenance planning model. Computational results of a set of benchmark cases are presented and discussed in Section 5. Finally, Section 6 summarizes the main findings of this research work and discusses some possible research directions.

2 THE INTEGRATED PRODUCTION AND IMPERFECT PREVENTIVE MAINTENANCE MODEL

In this section, the mathematical optimization model for integrated production and imperfect preventive maintenance problem described in (Aghezzaf et al., 2016) is briefly summarized. Then, the reformulation proposed by the authors for this production and imperfect preventive maintenance problem is shown and used as the underlying optimization model for the detailed subsequent discussion.

2.1 A Mathematical Optimization Model for the Integrated Production and Imperfect Preventive Maintenance Problem (IPImPMP)

In the IPImPMP problems, it is assumed that the systemś operating state is stochastically predictable, in terms of its operating age, and that it can accordingly be preventively maintained during preplanned periods. The preventive maintenance is assumed to be imperfect, so that after each maintenance action the manufacturing system is at an operating state that is between as bad as old and as good as new.

Along the same lines as in (Aghezzaf et al., 2016), we consider a planning horizon $H = \{1, ...T\}$ of T periods, each having a duration τ, and a set of products $j \in P = \{1, ...N\}$ to be planned during this horizon. Let d_{jt} be the demand for item j in period t, f_{jt} be the fixed cost of producing item j in period t, p_{jt} be the variable cost of producing item j in period t, and h_{jt} be the variable holding cost of item j in period t. The production system has a known maximum constant production capacity κ_{max} (given in time units) and the processing time of each unit of item j is given by ρ_j. The system can be maintained preventively or correctively when a failure occurs. The cost of carrying out a k^{th} preventive maintenance action is denoted by C_{PM}^k and the cost of performing a corrective maintenance action on the system when a failure occurred, right after k^{th} preventive maintenance, is denoted by C_{CM}^k. Finally, let δ_{PM}^k be the expected time required for the k^{th} preventive maintenance action, and δ_{CM}^k the expected time required to perform a corrective maintenance action on the system when a failure occurred, right after k^{th} preventive maintenance.

The variables of the model are: Q_{jt} the quantity of item j produced during period t; I_{jt} the inventory of item j at the end of period t; x_{jt} a binary variable set to 1 if item j is produced during period t and 0 otherwise; y_t a binary decision variable set to 1 if the machine is setup to production during period t and 0 otherwise; and finally z_{st}^k a binary variable set to 1 if the last preventive maintenance of the system before the time period t is the k_{th} one and has taken place during the time period s and 0 otherwise. By convention we assume that the manufacturing system is preventively maintained in the beginning of period 1, that is $z_{11}^1 = 1$ for $k \leq s \leq t$. The optimization model for the Integrated Production and Imperfect Preventive Maintenance Planning Problem (IPImPMP) is given by:

Minimize

$$Z_{ImPMP}^{IP} = \sum_{t=1}^{T}\sum_{j=1}^{N}(f_{jt}x_{jt} + p_{jt}Q_{jt} + h_{jt}I_{jt}) + \sum_{t=1}^{T}\sum_{k=1}^{t} C_{PM}^k z_{tt}^k + \sum_{t=1}^{T}\sum_{s=1}^{t}\sum_{k=1}^{s} C_{st}^k(\mathbf{y})y_t z_{st}^k$$

subject to:

$$Q_{jt} + \begin{cases} I_{j,t-1} & if\ t > 1 \\ 0, & if\ t = 1 \end{cases} - I_{jt} = d_{jt}, \quad \forall j \in P, \forall t \in H \tag{1}$$

$$Q_{jt} - \kappa_{max}x_{jt} \leq 0, \qquad \forall j \in P, \forall t \in H \tag{2}$$

$$x_{jt} - y_t \leq 0, \qquad \forall j \in P, \forall t \in H \tag{3}$$

$$\sum_{j=1}^{N} \rho_j Q_{jt} + \sum_{k=1}^{t} \delta_{PM}^{k} z_{tt}^{k} + \sum_{s=1}^{t}\sum_{k=1}^{s} \kappa_{st}^{k}(\mathbf{y}) y_t z_{st}^{k} \qquad (4)$$
$$\leq \kappa_{max}, \forall t \in H$$

$$\sum_{s=1}^{t}\sum_{k=1}^{s} z_{st}^{k} = 1, \qquad \forall t \in H \qquad (5)$$

