# Optimal Scheduling of an on-Demand Fixture Manufacturing Cell for Mass Customisation Production Systems Model Formulation, Presentation and Validation

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- Keywords: Mass Customisation, Reconfigurable Fixtures, Group Technology, Optimisation Model, Mixed Integer Linear Programming.
- Abstract: A focal point of mass customisation production systems (a significant aspect of the fourth industrial revolution) is the implementation of reconfigurable jigs and fixtures. Traditional methods for the treatment of conventional fixtures are inadequate for those of the reconfigurable type. This paper describes the implementation of an on-demand fixture manufacturing cell that would reside in a mass customisation production system. The focus, in particular, is on the behaviour and optimisation of this cell in relation to the production system. To achieve this, a multi-stage optimisation procedure was developed that involves cluster analysis and a mixed inter linear programming (MILP) model to minimise total idle time (and thus makespan) in the system.

## **1 INTRODUCTION**

The onset of Industry 4.0 has led to an increased research interest in mass customisation production systems (Yao and Lin, 2016). A primary focus for the successful implementation of such production systems is the topic of reconfigurable jigs and fixtures (Smith et al., 2013). Customised products are unique and this has to be accounted for by employing jigs and fixtures that can accommodate constantly varying geometries. Reconfigurable jigs and fixtures are workholding devices that can be adapted to suit the specifications of each customised product (Bi and Zhang, 2001). As such, the scheduling of such a production system has to consider the reconfigurable fixture as an active influence on the workflow, and not as a constant resource.

An on-demand fixture manufacturing cell that serves a mass customisation production system was developed as part of this research. This paper presents an optimisation procedure that schedules activities within the fixture manufacturing cell in tandem with a part processing cell. This was developed as a three-stage process, the last of which is the primary focus of this paper. The first two stages involve cluster analysis to optimally assign parts to fixtures. The third stage utilises a mixed integer linear programming (MILP) model that minimises the total idle time in the system caused by a lack of synchronisation between the two cells.

## 2 LITERATURE REVIEW

### 2.1 Fixtures

Fixtures are used to physically locate, hold and support parts during a manufacturing process. Mass customisation manufacturing requires fixtures that can hold parts of varying geometry, and be able to rapidly and cost effectively change configurations according to these variations. Reconfigurable fixtures are a low cost solution to this problem. Recent advancements include pin-array fixtures and phase-change materials. The most widely used reconfigurable fixtures, however, are modular fixtures. These consist of a constant fixture base upon which different modules can be attached to hold various parts (Bi and Zhang, 2001). An example of such a fixture is the Blüco-Technik® dowel fixture shown in Figure 1.

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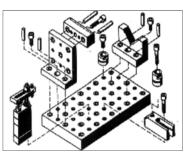


Figure 1: Blüco-Technik® dowel fixture (Bi and Zhang, 2001).

## 2.2 Group Technology

Mass customisation production systems aim to blend the advantages of both job shops (high variability but low volume) and dedicated manufacturing lines (high volume but low variability) while minimising their disadvantages (Fogliatto et al., 2012). Group Technology can play a major role in achieving this. Group Technology involves clustering similar parts into part families, which increases the efficiency of processing since the part family is then manufactured in a specialised cell. Group Technology has given rise to the cellular manufacturing paradigm. Modular fixtures can be effectively employed in cellular manufacturing systems, since the fixtures can be specialised for the part family associated with that cell. The fixtures are customised according to variations within the part family by adding and removing various modules (Groover, 2001).

The modular concept is applied in this research for implementing an on-demand fixture manufacturing cell. The fixtures and unfinished parts are handled separately until the two are assembled at the point where the part requires the fixture for it to be machined. The cellular manufacturing method is used so that modifications can be made to the same fixture base via fixture reconfigurations to serve numerous variations of the part type it is associated with.

### 2.3 Scheduling and Optimisation

A literary study of scheduling and optimisation models that considered fixtures as part of the system was conducted. This involved the typical job shop scheduling problem and numerous modifications thereof.

Thörnblad et al., (2013) conducted a study on a multi-task cell at GKN® Aerospace Engine Systems in Sweden. The problem was described as a flexible

job shop scheduling problem. A time-indexed formulation was used. The objective was to minimise the weighted tardiness, where the weighting increased as tardiness increased. The task was to assign a particular fixture to a job, and to limit the number of fixtures of each type.