$$z_{st}^{k} - z_{s,t+1}^{k} \geq 0, \quad \forall k,s,t \in H, k \leq s \leq t \leq T-1 \qquad (6)$$

$$z_{tt}^{k} - \sum_{s=k-1}^{t-1} z_{s,t-1}^{k-1} \leq 0, \quad \forall t,k \backslash \{1\} \in H, 1 < k \leq t \qquad (7)$$

$$\sum_{t=2}^{T}\sum_{s=2}^{t} z_{st}^{1} = 0 \qquad \forall s,t \geq 2 \in H, s \leq t \qquad (8)$$

$$\sum_{k=1}^{t} z_{tt}^{k} \leq y_t, \qquad \forall t \in H \qquad (9)$$

$$Q_{jt}, I_{jt} \geq 0, \quad x_{jt}, y_t, z_{st}^{k} \in \{0,1\}$$
$$\forall j \in P, s,t \in H, k \leq s \leq t$$

Constraints (1) are the flow conservation constraints. They guarantee that the available inventory augmented with the quantity produced in period t is sufficient to satisfy the demand d_{jt} of item j in that period. The remainder is stocked for the subsequent periods. Constraints (2) make sure that, when the production of an item is scheduled in certain period, the system is setup accordingly to produce that item in that period. These constraints force also disbursement of the fixed costs. Constraints (3) indicate whether the system is operating or not in each period, in which it is setup to produce some products. Constraints (4) are the capacity restrictions which are defined for each period $t \in H$. They guarantee that the quantity which is produced in a period t does not exceed the available capacity of the system, given its status in terms of the expected capacity loss during that period. Constraints (5) determine the periods during which the preventive maintenance activities take place. In order to assure the consistency, constraints (6) are established to guarantee that if the last preventive maintenance action, before a time period $t+1$, takes place in a period $s < t$ and it is the k^{th} one, then this preventive maintenance action must also be the k^{th} and the last one before a period t. Again, in order to keep the consistency, constraints (7) assure that the k^{th} preventive maintenance takes place in some period $t \geq k$, only if the $(k-1)^{th}$ preventive maintenance took place in some period before t. Constraints (8) assures that the system is always maintained for the first time in the first period. Constraints (9) ensure that the k^{th} preventive maintenance takes place only once and when the system is setup to production.

As in (Aghezzaf et al., 2016), the function $\kappa_{ts}^{k}(y)$ and $C_{ts}^{k}(y)$ are the expected production capacity loss and expected maintenance cost during the time period t when the k^{th} and the last preventive maintenance action before time period t has taken place in the time period s, with $k \leq s \leq t$. These parameters depend on the system's setup vector $\mathbf{y}$ and are given by

$$\kappa_{st}^{k}(y) = \begin{cases} \delta_{CM}^{k} \int_{0}^{\tau} \beta^{k} r_0 \left(u + \alpha^{k} \left[\sum_{t'=1}^{s-1} y_{t'} \right] \tau \right) du \\ \qquad \text{if } t = s, k \leq s, \\ \delta_{CM}^{k} \int_{0}^{\tau} \beta^{k} r_0 \left(u + \alpha^{k} \left[\sum_{t'=1}^{s-1} y_{t'} \right] \tau \right. \\ \qquad \left. + \left[\sum_{t'=s}^{t-1} y_{t'} \right] \tau \right) du \\ \qquad \text{if } s \leq t \leq T. \end{cases} \qquad (10)$$

$$C_{st}^{k}(y) = \begin{cases} C_{CM}^{k} \int_{0}^{\tau} \beta^{k} r_0 \left(u + \alpha^{k} \left[\sum_{t'=1}^{s-1} y_{t'} \right] \tau \right) du \\ \qquad \text{if } t = s, k \leq s, \\ C_{CM}^{k} \int_{0}^{\tau} \beta^{k} r_0 \left(u + \alpha^{k} \left[\sum_{t'=1}^{s-1} y_{t'} \right] \tau \right. \\ \qquad \left. + \left[\sum_{t'=s}^{t-1} y_{t'} \right] \tau \right) du \\ \qquad \text{if } s \leq t \leq T. \end{cases} \qquad (11)$$