A genetic algorithm was used by Wong et al., (2009) to solve a resource-constrained assembly job shop scheduling problem with lot streaming. The objective was to minimise total lateness cost. Resource constraints were used to place limits on the tools and fixtures used in the system, which were recyclable.

Yu et al., (2012) conducted a study on a reconfigurable manufacturing system with multiple process plans and limited pallets/fixtures. The problem was solved using a priority rule based scheduling approach, which compromised on optimality but improved ease of implementation. This simpler approach allowed the authors to consider multiple objectives: minimising makespan, minimising mean flow time, and minimising mean tardiness. The problem was constrained to only release jobs once the relevant pallet/fixture was available.

Literature has revealed that fixture utilisation in a production system was mostly limited to placing a constraint on the availability of fixtures as a resource. There was no research found that dealt with a system that could manufacture and reconfigure fixtures on-demand according to the manufacturing process demands.

# **3 PROBLEM STATEMENT**

## 3.1 **Problem Description**

The model presented in this paper describes a production system where two manufacturing cells exist to serve fixture reconfigurations and processing of parts, respectively. This represents a microcosm of a mass customisation production system that utilises cellular manufacturing principles to address the synchronicity required between reconfigurable fixtures and the customised parts that they serve. Pre-processed parts are to be processed in the part processing cell; the fixture configuration required to hold each of these parts is reconfigured on a fixture base in the fixture manufacturing cell and delivered to the part processing cell; each pre-processed part is then mounted to the fixture base configured for it (its fixture) so that it can be processed - this is a fixture-part mapping; the post-processed part is then

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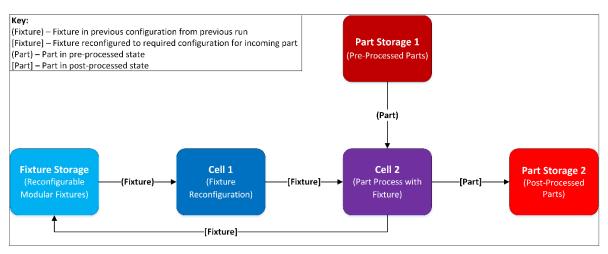


Figure 2: Workflow through the production system being considered.

removed from the fixture and released, while the fixture returns to the fixture storage system for it to be reconfigured for another part thereafter. The workflow through the production system is described in Figure 2.

The fixtures considered for this problem are of the reconfigurable modular type. The fixture base consists of an array of drilled holes. It is configurable with dowel pins as modules. The specifications are as follows:

- Array pattern: 8x8 holes (64 holes total) per fixture base;
- Pin range: 4-16 pins per fixture configuration.

The problem requires that parts be optimally assigned to fixture bases such that the interchange time between fixture configurations per fixture base, i.e. fixture reconfiguration times, are minimised. The problem also requires that fixtures be reconfigured and parts with fixtures be processed synchronously. These operations must be optimally scheduled such that the total idle time in the production system is minimised (thus minimising makespan). The MILP model presented in this paper focuses on this problem.

Total idle time was chosen to be the objective function of the model. This is because delays in the system would result from the idle time caused by one cell (either Cell 1 or Cell 2) being occupied after the other cell has completed its operation, thus halting workflow in the system. As such, emphasis has to be placed on ensuring that the operation times for Cell 1 and Cell 2, for every fixture and part combination, are as close to each other as possible for every operation.

## 3.2 Problem Formulation

The optimisation model presented in this paper is the final stage of a three-stage model. The three-stage model aims to solve the problems presented in Section 3.1 by separating the problem into three different stages. These are as follows:

- 1. Clustering Stage clusters similar parts to be assigned to the same fixture base by minimising the dissimilarity measure between the fixture configurations for those parts.
- 2. Intracluster Sequencing Stage sequences the clustered parts for each fixture base to be ordered such that the dissimilarities (and the reconfiguration times, by implication) between the fixture configurations on that fixture base are minimised.
- 3. Final Sequencing Stage minimises the idle time in the system by scheduling pairs of fixture-part mappings that yield a minimised time difference between their fixture reconfiguration operation (in Cell 1) and part processing operation (in Cell 2) for every time period.

The model presented in this paper isolates the third stage only. The first two stages will be briefly discussed.

The Clustering Stage computes a dissimilarity measure (an adaptation of the Sokal and Michener similarity measure (Choi et al., 2010)) for fixture configurations required for n number parts to be processed (set P). A comparative matrix is formed from these values. The measure is non-Euclidean, so multi-dimensional scaling is used to scale the comparative distances to a two-dimensional plane, where k-means clustering is used to cluster the parts to m number of fixtures (set Q). A fail-safe heuristic

is included to ensure that cluster sizes do not force infeasible solutions. These clusters form the ordered set I.