We adopt a hybrid failure rate model which was defined in (Aghezzaf et al., 2016). If after k^{th} preventive maintenance the failure rate function remains below some threshold function, $r_{k_{max}}(t)$, the system can again be preventively maintained. However, if it reaches or exceeds this threshold level, it is overhauled and will be returned to an "as-good-as-new" state. We considers a lifetime of a whole system is randomly distributed and for which the corresponding initial hazard rate function is given by the function $r_0(t)$. If the k^{th} preventive maintenance takes place $T_k\tau$ units of time after an overhaul, that is in the beginning of period T_k having fixed length τ, the hazard rate function $r_k(t)$ of the system is then defined as:

$$r_k(t) = \beta_k r_0(t + \alpha_k T_{AOT}^{k}),$$
$$t \in [0, (T_{k+1} - T_k)\tau], \forall k, 1 \leq k \leq k_{max} \qquad (12)$$

where T_{AOT}^{k} is the actual operating time of the system since the beginning of the planning horizon until the beginning of period T_k, the period during which the k^{th} preventive maintenance is taking place. The parameters α_k and β_k stand, respectively, for the age reduction coefficient and the hazard rate increasing coefficient (adjustment factor) such that $0 \leq \alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_{kmax} \leq 1$ and $1 \leq \beta_1 \leq \beta_2 \leq \ldots \leq \beta_{kmax}$.

2.2 Reformulation of the Problem (Re_IPImPMP)

The natural formulation of problem (IPImPMP) is nonlinear. It can be reformulated and modeled as a mixed-integer linear program as is shown in (Aghezzaf et al., 2016). We propose a slight variation of the mathematical reformulation proposed in (Aghezzaf et al., 2016) by adding the variables $v_{st}^{k}(p,q)$, with $p \leq s \leq t, k \leq s$ and $q \leq t-s+1$, to be a binary variable assuming value 1 if the system is setup to production with p time during the horizon $1,\ldots,s-1$ and q time during the period s,..,t, and the k^{th} maintenance takes place in period s and, $w_{st}^{k}(p,q)$, with $p \leq s \leq t, k \leq s$ and $q \leq t-s+1$, to be a binary variable assuming value 1 if the system is setup for production p time during the horizon $1,\ldots,s-1$ and q time during the period $s,..,t$ and the k^{th} maintenance takes place in period s and the system must be produced at period t.

$$(Re\_IPImPMP): Minimize \quad Z_{ImPMP}^{Re\_IP} = \sum_{t=1}^{T}\sum_{j=1}^{N}\left(f_{jt}x_{jt}+p_{jt}Q_{jt}+h_{jt}I_{jt}\right)+\sum_{t=1}^{T}\sum_{k=1}^{t}C_{PM}^{k}z_{tt}^{k} + \sum_{t=1}^{T}\sum_{s=1}^{t}\sum_{k=1}^{s}\sum_{p=0}^{s-1}\sum_{q=0}^{t-s}C_{st}^{k}(p,q)v_{st}^{k}(p,q)$$

subject to :

Eq. (1) - (3) and (5) - (9)

$$\sum_{j=1}^{N}\rho_{j}Q_{jt}+\sum_{k=1}^{t}\delta_{PM}^{k}z_{tt}^{k} + \sum_{s=1}^{t}\sum_{k=1}^{s}\sum_{p=0}^{s-1}\sum_{q=0}^{t-s}c_{st}^{k}(p,q)v_{st}^{k}(p,q) \leq \kappa_{max}, \forall t \in H \tag{13}$$

$$\sum_{p=0}^{s-1}\sum_{q=0}^{t-s}p.u_{st}(p,q)-\begin{cases}\sum_{s'=1}^{s-1}y_{s'}, & if\ s>1\\ 0, & if\ s=1\end{cases} \leq 0, \quad \forall t,s \in H, 1 \leq s \leq t \tag{14}$$

$$\sum_{p=0}^{s-1}\sum_{q=0}^{t-s}q.u_{st}(p,q)-\begin{cases}\sum_{s'=s}^{t-1}y_{s'}, & if\ t>1\\ 0, & if\ t=1\end{cases} \leq 0, \quad \forall t,s \in H, 1 \leq s \leq t \tag{15}$$

$$\sum_{p=0}^{s-1}\sum_{q=0}^{t-s}u_{st}(p,q)=1, \quad \forall t,s \in H, 1 \leq s \leq t \tag{16}$$

$$z_{st}^{k}+u_{st}(p,q)-v_{st}^{k}(p,q) \leq 1, \quad \forall k,s,t,p,q \in H, p \leq s-1, q \leq t-s \tag{17}$$

$$y_{t}+v_{st}^{k}(p,q)-w_{st}^{k}(p,q) \leq 1, \quad \forall k,s,t,p,q \in H, p \leq s-1, q \leq t-s \tag{18}$$

$$Q_{jt}, I_{jt} \geq 0, \quad x_{jt}, y_{t}, z_{st}^{k}, u_{st}\{p,q\}, v_{st}^{k}\{p,q\}, w_{st}^{k}\{p,q\} \in \{0,1\}$$
$$\forall j \in P, s,t \in H, k \leq s \leq t, p \leq s-1, q \leq t-s$$

where $u_{st}(p,q)$, with $p \leq s \leq t$ and $0 \leq q \leq t-s+1$, to be a binary variable assuming value 1 if the system is setup to production p times during the horizon 1,..., s-1 and q times during the periods s,..., t-1.

Constraints (1) - (3) and (5) - (9) are the same as before. Constraints (13) are revisited from (4). However constraints (14), (15), (16) determine the values of the variables u and v. The constraints (17) relate the variables z with u and v, meaning that if the k^{th} and last preventive maintenance before t takes places in period $s \leq t$ and if the system is setup to production p times during the horizon $\{1,\ldots,s-1\}$ and q times during the periods $\{s,\ldots t-1\}$ then $v_{st}^{k}(p,q)$ =1. The constraints (18) relate the variables v with y and w, meaning that if both the k^{th} and last preventive maintenance before t takes places in period $s \leq t$ and if the system is setup to production p times during the horizon $\{1,\ldots,s-1\}$ and q times during the periods $\{s,\ldots t-1\}$ and the system must be produced at period t then $w_{st}^{k}(p,q)$ =1.

Here again, as reported in (Aghezzaf et al., 2016), we let $C_{st}^{k}(p,q)$ and $\kappa_{st}^{k}(p,q)$ be respectively the expected maintenance cost and expected loss in production capacity of the system during period t, when the last preventive maintenance action before time period t has taken place in the beginning of period s, $s \leq t$. These parameters are given by:

$$C_{st}^{k}(p,q)=\begin{cases}C_{CM}^{k}\int_{0}^{\tau}\beta_{k}r_{0}(u+\alpha_{k}p\tau)du & if t=s,\\ C_{CM}^{k}\int_{0}^{\tau}\beta_{k}r_{0}(u+\alpha_{k}p\tau+q\tau)du & if s \leq t \leq T.\end{cases} \tag{19}$$

and

$$\kappa_{st}^{k}(p,q)=\begin{cases}\delta_{CM}^{k}\int_{0}^{\tau}\beta_{k}r_{0}(u+\alpha_{k}p\tau)du & if t=s,\\ \delta_{CM}^{k}\int_{0}^{\tau}\beta_{k}r_{0}(u+\alpha_{k}p\tau+q\tau)du & if s \leq t \leq T.\end{cases} \tag{20}$$

3 RELAX-AND-FIX WITH FIX-AND-OPTIMIZE HEURISTIC (RFFO) TO SOLVE THE Re_IPImPMP MODEL

The Relax-and-Fix (RF) heuristic solves MIP problem by sequentially resolving sub-problems in which some variable are fixed and others are relaxed. To