The Intracluster Sequencing Stage uses hierarchical clustering with single linkage to determine the optimal order of the elements within each of these clusters. Treating the dissimilarity measure as a distance, this order ensures that the shortest distance is traversed for each cluster. This should ensure that total reconfiguration time for each fixture base is minimised. The output of this stage is  $j \in i$  for each unordered set  $i \in I$ .

The Final Sequencing Stage can be isolated by artificially creating the outputs of the first two stages. This has no influence on demonstrating the effectiveness of the third stage. This stage only requires the input of the elements that comprise of the sets  $i \in I$ , i.e. *n* number of parts in *m* number of fixtures, distributed feasibly (with 0 representing empty slots when *n* is not a multiple of *m*).

#### 3.3 The Model

The problem is modelled as a mixed integer linear programming (MILP) problem and solved with a branch and bound algorithm. The notation for the entire problem is presented below.

## 3.3.1 Notation

$p; p \in P, P = \{1,, n\}$	<i>P</i> is the set of parts to be processed; <i>p</i> is an index of the ordered set <i>P</i> .
$q; q \in Q, Q = \{1,, m\}$	Q is the set of fixtures available; $q$ is an index of the ordered set $Q$ .
i; i€I, I={1,,m}	<i>I</i> is the set of <i>i</i> , i.e. a set of sets that holds all $p$ - $q$ mappings between sets <i>P</i> and <i>Q</i> ; <i>i</i> is an index of the ordered set <i>I</i> .
î; î <i>cI, I={1,,m}</i>	$\hat{i}$ is an alternate index of the ordered set <i>I</i> .
j; j€i, i={1,, i }	<i>i</i> is the unordered set of $p$ - $q$ mappings corresponding specifically to fixture $q$ , $j$ is an index of the set <i>i</i> ; $j$ denotes a part $p$ that is mapped to the fixture $q$ .
$\hat{j}; \hat{j} \in i, i = \{1,,  i \}$	$\hat{j}$ is an alternate index of the unordered set <i>i</i> .
k; keK, K={1,,n+1}	K is the set of time periods in which parts or fixtures are processed or reconfigured, respectively; $k$ is an index of the ordered set $K$ .
$\check{k}; \check{k} \in K, K = \{1,, n+1\}$	$\check{k}$ is an alternate index of the ordered set <i>K</i> .
$T_{ij}$	Part processing time; time for part $p$ corresponding to fixture-part mapping $j \in i$ to be processed; $T_{ij}$ is a parameter.
$R_{ij}$	Fixture reconfiguration time; time

for fixture  $\hat{i}$  to be reconfigured to fixture configuration corresponding to  $\hat{j}\in\hat{i}$  from fixture configuration corresponding to  $(\hat{j}-1)\in\hat{i}$ (implicitly), i.e. subsequent reconfiguration for fixture  $\hat{i}$ ;  $R_{ij}$  is a parameter.

A binary decision variable;  $X_{ijk} = 1$ if fixture *i* is reconfigured for the fixture-part mapping  $j \in i$  in time period *k*,  $X_{ijk} = 0$  otherwise.

A decision variable;  $\omega_{iijjkk} = 1$  if fixture-part mapping  $j \in i$  that was reconfigured in time period k is processed in time period k=k+1whilst fixture-part mapping  $j \in i$  is synchronously being reconfigured in time period k,  $\omega_{iijjkk} = 0$ otherwise.

A decision variable;  $\varphi_{iijjkk}$  is the absolute time difference between part processing time  $T_{ij}$  for fixturepart mapping  $j \in i$  reconfigured in time period k, and fixture reconfiguration time  $R_{ij}$  for fixturepart mapping  $j \in i$  reconfigured in time period k=k+1, i.e. the idle time for every time period where two operations are synchronous.