solve production planning problems, for example, the planning horizon is partitioned and the setup variables are fixed backward or forward. Fix-and-Optimize is an improvement heuristic also based on decompositions of the original problem into multiple sub-problems with smaller number of binary variables. The RFFO method is a framework designed to combine the Relax-and-fix (RF) and fix-and-optimize (FO) heuristics. In (Toledo et al., 2015), the authors propose an approach in which most binary variables are fixed or relaxed and only few of them are forced to be integer and are optimized. They named this small set of integer variables a "window" and suggested three window type strategies: row-wise, in which the window moves along rows; column-wise, in which the window moves along columns; and value-wise, in which the window selects the variables with relaxed values closest to 0.5. As in (Toledo et al., 2015), For both heuristics applied to the Re_IPImPMP model, we consider a matrix Y where each of its entries is the binary varaible y_t. The inputs of the RF are the set of binary variables ($sol.y$), the number of binary variables ($windowSize$) to be chosen, the selection criteria to choose variables ($windowType$), the overlap rate of binary variables to be re-optimized ($overlap$) and the execution time limit ($timeLimit$). Initially, all binary variables in the RF solution ($sol.y$) are relaxed, and a window is defined as a set that includes a fixed amount of ($windowSize$) variables. Then, those variables which are inside the window are enforced to be integer in the set y_{MIP}, while the others are kept in y_{LP}. We solve the problem to get the results of MIP. Next, a new set of variables ($window$) is defined by a subset of fixed integers (y_{fixed}), sets of optimized integer (y_{LP}) and relaxed variables (y_{MIP}). The $window$ moves forward by the $step$ parameter at each iteration on which each $step = round(|overlap * windowSize|), \quad overlap \in [0,1]$. All variables that leave the window are fixed in the next iteration, and the same number of relaxed variables are enforced to be integer. The algorithm proceed in this way until all variables are fixed. After the RF phase is complete, FO tries to improve this initial solution until the time limit has been reached. If the improvement achieved by a FO solution is not satisfactory, the window size is increased. The MIP subproblems become larger as an attempt to find better solutions.
The pseudo-code of the method can be provided to interested researchers upon request.

4 SINGLE DANTZIG-WOLFE BY PRODUCT DECOMPOSITION WITH FIX AND OPTIMIZE TO SOLVE THE MINLP IPImPMP MODEL

Dantzig-Wolfe decomposition (DWD) method is used in (Pimentel et al., 2010) to solve the multi-item capacitated lot sizing problem with setup times. The authors applied the standard DantzigWolfe decomposition (DWD) in three different ways. In the first the subproblems are defined by items (PIDWD), in the second they are defined by periods (PJDWD) and a third decomposition in which the subproblems of both types are integrated in the same model (MDWD). The three approaches were tested on the IPImPMP model defined in Section 2.1. Based on the results, the suitable approach seems to be the production decomposition which is then selected for the comparison of the results. We consider the capacity constraints 4, linking constraints the the variables associated with different products as the master problem for this decomposition.

4.1 Master Problem

The master problem includes the collections of production plans of each items and the maintenance planning. The decision variables are ϑ_j^m, corresponding to the items production plans m generated by the subproblems for products j, and z_{st}^k are the variables corresponding to the maintenance plan developed at the master problem level. The linear programming relaxation of the master problem of the production decomposition is given below:
(MPJ-PPM) : Minimize $Z_{IPImPPM}^{MPJ} =$

$$\sum_{j=1}^{N}\sum_{m=1}^{Mj}\left[\sum_{t=1}^{T}(f_{jt}\bar{x}_{jt}^m + p_{jt}\bar{Q}_{jt}^m + h_{jt}\bar{I}_{jt}^m)\right]\vartheta_j^m + \sum_{t=1}^{T}\sum_{k=1}^{t} C_{PM}^k z_{tt}^k + \sum_{t=1}^{T}\sum_{s=1}^{t}\sum_{k=1}^{s} c_{st}^k(\bar{y}).\bar{y}_t z_{st}^k$$

subject to:

$$\sum_{j=1}^{N}\sum_{m=1}^{Mj}\rho_j\bar{Q}_{jt}^m\vartheta_j^m + \sum_{t=1}^{T}\sum_{k=1}^{t}\delta_{PM}^k z_{tt}^k + \sum_{s=1}^{t}\sum_{k=1}^{s}\kappa_{st}^k(\bar{y})z_{st}^k \leq \kappa_{max}, \forall t \in H(\mu_t) \tag{21}$$

$$\sum_{m=1}^{Mj}\vartheta_j^m = 1, \qquad \forall j \in N(\pi_j) \tag{22}$$

$$\sum_{m=1}^{Mj} \bar{x}_{jt}^{m} \vartheta_j^m <= \bar{y}_t, \quad \forall j \in N; t \in H \quad (\eta_{jt}) \tag{23}$$

$$\vartheta_j^m \geq 0, \qquad \forall j \in N \tag{24}$$

$$z_{st}^k \geq 0, \qquad \forall j \in N; t,s \in H \tag{25}$$

and Eq. (5) - (9)

where $\bar{y}_t = max_{j,m} \left\{ \bar{x}_{jt}^m \right\}$ and the objective function minimizes the overall production and maintenance costs. The set of constraints (21) are capacity constraint. These constraints impose that the combination of the chosen production plans satisfies the available capacity in each period based on the maintenance plans. Constraints (22) are the convexity constraints. The mixing of the chosen production plans is forced by constraints (23) to satisfy the production setup requirements. Constraints (24) and (25) force the decision variables to take nonnegative values.