## 3.3.2 Assumptions

 $X_{ijk}$ 

 $\omega_{i\hat{i}\hat{i}\hat{k}\hat{k}}$ 

 $\varphi_{i\hat{j}\hat{j}k\check{k}}$ 

The assumptions used to describe and simplify the production system for this model are as follows:

- Fixture reconfiguration times are known.
- There are fewer fixture bases than parts; |Q| < |P|.
- The required number of fixtures are already manufactured and stored, so that only reconfigurations are now necessary.
- Transportation time between fixture manufacturing cell and part processing cell is negligible.
- Once a part or fixture is assigned to a period *k*, it is processed or reconfigured, respectively, without interruption.
- Flow is synchronised between Cell 1 and Cell 2; a job does not exit Cell 1 until Cell 2 is available, Cell 1 does not start a new job until the previous job has exited the cell.
- Cell 1 and Cell 2 have a just-in-time workflow policy (i.e. unit workflow).
- The fixture reconfigured in Cell 1 in time period k is used to process the part assigned to it in Cell 2 in the next time period k = k+1.

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Time Period	Time Period Index		Cell 1	Cell 2	
(Absolute)	k	Ř	Fixture Reconfiguration $(R_{\hat{i}})$	Part Process with Fixture (T <sub>ij</sub> )	
1	-	-	X <sub>111</sub>		
2	1	2	X <sub>212</sub>	X <sub>111</sub>	$\phi_{iijjkk} = \phi_{121112}$
3	2	3	X <sub>123</sub>	X <sub>212</sub>	φ <sub>iĭjjkk</sub> = φ <sub>211223</sub>
4	3	4	X <sub>224</sub>	X <sub>123</sub>	φ <sub>iĭjjkk</sub> = φ <sub>12<b>2</b>2<b>3</b>4</sub>
5	-	-		X <sub>224</sub>	

Figure 3: Example of how the final  $\varphi_{iijjkk}$  decision variables are generated, based on the flow and synchronisation of fixture reconfiguration operation and part processing operation in either cell.

#### 3.3.3 Mathematical Model

$$\operatorname{Min} \sum_{i}^{m} \sum_{i\neq i}^{m} \sum_{j}^{|i|} \sum_{j}^{|i|} \sum_{j}^{n-1} \sum_{k}^{n-1} \varphi_{iijkk}$$
$$(\forall \check{k} = k+1)$$

$$\varphi_{i\hat{\imath}j\hat{\imath}k\check{k}} = |(T_{ij} - R_{\hat{\imath}}) * \omega_{i\hat{\imath}j\hat{\imath}k\check{k}}| \begin{pmatrix} \forall i, \forall \hat{\imath} \neq i, \forall j \in i, \forall \hat{\imath} \in \hat{\imath}, \forall k < n, \\ \forall \check{k} = k + 1 \end{pmatrix}$$
(1)

$$-\varphi_{i\hat{\imath}j\hat{\jmath}k\check{k}} + (T_{ij} - R_{\hat{\imath}j}) * \omega_{i\hat{\imath}j\hat{\jmath}k\check{k}} \le 0$$
(1a)

$$-\varphi_{\hat{\imath}\hat{\imath}\hat{j}\hat{j}k\check{k}} - (T_{ij} - R_{\hat{\imath}\hat{j}}) * \omega_{\hat{\imath}\hat{\imath}\hat{j}\hat{j}k\check{k}} \le 0$$
(1b)

$$\omega_{iijjk\bar{k}} = X_{ijk} * X_{ij\bar{k}}$$

$$\begin{pmatrix} \forall i, \forall \hat{i} \neq i, \forall j \in i, \forall \hat{j} \in \hat{i}, \forall k < n, \\ \forall \bar{k} = k + 1 \end{pmatrix}$$
(2)

$$-\omega_{i\hat{\imath}\hat{\jmath}\hat{j}k\check{k}} + X_{ijk} + X_{\hat{\imath}\hat{j}\check{k}} \le 1$$
 (2a)

$$\omega_{i\hat{i}j\hat{j}k\check{k}} - X_{ijk} \le 0 \tag{2b}$$

$$\omega_{i\hat{\imath}j\hat{j}k\check{k}} - X_{\hat{\imath}\tilde{j}\check{k}} \le 0 \tag{2c}$$

$$\sum_{i}^{m} \sum_{\hat{i} \neq i}^{m} \sum_{j}^{|i|} \sum_{\hat{j}}^{|\hat{i}|} \sum_{\hat{j}}^{\hat{i}} \sum_{k}^{n} \omega_{i\hat{i}j\hat{j}k\check{k}} = n - 1$$

$$(\forall \check{k} = k + 1)$$

$$(3)$$

$$X_{ijk} + X_{ij\tilde{k}} \le 1$$

$$(\forall i; \forall j \in i; \forall \hat{j} \in i < j \in i; \forall k; \forall \check{k}$$