Of course at the end of the process, the problem is solved again but then with the variables ϑ_j^m and z_{st}^k satisfying the following conditions:

$$\vartheta_j^m, z_{st}^k \in \{0,1\} \forall j \in N, s,t \in H, k \leq s \leq t, m \in Mj$$

To recover solution of problem IPImPMP, in terms of the original variables, we can obtain the value of (Q_{jt}, x_{jt}) from a solution of master problem (MPJ_PPM) as follows:

$$Q_{jt} = \sum_{m=1}^{Mj} \bar{Q}_{jt}^m \vartheta_j^m, \qquad \forall j \in N, \quad \forall t \in T \tag{26}$$

$$x_{jt} = \sum_{m=1}^{Mj} \bar{x}_{jt}^m \vartheta_j^m, \qquad \forall j \in N, \quad \forall t \in T \tag{27}$$

4.1.1 Subproblem

Assuming that μ_t is the vector of dual variables associated with the constraints (21), indexed by t, the π_j the vector of dual variables associated with the set of convexity constraints (22) and the η_{jt} is a dual associated variables with constraints (23). Each subproblem is one of following types:

$$Z_{IPImPMP}^{SPJ} = \sum_{t=1}^{T} (f_{jt} x_{jt} + p_{jt} Q_{jt} + h_{jt} I_{jt}) - \sum_{t=1}^{T} \rho_j Q_{jt} \pi_j - \mu_t - \sum_{t=1}^{T} \eta_{jt} x_{jt}, \forall j \in N$$

or

$$Z_{IPImPMP}^{SPJ} = \sum_{t=1}^{T} [(f_{jt} - \eta_{jt}) x_{jt} + p_{jt} Q_{jt} + h_{jt} I_{jt}] - \sum_{t=1}^{T} \rho_j Q_{jt} \pi_j - \mu_t, \forall j \in N$$

subject to:
Eq. (1) - (2)

$$Q_{jt}, I_{jt} \geq 0, \quad x_{jt}, y_t \in \{0,1\}$$
$$\forall j \in N, s,t \in H, k \leq s \leq t, p \leq s-1, q \leq t-s$$

variables :$Q_{jt}, I_{jt} \geq 0, \quad x_{jt}$
parameters: $d_{jt}, \mu_t, \pi_j, \eta_{jt}$

The pseudo-code of the proposed DWD approach can be provided to interested researchers upon request.

5 RESULTS AND DISCUSSIONS

In order to evaluate the effectiveness of the Re_IPImPMP model and the developed algorithms, the following paragraphs present the results of the computational experiments on some test instances, available in the literature. In particular, a collection of test instances from the LOTSIZELIB (Trigeiro, 1989) is used to evaluate the performance of the model (Re_IPImPMP) and developed algorithm. Of course, the instances from the LOTSIZELIB were extended and adapted to the integrate maintenance optimization aspect as done in (Aghezzaf et al., 2016).

5.1 The Test Instances

The algorithms presented above are coded in AMPL using the callable CPLEX 12.6 library to solve the MILP problems. The computation tests were carried out on an Intel(R) Core(TM) i7-3770 CPU @ 3.40 GHz, 3401 MHz, 4 Core(s), 8 Logical with 32 GB RAM. under windows 7. CPU times are given in seconds. For the maintenance part, we assumed that the machine is subject to the random failures according to a Gamma distribution $\Gamma(m=2, \nu=2)$ with a shape parameter m = 2 and a rate parameter ν = 2 as in (Aghezzaf et al., 2016). We also assume imperfect preventive maintenance with $\alpha_k = k/(3k+7)$ and $\beta_k = (12k+1)/(11k+1)$ for all k.

Table 1: Initial value of *WindowSize* and *Overlap* chosen in RFFO Algorithm.

Instances	*WindowSize*	*Overlap %*
A2007	5	60
tr6_15	8	60
tr6_30	12	60
tr12_15	8	60
tr12_30	12	60
tr24_15	8	60
tr24_30	12	60
set1ch	8	60

Table 2: A Summary of the experimental results for comparision between the Re_IPImPMP and RFFO Heuristic algorithm.