$$> k)$$

$$(4)$$

$$\sum_{i}^{m} \sum_{j}^{|l|} X_{ijk} = 1$$

$$(\forall k)$$
(5)

$$\sum_{k}^{n} X_{ijk} = 1$$

$$(\forall i, j \in i)$$
(6)

$$X_{ijk} \in \{0,1\}$$

$$(\forall i, \forall j \in i, \forall k < n)$$
(7)

$$\varphi_{i\hat{i}\hat{j}\hat{j}k\hat{k}} \ge 0$$

$$\begin{pmatrix} \forall i, \forall \hat{i} \neq i, \forall j \in i, \forall \hat{j} \in \hat{i}, \forall k < n, \\ \forall \tilde{k} = k + 1 \end{pmatrix}$$
(8)

### 3.3.4 Model Description

The objective function aims to optimally match the part processing time for fixture-part mapping  $j \in i$  and fixture reconfiguration time for another fixture-part mapping  $\hat{j} \in i$  such that the difference between them is minimised for time period  $\tilde{k}=k+1$  (which determines the fixture-part mapping  $j \in i$  to be scheduled for fixture reconfiguration in time period k). This minimises the idle time for either cell for every time period k.

Constraint (1) calculates the absolute difference between the part processing time related to fixture-part mapping  $j \in i$  in Cell 2 and the fixture reconfiguration time related to fixture-part mapping  $j \in i$  in Cell 1 for time period k = k+1 for every  $\omega_{iijjkk}$ . As this constraint is non-linear, Constraints (1a) and (1b) are used instead of (1) to linearise the absolute value.

Constraint (2) ensures that the idle times calculated in Constraints (1a) and (1b) are valid. This is determined by ensuring that the binary decision variables related to fixture-part mappings  $j \in i$  and  $j \in i$  for time periods k and k = k+1,

respectively (i.e.  $X_{ijk}$  and  $X_{ijk}$ ), must both be active (equal to 1) for  $\omega_{iijjkk} > 0$ . As this constraint is quadratic, Constraints (2a) to (2c) are used instead of (2) to linearise the non-linearity of (2).

Constraint (3) ensures that the number of  $\omega_{iijjkk} > 0$  corresponds to the number of time periods in which Cell 1 and Cell 2 perform operations synchronously, i.e. one less than the total number of jobs (*n*-1) since the first time period hosts an operation in Cell 1 only (the first fixture reconfiguration).

Constraint (4) imposes the intracluster order on the final sequence by ensuring that two fixture-part mappings for the same fixture ( $j \in i$  and  $\hat{j} \in i$ ) must appear in time periods relative to each other that correspond to the intracluster order ( $\check{k} > k$ ).

Constraint (5) ensures that there is only one fixture-part mapping  $j \in i$  assigned to each time period k. Constraint (6) ensures that each fixture-part mapping  $j \in i$  is assigned to a time period k only once in the schedule.

Constraint (7) is a bound stating that  $X_{ijk}$  is a binary variable. Constraints (8) and (9) are bounds restricting  $\varphi_{iijjkk}$  and  $\omega_{iijjkk}$ , respectively, to be non-negative. This ensures that the linearising constraints for these decisions variables perform their desired function.

These constraints and bounds limit the problem search space to remain within the behavioural boundaries associated with the production system described in Section 3.1 and the assumptions presented in Section 3.3.2.

Figure 3 shows an example to demonstrate how the binary decision variable associated with a given fixture-part mapping takes on the form of both  $X_{ijk}$ when in Cell 2 and  $X_{ijk}$  when in Cell 1. Please note that the time period index in this figure only describes the time period value assigned to the binary decision variable for that absolute time period - based on the indices of the binary decision variable  $(ijk \text{ or } \hat{i}jk)$  for either cell. This is because for fixture-part mapping  $l \in l$  to be assigned to time period *I*, the operation time in Cell 2 ( $T_{ij}$ ) has to be considered alongside that for  $l \in 2(R_{ij})$  when both are synchronously operated on in time period 2. This process produces the final decision variable  $\varphi_{iijjkk}$ , from which the workflow can be easily interpreted from the indices, as shown in Figure 3.

#### 3.4 Results

The model was solved using the MILP solver integrated into MATLAB® 2016a. The solver used a branch and bound algorithm to solve the problems presented to it.

Problems with a fixture range of 2-4 fixtures and a part range of 4-16 parts were formulated. The operation time values were randomised integers within a range of 15-45 seconds for fixture reconfiguration operations ( $R_{ij}$ ) and 30-90 seconds for part processing operations ( $T_{ij}$ ).

These problems and their solution results are presented in Table 1. The problems were solved to optimality. This is shown by the graphs of convergence for the branch and bound algorithm (Figure 4 to Figure 6 for a selection of the problems presented in Table 1).

The test was executed on an Intel® Xeon® CPU E3-1270 v3 at 3.50 GHz with 16 GB RAM on a 64-bit operating system.

Table 1: Sample problems and results.

Number of Fixtures	Number of Parts	Number of Variables	Variable Creation Time (s)	Solution Time (s)	Convergence
2	4	64	0.044	0.719	Yes
2	6	216	0.111	0.611	Yes
2	8	512	0.424	1.027	Yes
2	10	1000	1.474	5.551	Yes
2	12	1728	4.305	49.12	Yes
3	6	276	0.142	0.752	Yes
3	9	945	1.156	8.583	Yes
3	12	2256	6.510	842.6	Yes
4	8	736	0.762	4.243	Yes
4	12	2520	7.407	6394	Yes

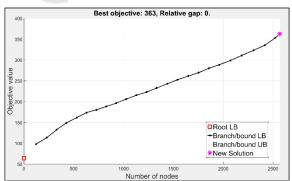


Figure 4: Convergence of 2 fixture/12 part problem.

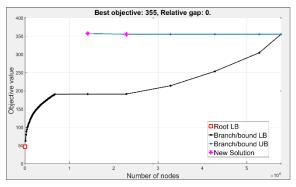


Figure 5: Convergence of 3 fixture/12 part problem.

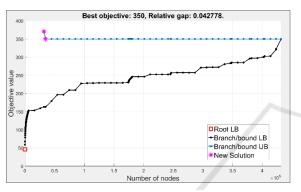


Figure 6: Convergence of 4 fixture/12 part problem.

The sample problem size was not large. This was due to limitations that resulted from both the variable size increase and the solution time increase for larger problems.

The results show that the variable size increases by a decreasing factor for the linearly increasing number of parts on a constant number of fixtures (logistical growth). This growth in variable size resulted in an exponential growth in solution times. A similar observation of logistical growth was made for the variable sizes that increased due to a linearly increasing number of fixtures for a constant number of parts. However, the solution times for this case appear to increase logistically as well; as opposed to the previous case, where solution times increased exponentially.

The sharp growth in solution times from the solver meant that limited fixture/part combinations could be tested within a reasonable timeframe. As the problem is NP-hard, it is expected that finding exact solutions via the branch and bound algorithm would be computationally expensive. The problem is exasperated by the MATLAB® 2016a branch and bound solver's inability to utilise parallel processing for this application, despite the multicore processor of the machine used.

Optional parameters on the solver were adjusted to yield solutions in minimum time. These included the branch rule used (most fractional), node selection criterion (minimum objective) and algorithm used (primal-simplex), amongst others. The tolerance parameters were also adjusted to cater for the integer values used in the dataset.

The results from this sample problem set confirm that the MILP model does create a schedule that minimises the total idle time in the system. The solver reached convergence for the sample set and it was confirmed (by inspection) that the resultant schedules from this algorithm were those of minimum idle time.

## 4 CONCLUSIONS

This paper presented a three-stage procedure for the optimal and combined scheduling of a synchronised fixture and part manufacturing cell. The paper focused on the third stage of the procedure where a mixed integer linear programming (MILP) model was used to optimally schedule the production system The results demonstrated that the model minimises the total idle time in the system, thus saving on operating costs and tardiness penalties in practice.

This is useful for mass customisation production systems, where the use of reconfigurable fixtures in the manufacturing process cannot be optimised with conventional approaches.

Despite the logistical and exponential increases in solution time (depending on which variable is held constant – fixtures or parts), the MILP model is valid for the production system described for a problem of any reasonable size.

The MILP model is limited by the assumptions listed in Section 3.3.2. Most of these are somewhat redundant, as production systems would exhibit such behaviour in most practical cases anyway. The unit workflow requirement is a limiting factor, but this could be edited to represent batch workflow quite easily. The requirement that fixtures are already made and waiting, is another limiting factor that is not as readily solved.

Further work on this research topic involves creating a heuristic to cope with larger-sized problems more efficiently – producing sub-optimal but good solutions with smaller variable sets and reduced solution times. Other factors, such as the influence of manufacturing new fixtures and maintaining an optimal fixture inventory, can be addressed in future research endeavours.

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