Instances	Re_PPImPMP	CPU Time (sec)	RFFO	CPU Time(sec)	GAP %
A2007	815.435	1.05	815.435	51.25	0.00
tr6_15	337,355.000	4,339.16	337,355.000	210.59	0.00
tr6_30	675,607.000	2,103,730.00	676,080.000	642,130.00	0.07
tr12_15	1,245,920.000	2,710.00	1,252,120.000	520.00	0.50
tr12_30	4,387,470.000	2,088,910.00	4,387,470.000	1,126,980.00	0.00
tr24_15	2,502,640.000	2,920.00	2,502,640.000	245.89	0.00
tr24_30	8,272,760.000	2,031,020.00	8,272,760.000	752,230.00	0.00
set1ch	107,532.000	130.00	107,532.000	244.69	0.00

Table 3: A summary of the experimental results of the DWD applied algorithm.

Instance	Value DWD	CPU Time (sec)	GAP %
A2007	875.520	1.95	7.37
tr6_15	337,787.000	1.65	0.13
tr6_30	859,051.000	1.29	27.15
tr12_15	2,159,410.000	1.05	73.32
tr12_30	8,917,970.000	3.59	103.26
tr24_15	4,829,470.000	1.01	92.98
tr24_30	16,945,000.000	3.76	104.83
set1ch	172,504.000	1.72	60.42

In this study, to evaluate the effect of *windowSize* and *overlap* for the RFFO heuristic method, we tested all the *windowSize* parameters from 1 to the last period of the planning horizon, and increasing *overlap* by *step* from 1 to *WindowSide* for instances $A20007$ and $tr6\_15$ to get initial value of the problem as shown in table 1.

5.2 Analysis and Discussions about the Experiments

Table 2 summarizes the results of the experiments which were carried out to compare the Re_IPImPMP (Aghezzaf et al., 2016) and the RFFO heuristic method. The first column of the table identifies the solved instances. The second column reports the optimal value of each instance which was obtained from the Re_IPImPMP model and the third column reports the resulted CPU running time. The fourth column describes the value of each instance which was obtained by the proposed RFFO heuristic algorithm and the fifth column presents the obtained CPU running time. The last column shows the GAP between the RFFO heuristic value and the Re_IPImPMP value. When comparing the results of the proposed RFFO with the results of the Re_IPImPMP model, it is clear that the proposed RFFO algorithm has the same optimal value with a considerable saving of the CPU (solve) time; it is faster from 4 to 10 times for the cases of the medium and/or large scale problems. However, in the small scale problems (A2007 and set1ch instances) was increasing large CPU time by the inner loop algorithm.

Table 3 summarizes the results of the computational experiments carried out for the DWD decomposition method. The first column of the table identifies the instances solved. The second column presents the value of each instance which was obtained via the proposed DWD decomposition algorithm; and the third column reports the CPU (solve) time. The last column describes the GAP between the DWD decomposition value and the Re_IPImPMP value by

$$GAP\% = \frac{(valueDWD - valueRe\_IPImPMP)}{valueRe\_IPImPMP} * 100.$$

As shown in Table 3, the proposed DWD provides solutions which are not so far from the optimal Re_IPImPMP value, with the less CPU time and memory which is used to reach the optimality for the same instance. However, the GAP, which is defined as the ratio of the difference between the value of the DWD algorithm and the value of the Re_IPImPMP model, showing it is just suitable for small and medium scale instances.

6 CONCLUSIONS

In this paper, we investigated the optimization model of an integrated production planning and imperfect preventive maintenance. The natural optimization model for this problem is as a nonlinear mixed integer problem. We slightly modified the reformulation (Re_IPImPMP) the problem proposed in (Aghezzaf et al., 2016) that solves the problem as a linear mixed integer program. There are a few major limitations for this reformulated model, including the time consuming as well as the increased number of variables and constraints for the large and medium-sized problems. We applied the Relax-and-Fix/Fix-and-Optimize heuristics and the Dantzig-Wolfe Decomposition (DWD) methods to select the suitable strategies to solve the proposed optimization models. The developed algorithm are tested and compared for CPU time and gap. The results from the numerical examples and computational experiments showed that the developed algorithm for solving the Re_IPImPMP problem has a very good solution quality with reduced computational time. Further studies are currently investigated to improve the DWD method in order to obtain better quality solutions and increase computational time savings, especially for large scale, block structured, and linear programming problems of integrated production planning and imperfect preventive maintenances.

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